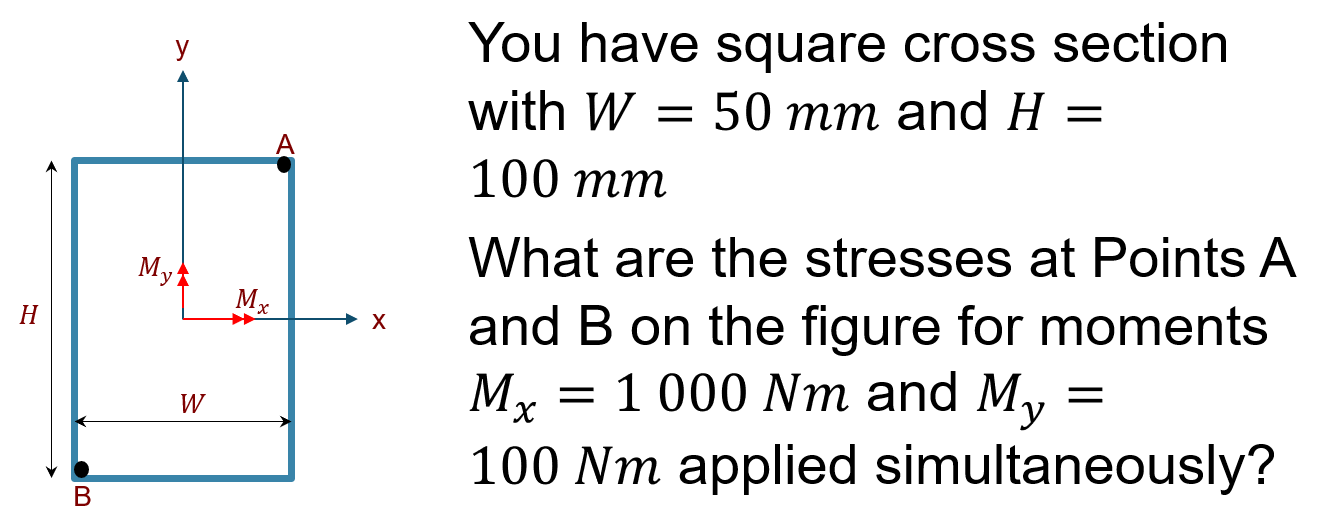
# INTRODUCTION

This document will be updated with the notes made during class. The document will be made available in MS WORD format to allow students access to the Excel spreadsheets that were included during class. Please note that this document was compiled during class and was not proof read for typing, language, and calculation errors. Please inform the lecturer should you detect any serious errors.

# BENDING STRESS ON SQUARE CROSS-SECTION

## Problem statement



## Solution

The second moments of area for the cross section are:

W=0.050;H=0.100;Ixx=1/12\*H\*W^3,Iyy=1/12\*W\*H^3,

Ixx = 1.0417e-06

Iyy = 4.1667e-06

The bending stress at any location in the x-y plane is:

sigb=Mx\*xy(:,2)/Ixx-My\*xy(:,1)/Iyy

sigb = 1.0e+07 \*[0.9600 -0.9600 1.200**]**

# BENDING MOMENT DISTRIBUTION

## Principle

Steps to calculate the bending moment at any position :

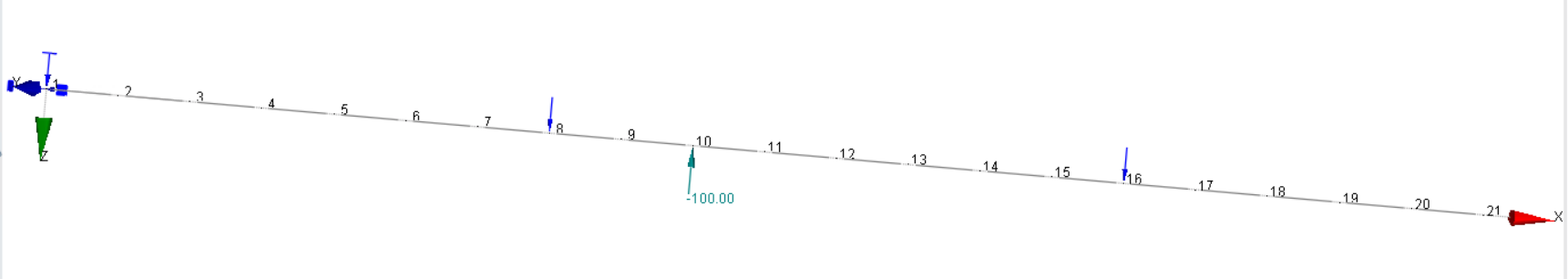
1. At position , formulate the bending moments by just looking towards the origin of the coordinate system.
   1. For :
   2. For :

z

## Example

Consider the beam loaded in the -z direction with force 100 kN and constrained at Points A, B and C as shown in the figure below. The bending moment distribution is also shown below. From the bending moment distribution, if possible, the best point for splicing is where the bending moment is zero.

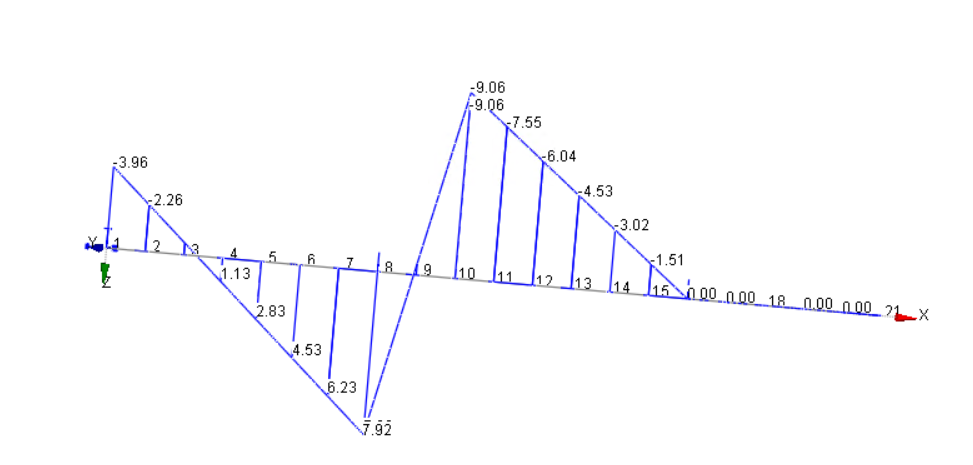
Point A



Point C

Point B

Figure 2: Bending moment diagram



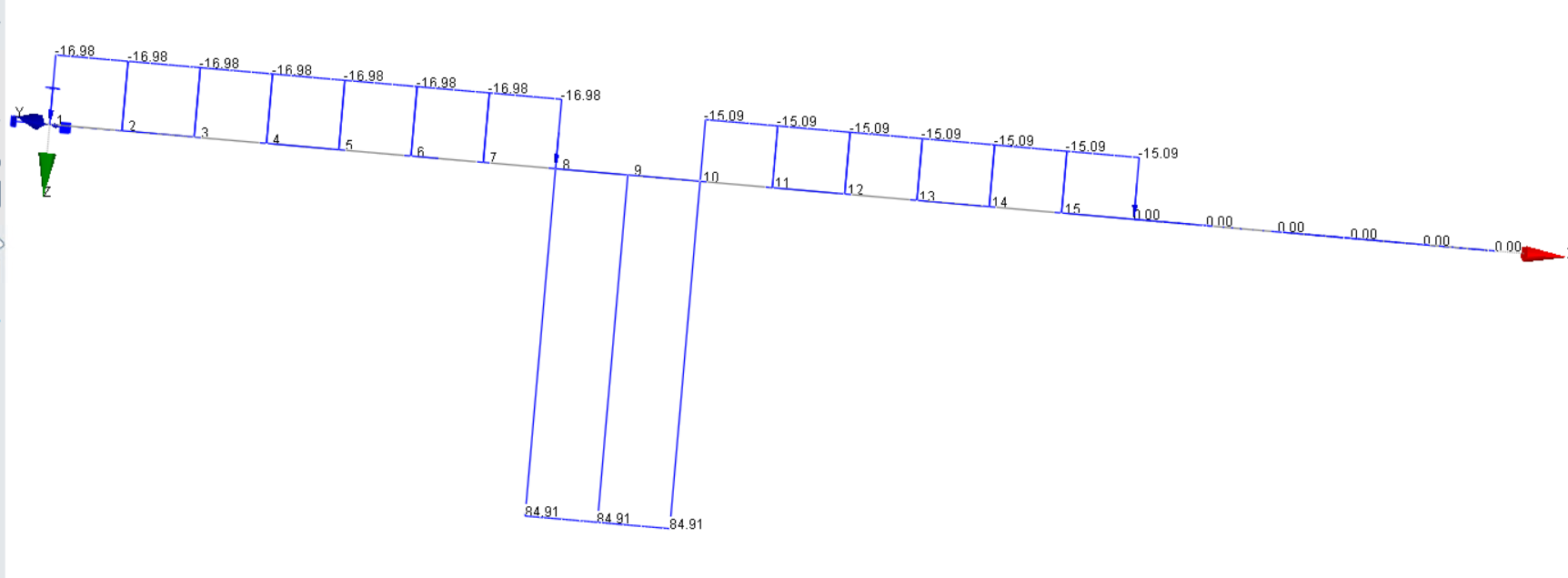
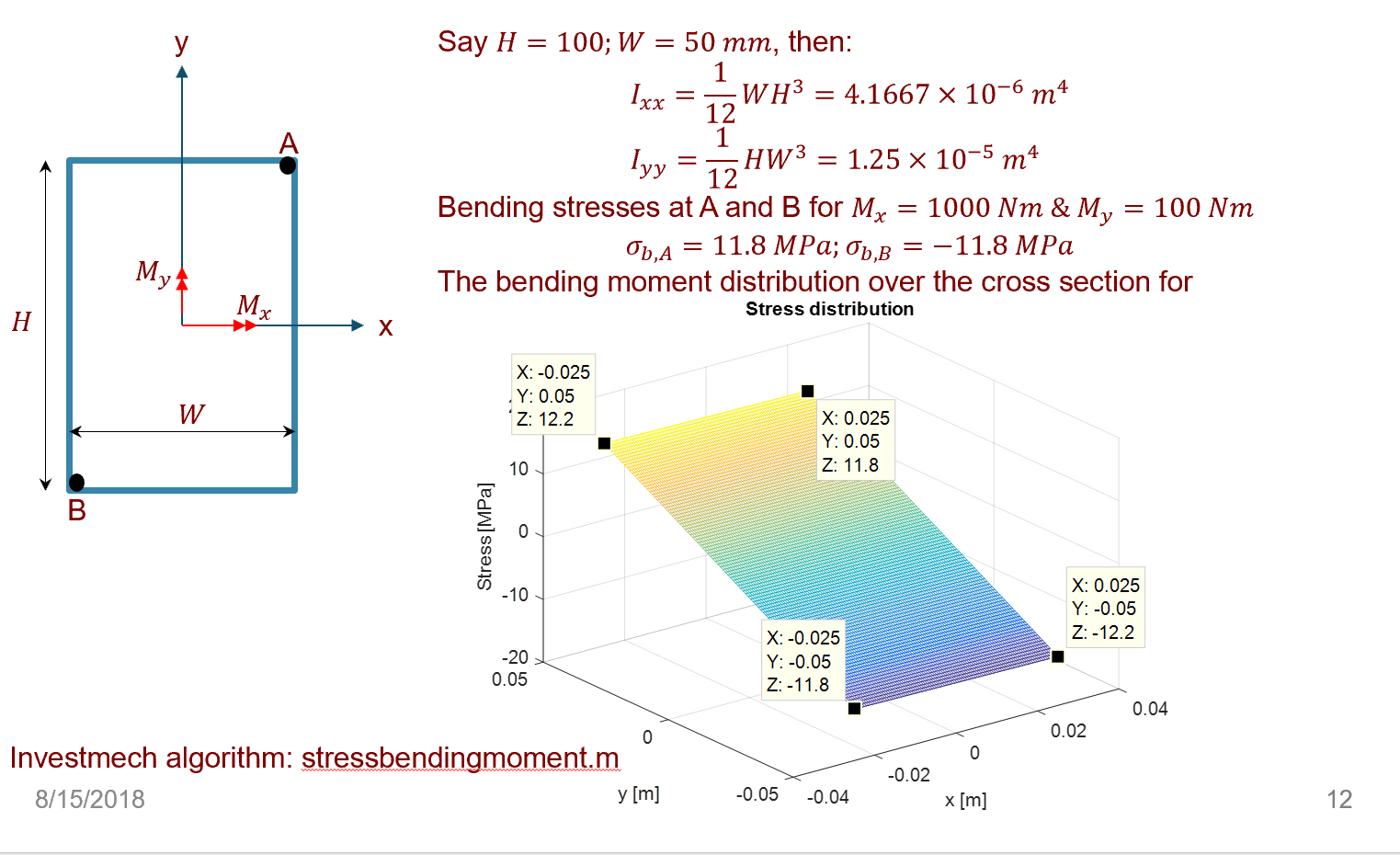


Figure 3: Shear force diagram

# BENDING STRESS DISTRIBUTION



The bending stress distribution is given by:

The second moment of area for this problem is:

For Point A:



If a tensile force is also applied, the stress at any point is given by:

Say a force of 1 kN is applied, what is the stress at point A?



# STRESS LIFE PROBLEM

## Problem statement

300WA structural steel has the following material properties:

*E* = 206 GPa, *fy* = 300MPa,

*fut* =450MPa

Assume a notch fatigue factor of

What is the endurance limit for ?

How many cycles to failure at:

1. *Sa*= 200MPa
2. *Sa*= 300MPa

## Solution

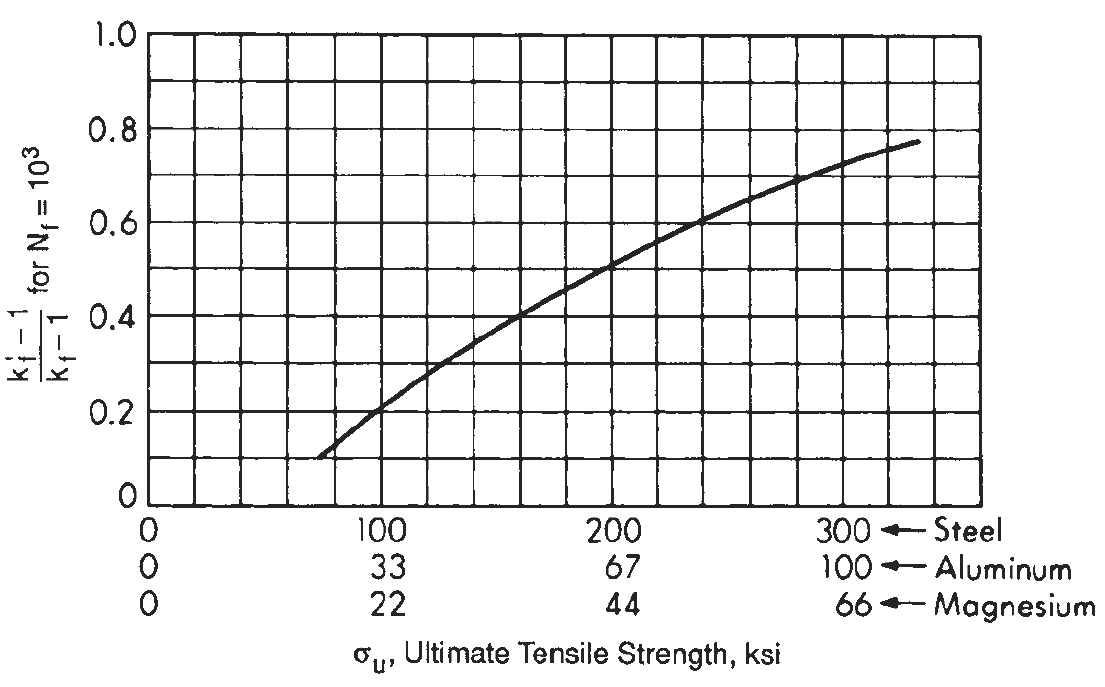
Step 1: Determine the unnotched S-N curve

This is steel, and we know that the endurance ratio is given as follows:

For the stress at 1000 cycles, we have:

Step 2: Fatigue notch factors

The fatigue notch factor at endurance was given as . The figure below shows that the fatigue notch factor at 1 000 cycles is:



Step 3: Modification factors and S-N curve

The fatigue strengths at 1 000 and 1 000 000 cycles are as follows:

**Size:**

Nothing was mentioned,

**Load:**

, because it is assumed that the part will be subject to reversed bending.

**Surface:**

Nothing is said, and

**Temperature:**

Nothing is said, and

**Reliability:**

Nothing is said, and it is assumed that calculations is to be performed for 50% probability of survival.

**S-N curve:**



# S-N CURVE STRESS LIFE EXAMPLE – same as previous problem with different S-N curve & notch fatigue factor

## Problem statement

300WA structural steel has the following material properties:

*E* = 206 GPa, *fy* = 300MPa,

*fut* =450MPa

Assume a notch fatigue factor of

What is the endurance limit ?

How many cycles to failure at

1. = 200MPa
2. = 300MPa

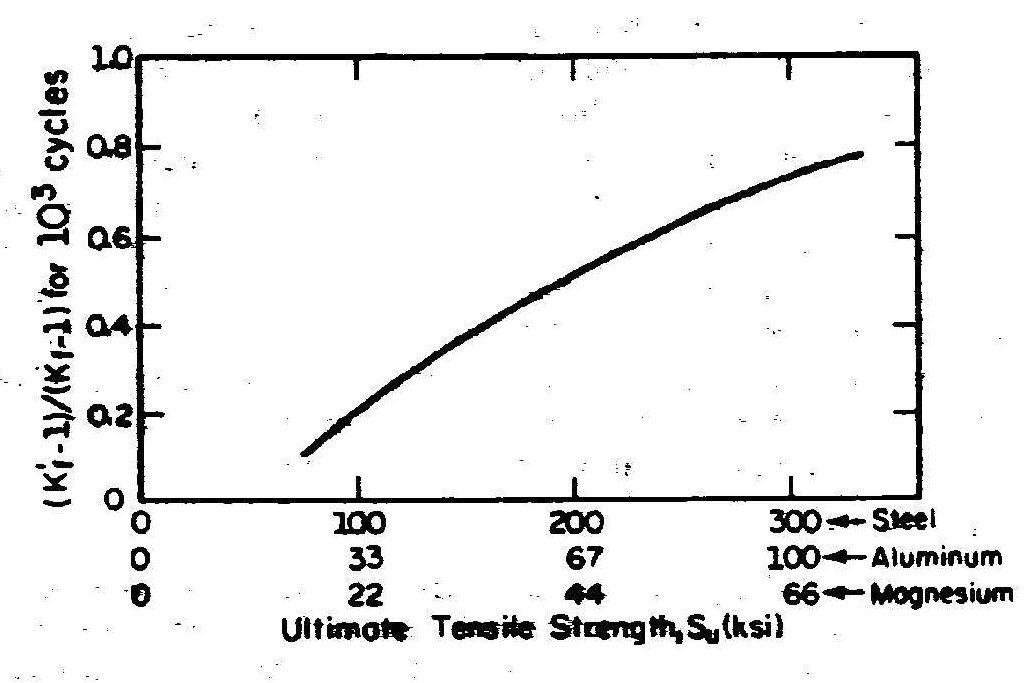
## Solution

Step 1: Define the equation for the S-N curve

In this case the yield strength is below 700 MPa, and we will assume that the S-N curve need to be modified for a notch fatigue factor at 1 000 cycles. For the general unmodified, unnotched material, the S-N curve has the following two points:

However, we need to modify the S-N curve for the notch by the following factors:

The notch fatigue factor at 1 000 cycles, is determined as as shown below.



For , , therefore,

Figure 4: Calculation of notch fatigue factor at 1 000 cycles

The limits on the S-N curve, modified for the notch, is:

The equation for the S-N curve is now:



Figure 5: S-N curve for the notched specimen

The exponent (slope), is:

Step 2: Calculate endurance at requested stress amplitudes

1. For (larger than ), the endurance is:
2. For (larger than ), the endurance is:

Step 3: General remarks

If all the stress amplitudes where endurance was required are below the endurance limit, you do not need to model the S-N curve, because, the answer is infinite life.

# S-N CURVE EXAMPLE

## Problem statement

Construct the S-N curve for a material with completely reversed stress amplitude at cycles and endurance limit at cycles. Then, calculate the endurance for the following completely reversed stress amplitudes: 300 MPa, 200 MPa, and, 100 MPa.

## Solution

The equation for the S-N curve is assumed to be: .

Therefore:

Say I need to calculate the endurance (fatigue life) at any stress amplitude, the equation becomes:

The other option is to find the equation for the S-N curve in the form . For this, we use the points provided:

In this case, the endurance at any stress is then:

From which it is clear that



# RAINFLOW COUNTING

The explanation that was done in class.

Table 1: Rainflow counted stress spectrum over a period of 1 year

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 45 | 35 | 90 | 0.5+0.5=1 |
| 90 | 10 | 180 | 0.5+0.5=1 |
| 25 | 25 | 50 | 0.5+0.5=1 |
| 35 | 25 | 70 | 0.5+0.5=1 |



(100;-80;180)

(80;-10;90)

(60;-10;70)

(50;0;50)

(-10;60;70)

(0;50;50)

(-80;100;180)

(-10;80;90)

Figure 1: Stress signal over a period of 1 year

# MEAN STRESS CORRECTION

## Problem Statement

Check sensitivity for the completely reversed bending stress amplitude, , for the mean stress of a signal using the following mean stress correction relationships:

* Modified Goodman
* Gerber
* Smith, Watson & Topper (WST)
* Walker

Assume a material ultimate tensile strength and consider the following stress states:

|  |  |
| --- | --- |
| **Stress amplitude**  **[MPa]** | **Stress mean**  **[MPa]** |
| 100 | -100 |
| 100 | 0 |
| 100 | 100 |

## Solution

From the notes, the relevant equations are:

|  |  |
| --- | --- |
| **Approach** | **Equations** |
| Modified Goodman |  |
| Gerber |  |
| SWT |  |
| Walker |  |

Programmed in Excel, the results are as shown in the table below. Can you explain and make recommendations? See your text book.



# STRESS LIFE EXAMPLE WITH MEAN STRESS CORRECTION

## Problem statement

Component undergoes cyclic stress with:

σmax = 770MPa

σmin = 70MPa

Material is steel with = 1,050MPa and = 420MPa. The fully reversed stress at = 770 MPa.

How many cycles can be loaded on the component until fatigue crack initiation? That is, what is the fatigue life of the component?

## Solution

### Calculate stresses and compare with endurance limit

The stress amplitude is:

The mean stress is:

Do mean stress correction. Use Goodman:

The Goodman mean stress corrected equivalent completely reversed stress amplitude is , therefore, we have finite life and the S-N curve must be calculated.

### S-N curve

The S-N curve is given as:

For which:

### Calculate endurance at specified stress amplitude

is larger than the endurance limit . For this completely reversed stress amplitude, the endurance is the:

# STRAIN-LIFE EXAMPLE WITH STRAIN INPUTS

## Problem statement

The cyclic stress-strain and strain-life parameters for a steel is:

E=30 x 103 ksi; K’ = 154 ksi, σ’f = 133ksi, ε’f = 0.26, n’ = 0.202, b = -0.095,

c=-0.47

Determine the life for the histories shown on the next slide (use Morrow).

## Solution

The equations needed:

Ramberg-Osgood:

Strain-life equation:

See the attached Excel spreadsheet.

# DAY 2: YOUR FIRST DAMAGE CALCULATION

## Problem Statement

A component undergoes axial cyclic loading as summarized in the table below, which was obtained from Rainflow cycle counting.

Table 2: Stress spectrum for one repetition



Material is steel with Sut = 1,050 MPa with hardness 350 BHN. The theoretical stress concentration factor at a notch on the part is . The notch radius is . The surface finish is machined. The shaft has a radius of 100 mm and operates at temperature .

How many blocks/repetitions of loading can be loaded on the component for a 1 % probability of fatigue crack initiation? That is, what is the fatigue life of the component for a probability of survival of 99%?

## Solution

The steps that I will follow:

1. Calculate notch fatigue factors:
2. Calculate the influencing factors and the fatigue strength at 1 000 and 1 000 000 cycles as shown below:
3. Mean stress correction to calculate the equivalent completely reversed stress amplitude. Use Goodman:
4. Calculate damage using the Palmgren-Miner rule
5. Calculate the number of repetitions:
6. Calculate life which is (not applicable in this case):

### Calculate S-N curve for the notched specimen

#### Modelling the notch effect

The theoretical stress concentration factor is .

The ultimate tensile strength: .

Notch root radius:



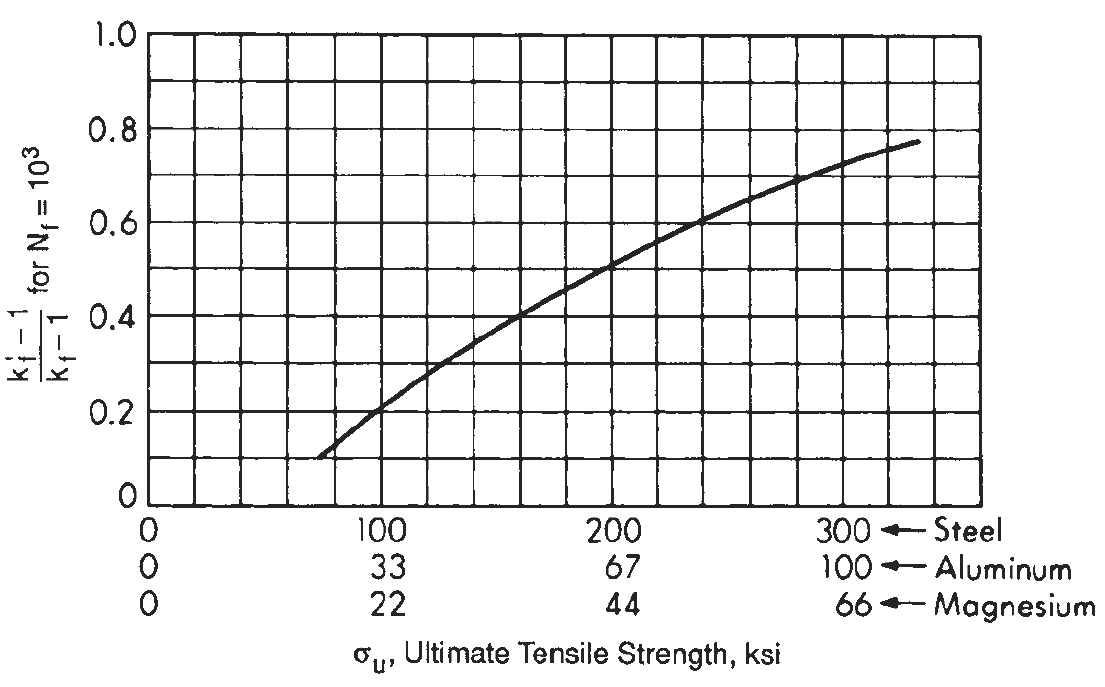
For this we have the follow equations:

At endurance limit at 1 million cycles:

* Approximations for :

Therefore:

Fatigue notch factor at 1 000 cycles:



From the figure above, we have:

### Modification factors

Size modification factor

The recommended modification factor for size is:

In this case, the component diameter is . The modification factor for size is then:

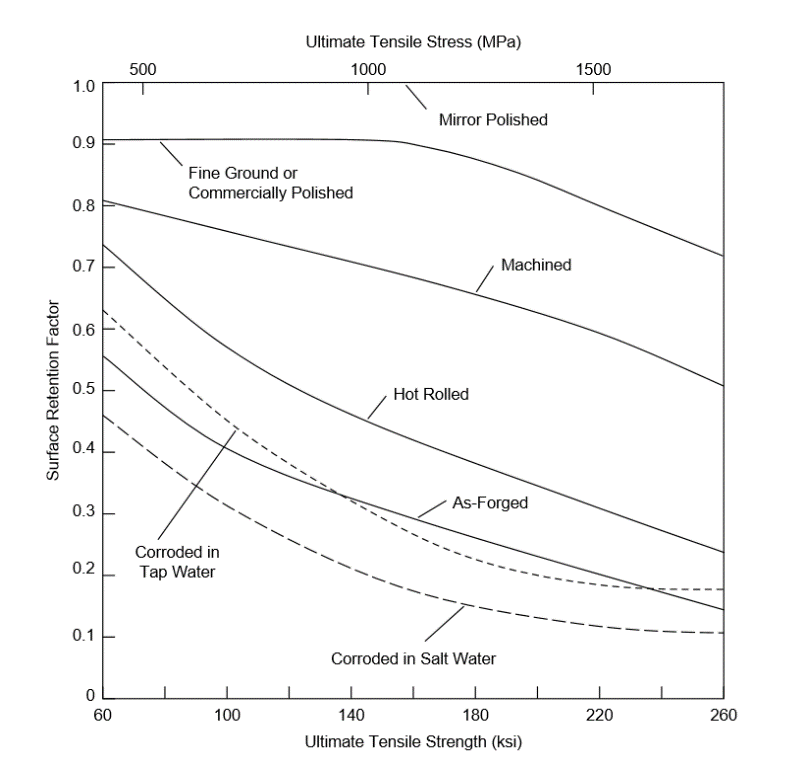
Load modification factor

In this problem, the specimen is subject to axial loading. The S-N curve were estimated using equations for rotating bending test. Therefore, a modification must be made for load in this case.

In this case,

Surface finish modification factor

From the figure below, .

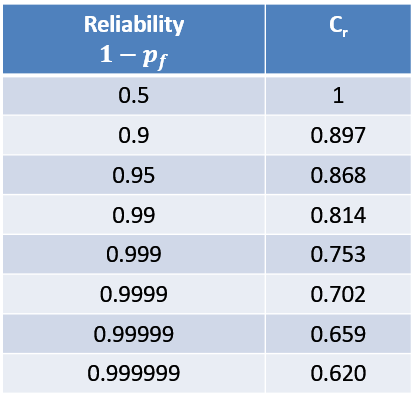


Temperature modification factor

From the equation below, ().

Reliability modification factor

From the table below, .



### Mean stress correction

|  |  |
| --- | --- |
| **Approach** | **Equations** |
| Modified Goodman |  |
| Gerber |  |
| Morrow |  |
| SWT |  |
| Walker |  |

### Equation for the S-N curve

In this case, the slope of the S-N curve is calculated from the applicable fatigue strengths at 1 000 and 1 000 000 cycles as follows:

The slope of the S-N curve:

### Damage for one repetition



Goodman mean stress correction:

Endurance/fatigue life:

Damage:

Goodman mean stress correction:

Endurance/fatigue life:

Damage:

Goodman mean stress correction:

Endurance/fatigue life:

Damage:

Goodman mean stress correction:

Endurance/fatigue life:

Damage:

#### Total damage per repetition

The total damage according to the Palmgren-Miner rule is:

The number of repetitions to failure is then 3.8:

### Check with Excel

For the supplied stress spectrum, the life is 12 Blocks for a 1% probability of failure (99% probability of survival).

Table 4: Stress spectrum for one block



### Conclusion

The fatigue life of the component for 99% probability of survival is 3.8 repetitions.

# Strain-life: Example 1

Problem statement

One repetition of the nominal stress on a Ti-6Al-4V alloy is shown in the table below. The elastic stress concentration factor (also called the theoretical stress concentration factor) is . Estimate the number of repetitions required to cause fatigue cracking at the notch. Use the Ramberg-Osgood stress-strain relationship and the Neuber equation for notch response.

* The stress history is shown in Figure 59.
* Which was then reordered to start and end with the first peak or valley with the maximum absolute value as shown in Figure 60.
* In this case, no extrema were calculated, and exact values are used. The stress history is short enough for this.



Figure 59: Original stress history - 1 repetition



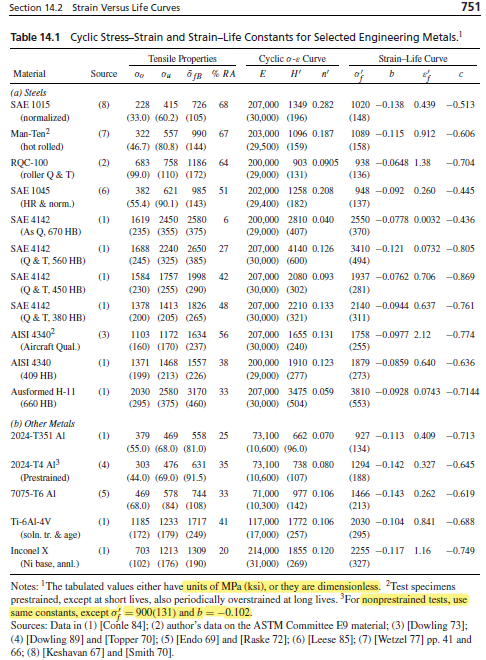
Figure 60: Reordered stress history - for one repetition



Figure 61: Stress history with stresses indicated

Solution

The material properties for Ti-6Al-4V is as follows from Dowling Table 14.1:



Evaluate the stress history

Rainflow counting was performed on the stress signal as shown below:

1. The first step was to draw the start and stops for each reversal.
   1. Note, that a few points, and one turning point could have been left out if peak-valley reduction (signal to extrema) was done before the Rainflow counting.
2. Then number the peaks and valleys of the relevant cycles. In this case the time axis will be used as was calculated in Matlab. Otherwise use A, B, C, …. as per the examples in Dowling.



The stress-strain response is then as follows:



### Stress-strain response at the notch

The purpose of this section is to calculate the stress and strain response at the notch, and to calculate the stress and strain at every point of interest. The points of interest are indicated in yellow in the figure below.



9

8

7

6

5

4

3

2

1

Reversal 1: From 0 to 1, monotonic stress-strain curve

For the first stress cycle, the nominal monotonic stress amplitude is . The notch stress response is solved in Matlab as:

The notch strain amplitude for the stress reversal is then:



1:[0.0338, 1191]

Reversal 2: 1 to 2, cyclic stress strain curve

The nominal monotonic stress amplitude is .

Direction: .

The notch stress response is solved in Matlab as:

The notch strain amplitude for the stress reversal is then:

Therefore, the stress and strain at the end of the reversal will be:



2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 3: Reversal 2 to 3, cyclic stress strain curve



3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 4: 3 to 4, cyclic stress strain curve



4:[0.000845, 61]

3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 5: 2 to 5, cyclic stress strain curve



5:[0.02355, 1 131]

4:[0.000845, 61]

3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 6: 5 to 6, cyclic stress strain curve



6:[-0.00927, -975]

5:[0.02355, 1 131]

4:[0.000845, 61]

3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 7: 6 to 7, cyclic stress strain curve



7:[0.00722, 723]

6:[-0.00927, -975]

5:[0.02355, 1 131]

4:[0.000845, 61]

3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 8: 7 to 8, cyclic stress strain curve



8:[0.00477, 437]

7:[0.00722, 723]

6:[-0.00927, -975]

5:[0.02355, 1 131]

4:[0.000845, 61]

3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

Reversal 9: 2 to 9, cyclic stress strain curve

9:[0.0338, 1 191]



8:[0.00477, 437]

7:[0.00722, 723]

6:[-0.00927, -975]

5:[0.02355, 1 131]

4:[0.000845, 61]

3:[-0.0090, -1099]

2:[-0.0318, -1181]

1:[0.0338, 1191]

### Matlab commands: notch stress and strain at every point

Filename: strain\_life\_classproblem\_2020\_04\_23.mlx

Class problem in strain-life

Date: 2020-04-23 done during class time.

E=117e3;

Ha=1772;na=0.106;sigfa=2030;b=-0.104;

epsfa=0.841;c=-0.688;

kt=2.6;

ferror=@(Sa,kt,E,sa,Ha,na) (kt\*Sa)^2./sa/E-(sa/E+(sa/Ha).^(1/na));

epsa=@(sa,E,Ha,na) sa/E+(sa./Ha).^(1/na);

% Cycle 1: 0 to 835 MPa, Monotonic

Sa=835;sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=1

sa1=sa

epsa1=epsa(sa,E,Ha,na)

s1=0+psi\*sa1

eps1=0+psi\*epsa1

% Cycle 2: 835 to -807

Sa=(835--807)/2;sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=-1

sa2=sa

epsa2=epsa(sa,E,Ha,na)

s2=s1+psi\*2\*sa2

eps2=eps1+psi\*2\*epsa2

% Cycle

Sa=(440--807)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=1

sa3=sa

epsa3=epsa(sa,E,Ha,na)

s3=s2+psi\*2\*sa3

eps3=eps2+psi\*2\*epsa3

Sa=(440-72)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=-1

sa4=sa

epsa4=epsa(sa,E,Ha,na)

s4=s3+psi\*2\*sa4

eps4=eps3+psi\*2\*epsa4

Sa=(682--807)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=1

sa5=sa

epsa5=epsa(sa,E,Ha,na)

s5=s2+psi\*2\*sa5

eps5=eps2+psi\*2\*epsa5

Sa=(681--412)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=-1

sa6=sa

epsa6=epsa(sa,E,Ha,na)

s6=s5+psi\*2\*sa6

eps6=eps5+psi\*2\*epsa6

Sa=(283--412)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=1

sa7=sa

epsa7=epsa(sa,E,Ha,na)

s7=s6+psi\*2\*sa7

eps7=eps6+psi\*2\*epsa7

Sa=(283-173)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=-1

sa8=sa

epsa8=epsa(sa,E,Ha,na)

s8=s7+psi\*2\*sa8

eps8=eps7+psi\*2\*epsa8

Sa=(835--807)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);

psi=1

sa9=sa

epsa9=epsa(sa,E,Ha,na)

s9=s2+psi\*2\*sa9

eps9=eps2+psi\*2\*epsa9

### Fatigue

From the figure below, the nominal stress cycles with number of cycles are as follows:

|  |  |  |
| --- | --- | --- |
| **From** | **To** | **Number of cycles** |
| 0 | 1 | 1 |
| 1 | 2 | 1 |
| 3 | 4 | 1 |
| 5 | 6 | 1 |
| 7 | 8 | 1 |



9

8

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6

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4

3

2

1

The steps to calculate the mean stress compensated fatigue life at the notch is as follows:

STEP 1: The zero-mean-stress-equivalent fatigue life is given by:

STEP 2: The mean-stress compensated fatigue life is:

This was programmed in the excel sheet on the next page.

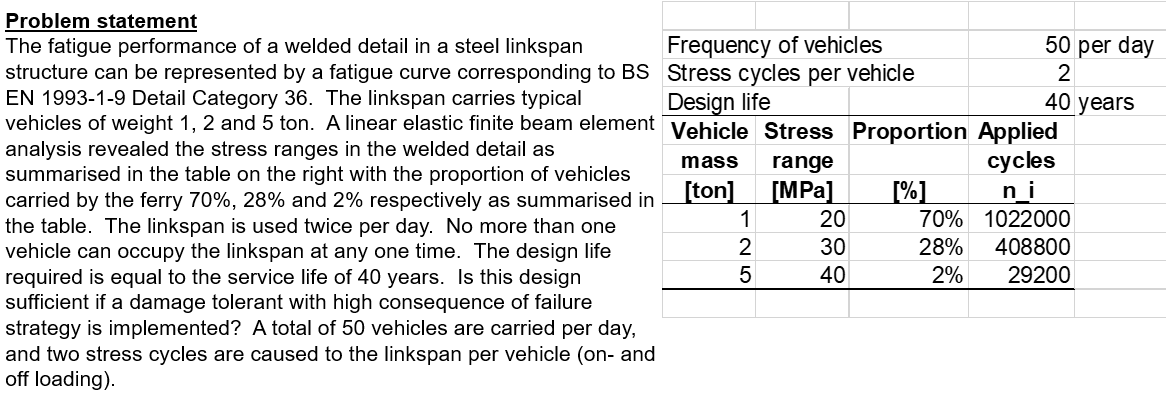
The filename of the Excel Workbook is: Strainlife.xls

The use of the solver was demonstrated in class, as well as the use of Matlab to find the damage.



# EXAMPLE 2 IN THE NOTES

## Problem statement



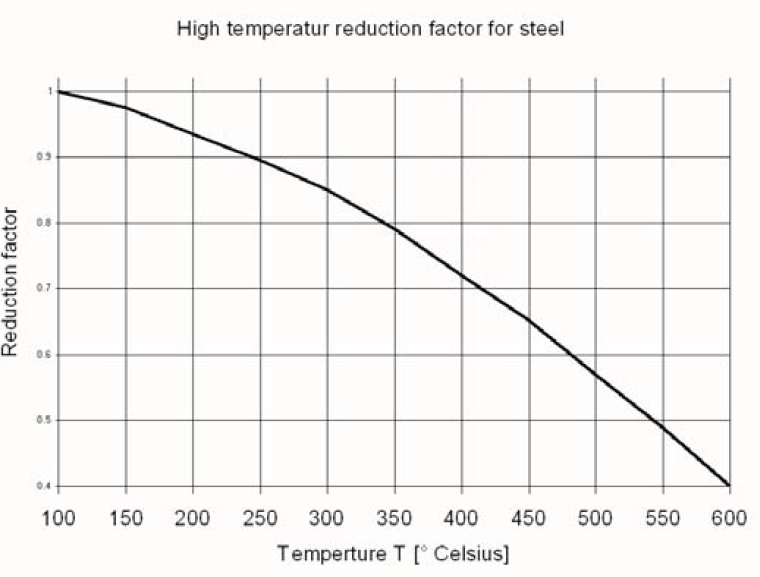
## Solution

### Step 1: Define the fatigue curve

The modified characteristic strength at 2 million cycles is given by:

The design requires damage tolerant design with low consequence of failure, that means .

For temperature, the modification factor at 300 °C is .



The modified characteristic strength is then:

The constant amplitude fatigue limit is:

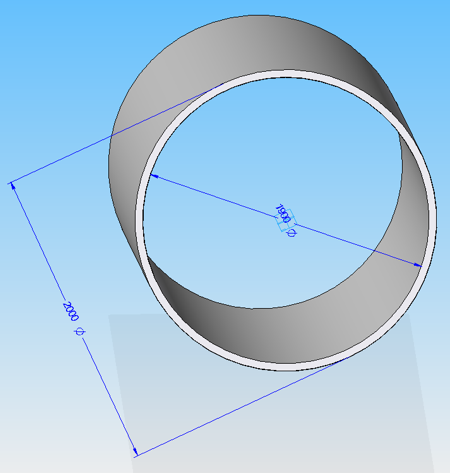
The cut-off limit is:

### Endurance calculation

There are stress ranges exceeding the constant amplitude fatigue limit, therefore, finite life and the Sr-N curve is given by:

**For**

# THIN WALL PRESSURE CYLINDER

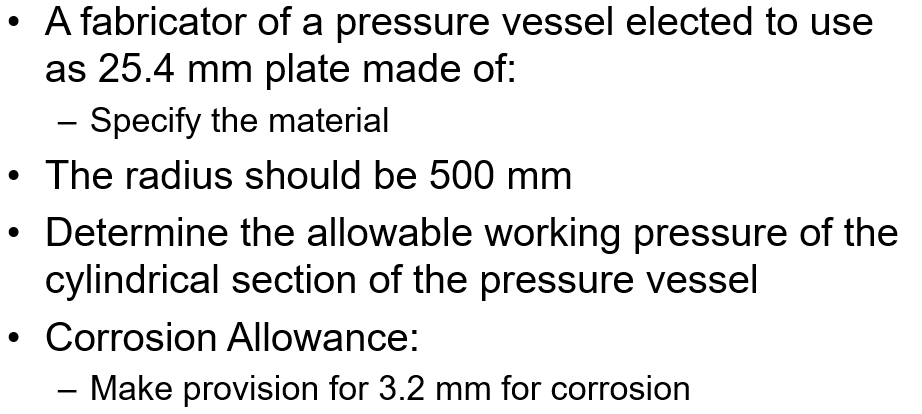


The force that holds the longitudinal loads:

Cross-section area of steel:   
Area of pressure force:   
Stress produced in steel:

# PRESSURE VESSEL DESIGN EXAMPLE

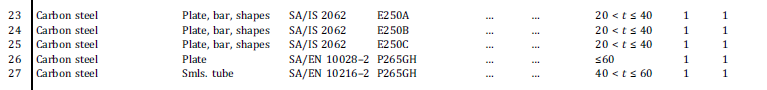
## Problem statement



## Solution

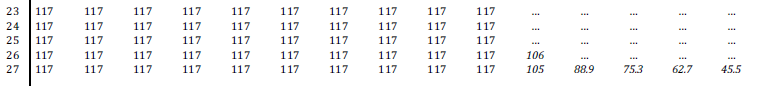
### Material selection

Choose plate steel to SA2062. Line 23 below.



The allowable stress at the operating temperature, assume room temperature, is: .





### Joint efficiency

Assume no radiography. Assume welds are all butt welds according to the description below, from which the joint efficiency is .

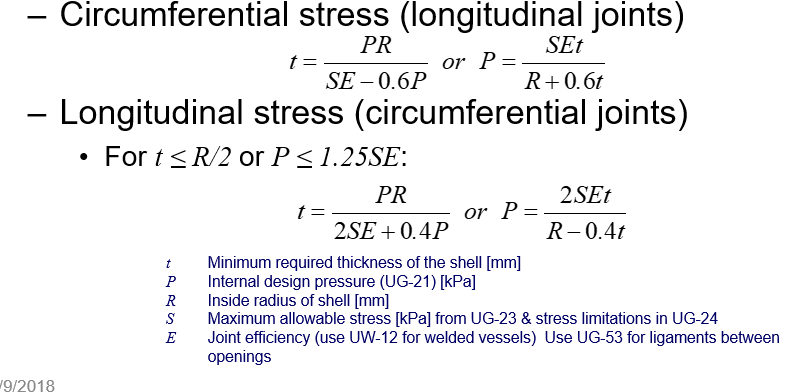
### 

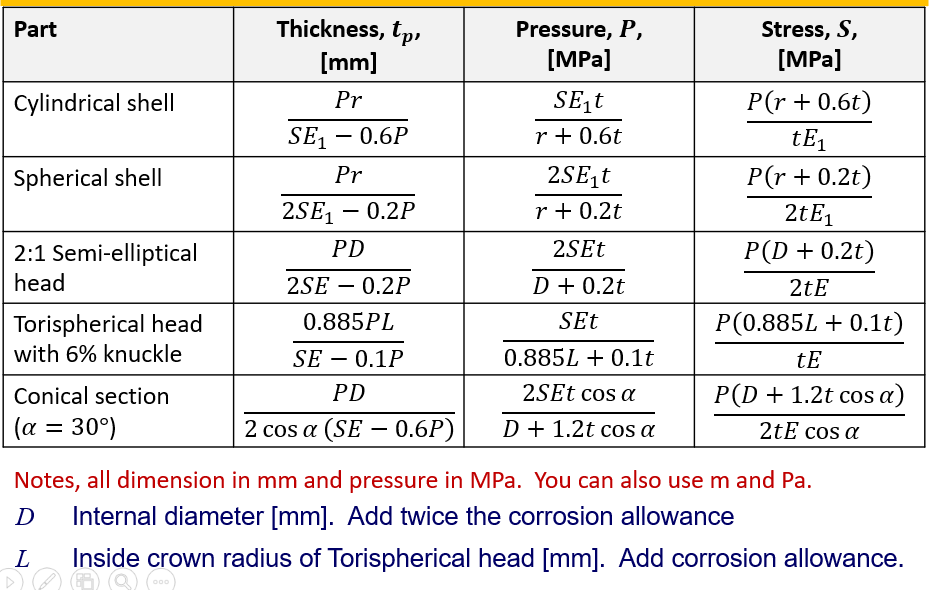
### Thickness to use in calculation of design pressure

The thickness of 25.4 mm plate, specified by the client, includes a corrision allowance of 3.2 mm. Therefore, the thicknes to use in the determining the design pressure shall be

### Maximum allowable operating pressure

We know that the longitudinal (Category A) welds are loaded to the highest stress. Or, in other words, the circumferential stress will govern the maximum allowable pressure.





The maximum allowable operating pressure:

# Acceptable fatigue design question

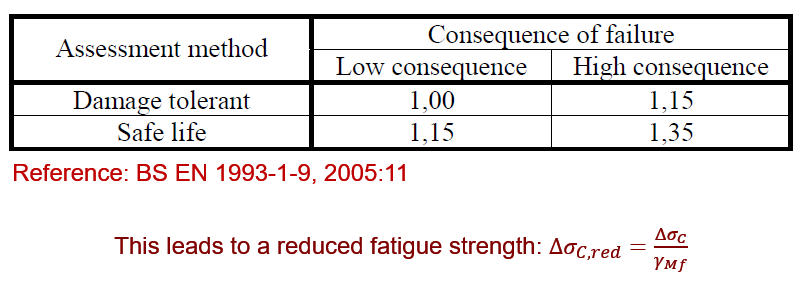
## Problem Statement

The flange of a welded steel girder is classified as Detail category 125 according to BS EN 1993-1-9. The component is subject to 500 000 cycles for stress range 200 MPa. Adopt a safe life strategy with low consequence of failure. The partial factor for equivalent constant amplitude stress range is . Is this design acceptable?

## Solution

Partial factor for fatigue:

From the table below, the partial factor for fatigue strength for safe life and low consequence of failure is .



Modification factors

No modification effects requested. So, none applicable.

Fatigue curve

For the stress range given above, the S-N curve equations can be manipulated as below to calculate the endurance at any stress range:

. Therefore, the endurance at 200 MPa is 321 000 cycles, which is below the required 500 000 cycles, and, the design do not comply.

ALTERNATIVELY

Steps:

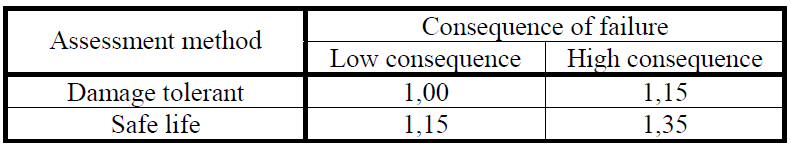
1. Calculate the stress range at 500 000 cycles, for the partial factor for fatigue modified Sr-N curve.
2. Check if the applied stress range is below the stress range calculated above, if so, the design is acceptable.

Could also:

1. Calculate the endurance at stress range 200 MPa from the partial factor modified Sr-N curve.
2. If the applied number of cycles is < than this endurance, the design is acceptable.

Modification factors

In this case we only need to modify for the safe life strategy with high consequence of failure, for which .



### Check 1

The stress range that the Sr-N curve allows at endurance 500 000 cycles is:

Assume the slope of the Sr-N curve is .

Therefore:

This stress range is BELOW the applied stress range, and the design is not acceptable.

### Check 2: Calculate endurance

The endurance at 200 MPa stress range is 321 000 cycles, which is less than the required 500 000 cycles and the design is not acceptable:

# WELD FATIGE EXAMPLE ON 2017-08-15

## Problem statement

The fatigue performance of a welded detail in a steel linkspan structure can be represented by a fatigue curve corresponding to BS EN 1993-1-9 Detail Category 36. The linkspan carries typical vehicles of weight 1, 2 and 5 ton. A linear elastic finite beam element analysis revealed the stress ranges in the welded detail as summarised in the table on the right with the proportion of vehicles carried by the ferry 70%, 28% and 2% respectively as summarised in the table. The linkspan is used twice per day. No more than one vehicle can occupy the linkspan at any one time. The design life required is equal to the service life of 40 years. Is this design sufficient if a damage tolerant with high consequence of failure strategy is implemented? A total of 50 vehicles are carried per day, and two stress cycles are caused to the linkspan per vehicle (on- and off loading).

The exhaust pipe blows are at temperature 350 °C on the link. No post-weld treatment was done, as it was reserved for when failures occur.



## Solution

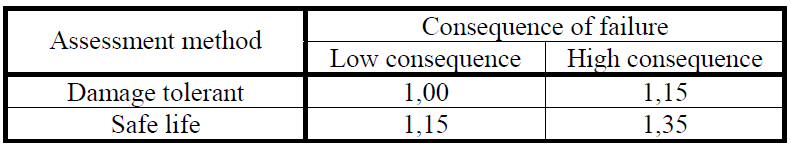
### Steps

The following steps will confirm the design:

1. Calculate the partial factor for fatigue.
2. Calculate temperature modification factor.
3. Calculate total damage and fatigue life.

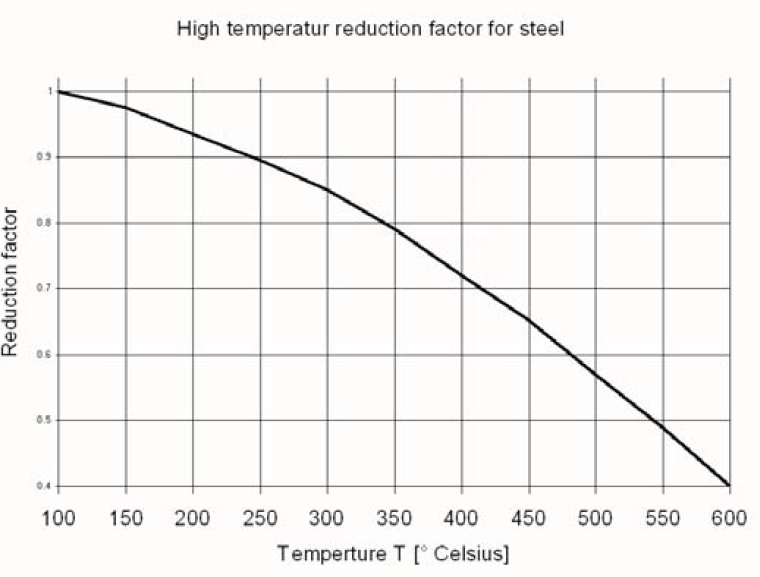
### Partial factor for fatigue

For a damager tolerant and high consequence of failure strategy, the partial factor for fatigue is .



### Temperature modification factor

The modification factor for temperature for operating at 350°C is



### Equation for the S-N curve

The endurance is given as follows:

The important points on the S-N curve is then:

### Damage calculation

For 20 MPa:

For 40 MPa:

For 100 MPa:

The total damage is then:

The total damage is then:

Therefore the design life of the link for the specified stress spectrum is:

### The stress spectrum

To calculate the applied cycles over the design life of 40 years, the following formula was used:

## Steps

1. Construct the Sr-N cure. Modify for
   1. Partial fatigue strength factor, .
   2. Temperature.
   3. Post-weld treatment.
2. Calculate damage.
3. Calculate life.
4. Make recommendations on the use of post—weld treatments if the fatigue life is < design life required.

## Calculate damage

The equation of the Sr-N curve that will be used to calculate endurance, is as follows:



## What about post-weld treatment

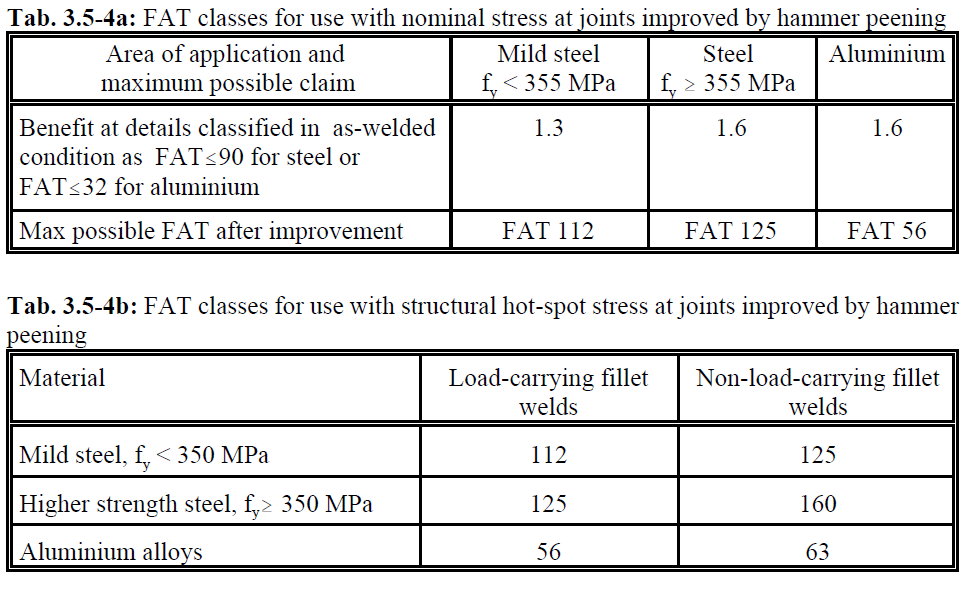
The first to verify, is if the post-weld treatment will be of any benefit for the detail.

When we do peening.

* Special requirements
  + Maximum of nominal compressive stress including proof loading
    - Assume we comply
  + Dependent on stress ratio:
    - effective stress range =
    - effective stress range = maximum applied stress
    - no benefit

It is assumed that the link is only loaded on one direction, from zero stress to a maximum value equal to the range. In this case . Therefore, the effective stress range is as given in the table.

Assume the material is structural steel 350W. For this material, the benefit from peening is as follows:





# WELD FATIGE CLASS PROBLEM INCLUDING ALL

An 8 mm double fillet weld was used in a T-joint as shown below. The stress spectrum was calculated over a period of 5 years and is summarized in the table below. The surface is corrosion protected. The component operating temperature is at 250 °C. Because of the small weld that was used in the construction, the weld toes were burred to specification, and peened to specification afterwards. The stress range has in all cases a stress ratio of R=-0.2. The component was heat treated before post-weld treatment. The design strategy is damage tolerant with low consequence of failure. What is the fatigue life of the component?

Table 6: Stress spectrum over a period of 5 years



50

50

Width:

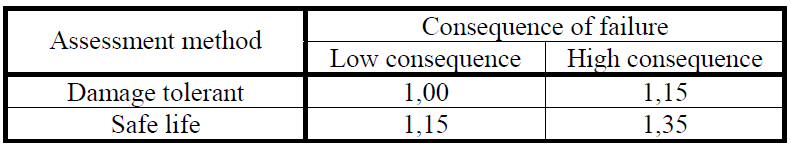
## Solution

The following procedure will be used:

1. Find the partial factor for fatigue.
2. Calculate modification factors:
   1. Temperature.
   2. Size.
   3. Post-weld treatment.
   4. Heat-treatment effects.
3. Determine detail categories.
4. Determine the equation for the S-N curve for crack initiation at the:
   1. Weld toe.
   2. Weld root.
5. Calculate damage.
6. Calculate life.

### Partial factor for fatigue

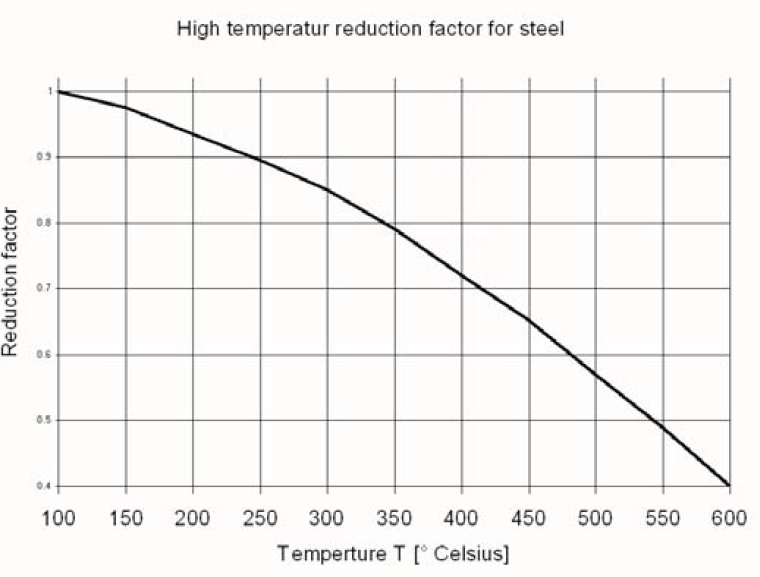
For a damage tolerant and low consequence of failure, the partial factor for fatigue is



### Modification factors/effects

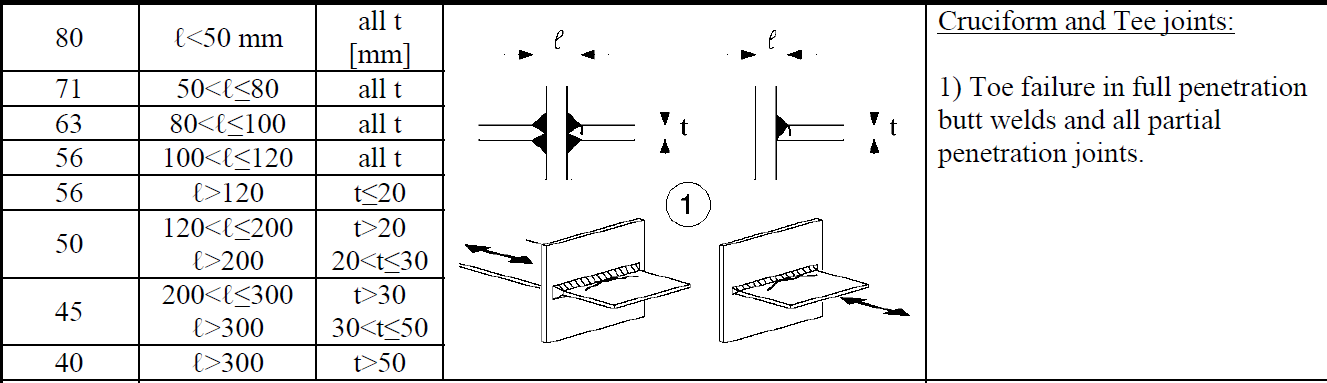
#### Temperature

The modification factor for operation at 250 °C is



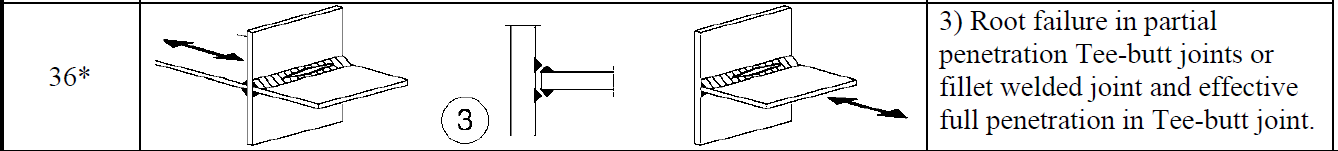
### Detail category: crack initiation at the weld toe

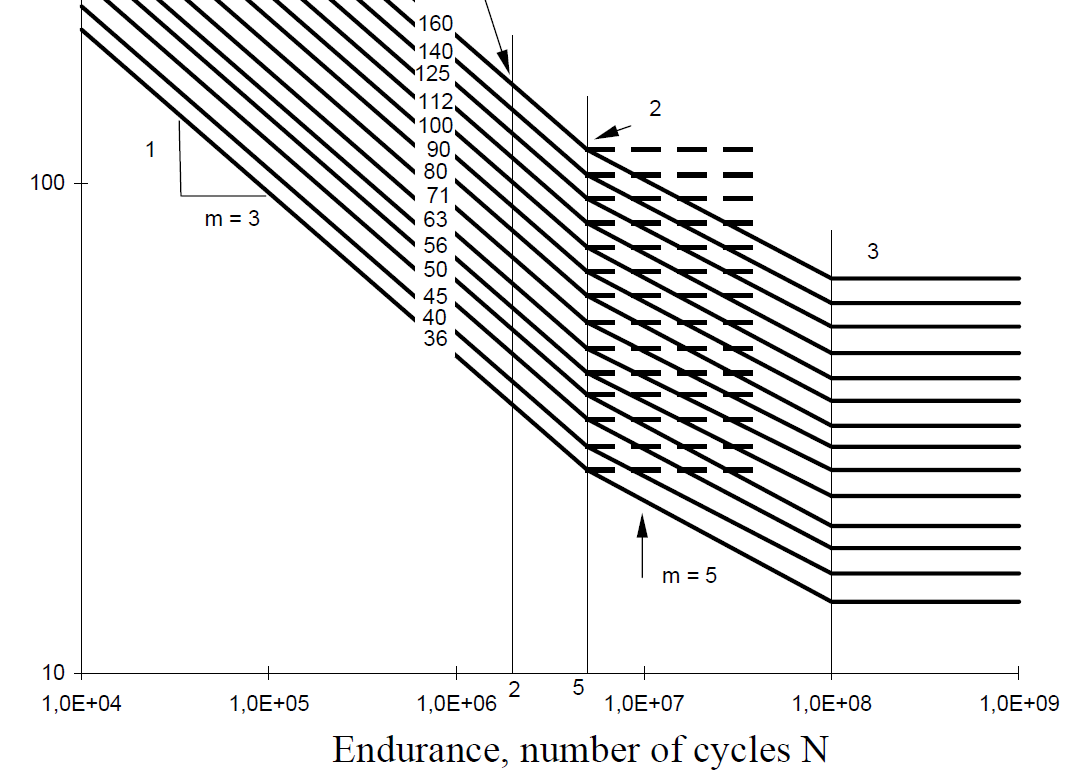
The detail category for crack initiation at the weld toe is . This detail will benefit from heat treatment, weld toe treatment, and, peening.



### Detail category: crack initiation in the weld root

In this case, the detail category is marked with an \*, and can be moved one category up. In this case conservativeness is not so important and the detail category was raised by one level to (see S-N curves below). This detail will NOT benefit from weld toe treatment or peening. But, it will benefit from heat treatment.





### Heat treatment

In this case heat treatment allows the use of 60% of the compressive part in the stress signal for range calculation. The stress spectrum only provides the stress range and stress ratio. To calculate the compressive stress and effective stress range for every given stress range, the following equation was used:

Steps to follow:

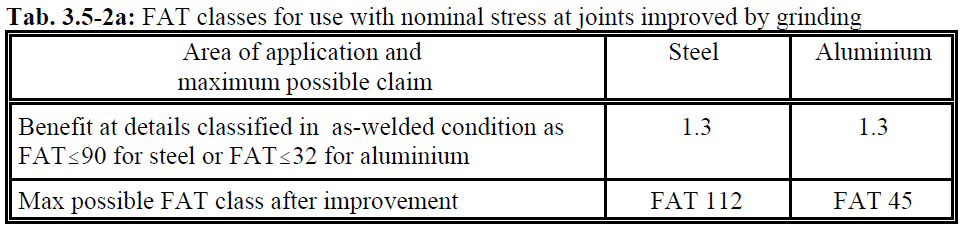
1. Calculate
2. Calculate new stress range from:

### Grinding and peening for improvement at the weld toe

The problem statement mentioned grinding, then peening afterwards. In this case, we can have the benefit of both provided that we do not exceed the limits as specified.

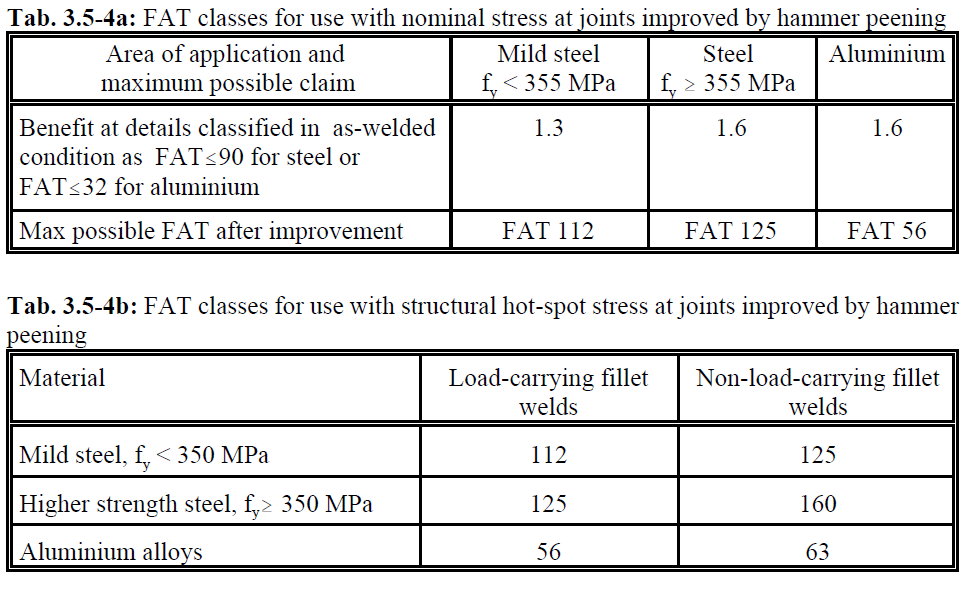
#### Grinding

For toe grinding the benefit and finale characteristic strength is:



#### Peening

The benefit from peening after grinding is:



### Stress in the weld throat

The stress in the weld throat from the nominal stress as given will be calculated as follows:

### Damage calculation and fatigue life assessment

The equation for the S-N curve will calculated endurance for any stress range as follows:

### Fatigue life for crack initiation at the weld toe

The fatigue life for 95% probability of survival for crack initiation at the weld toe is 16.9 years.



### Fatigue life for crack initiation at the weld throat

The fatigue life for 5% probability of crack initiation in the weld throat is 2.16 days! Please redesign this thing.



# PRESSURE VESSEL EXAMPLE 1

## Problem statement

* A fabricator of a pressure vessel elected to use as 25.4 mm plate made of:
  + Specify the material
* The radius should be 500 mm
* Determine the allowable working pressure of the cylindrical section of the pressure vessel
* Corrosion Allowance:
  + Make provision for 3.2 mm for corrosion

Answer in class in the class notes document

## Solution

Methodology:

1. Select a material that can handle the operating temperature and environment.
2. Assume no non-destructive testing. Find joint efficiency for category A welds with no NDT.
3. Calculate a thickness of t=25.4 – 3.2.
4. Calculate pressures for this t.

### Select material

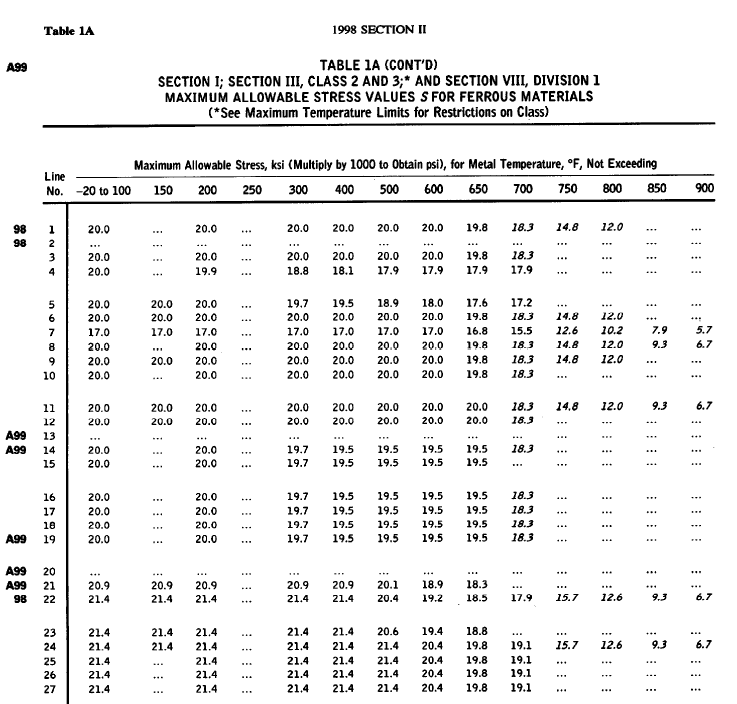
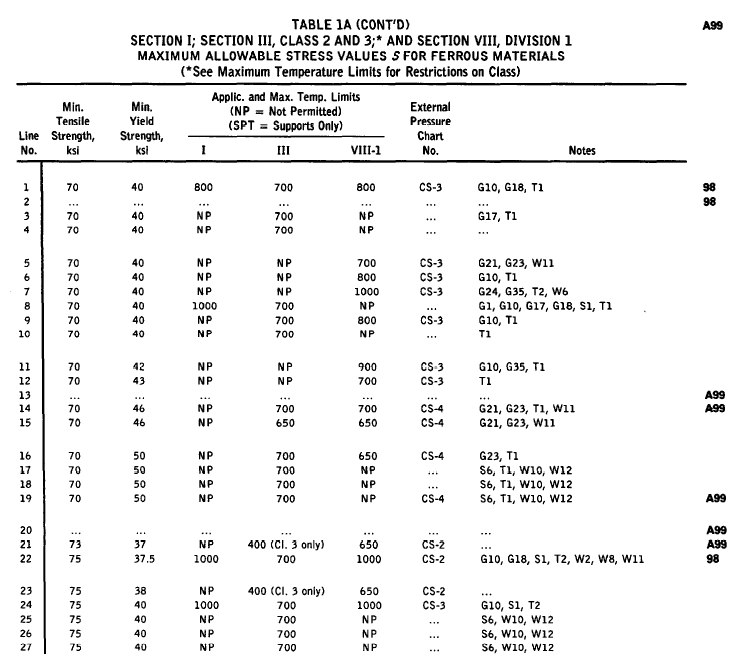
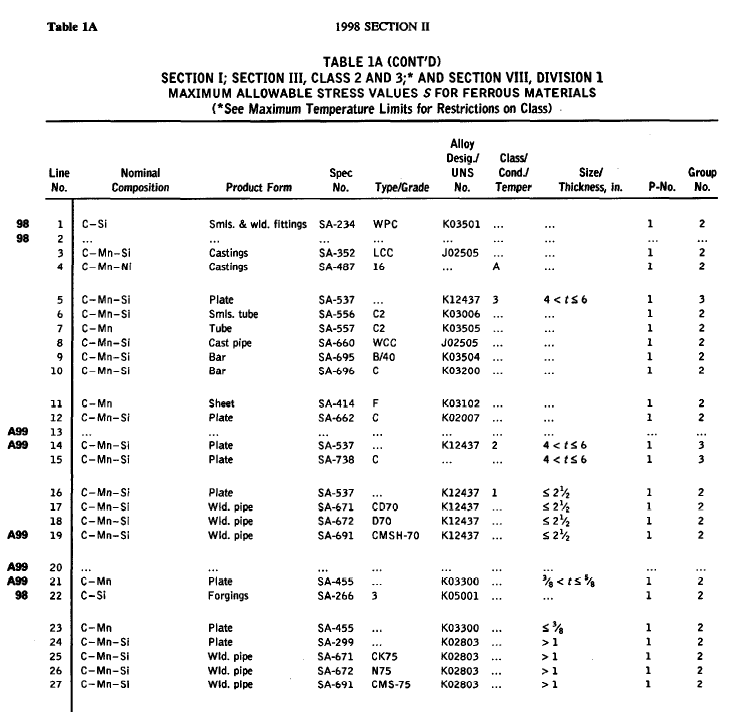
No temperature was specified and it is assumed that the vessel will operate under room temperature. The selection of the material was done as follows:

1. Selected materials that can offer a wide thickness range. I can always iterate this process if the plate selected now is not the best option.
2. Confirmed that the material can be used at the operating temperature.

This resulted in the selection of the following material:

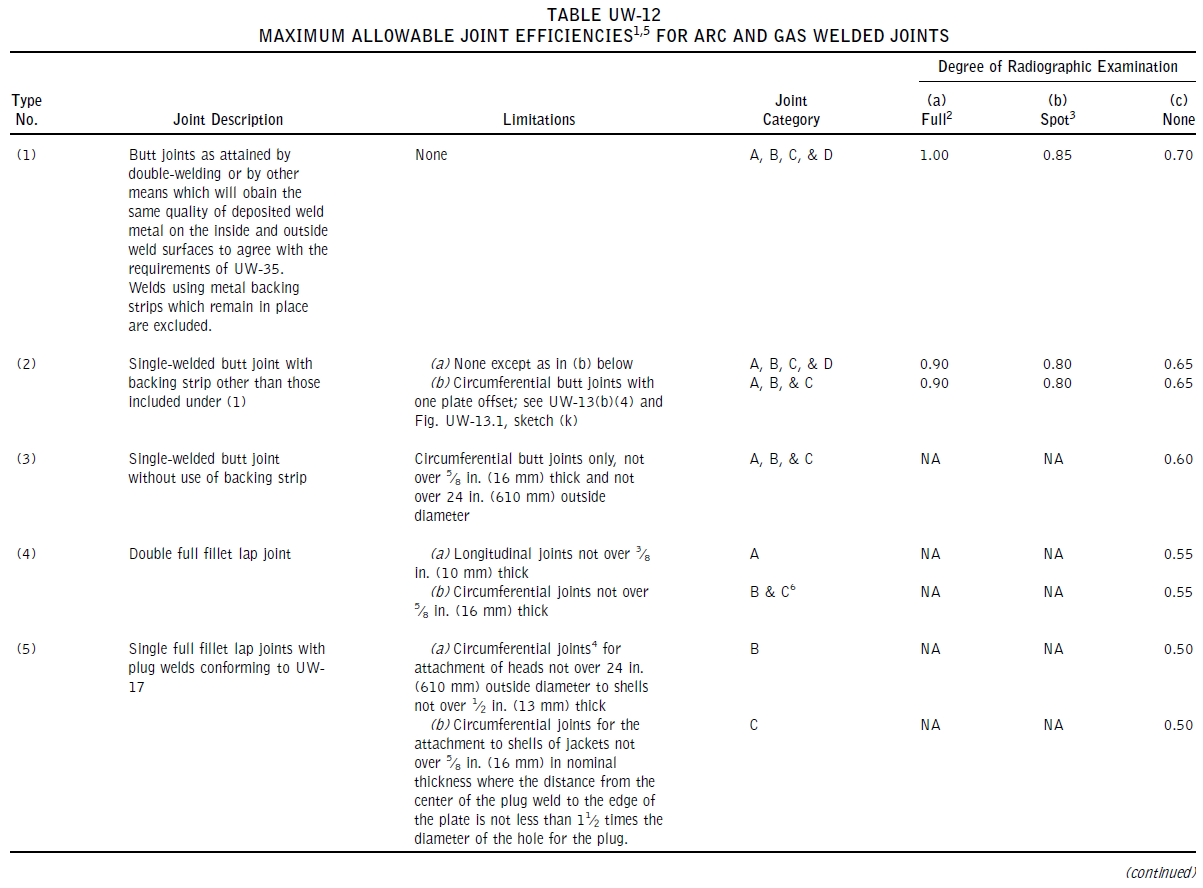
SA-537 Plate, S = 20 ksi at room temperature

The maximum allowable stress on this material at room temperature is 140 MPa.



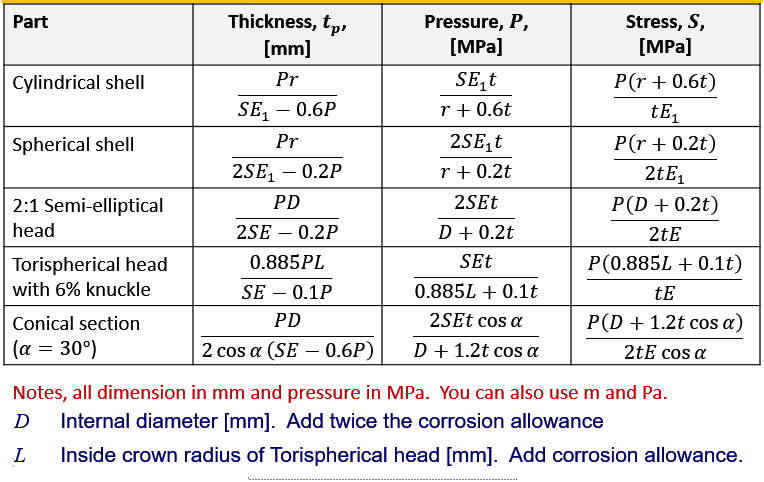
### Joint efficiency

The circumferential stress is the highest. Therefore, joint efficiencies for Joint A is required. No information on the welds were given. An instruction will be given for the WPS to specify a welding from one side without a backing strip. For this weld with no post-weld NDT, the joint efficiency is .



## Applicable equations

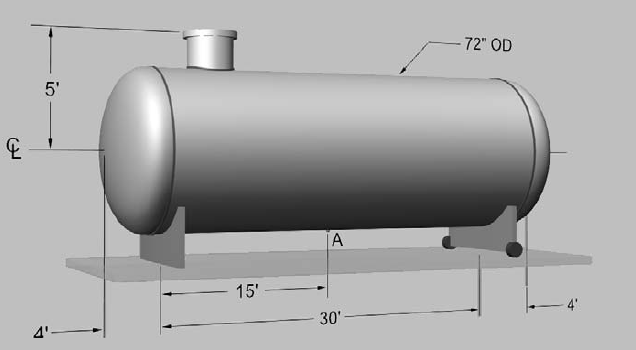
From the table below, the pressure for the cylindrical shell is given by (the thickness ):



# PRESSURE VESSEL AND CHECK FOR BUCKLING

## Problem statement

* A horizontal vessel with inside diameter 1,500 mm is to be fabricated from SA-516 Grade 70 material. The design pressure at the top of the vessel is 3,378 kPa (3.4 MPa) at 216 °C. This measurement point was at a height of 1.5 m above the centerline of the cylinder.
  + All longitudinal joints shall be Type 1 and spot radiographed in accordance with UW-52
  + Circumferential joints are Type 1 with no radiography
* Vessel operates full of liquid with density 998 kg/m3. Distance from the centerline to the uppermost part of vessel is 1.5 m.
* Determine the required thickness at Point A
  + Neglect the weight of the vessel in the calculation
* UG-22 states that the static head of the liquid must be included in the pressure P



## Solution

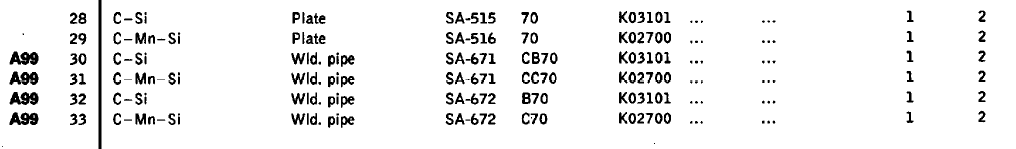
1. Calculate the design pressure.
2. Find the maximum allowable design stress.
3. Circumferential stress
   1. Find joint efficiency.
   2. Calculate
4. Longitudinal stress at the bottom
   1. Find joint efficiency
   2. Calculate bending stress
   3. Calculate
   4. Calculate stress at the top of the vessel.
      1. If < 0, check for buckling.
5. Add thickness for corrosion

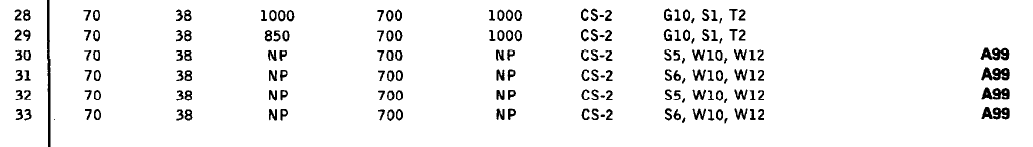
### Design pressure

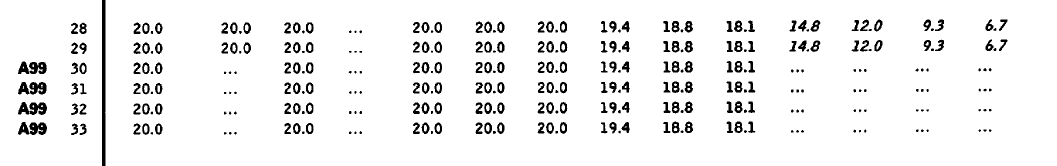
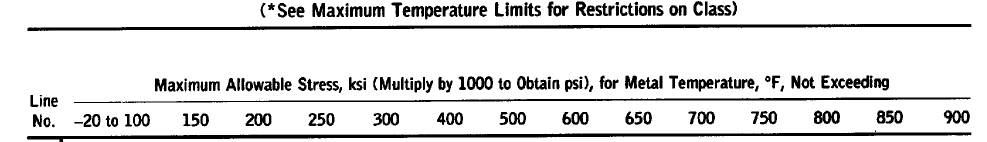
The pressure at the top of the vessel was given 3 378 kPa. However, hydrostatic pressure need to be included and from this the design pressure is:

### Maximum allowable design stress

The temperature is 443 °F. For this, the maximum allowable stress is 20 ksi, 140 MPa.





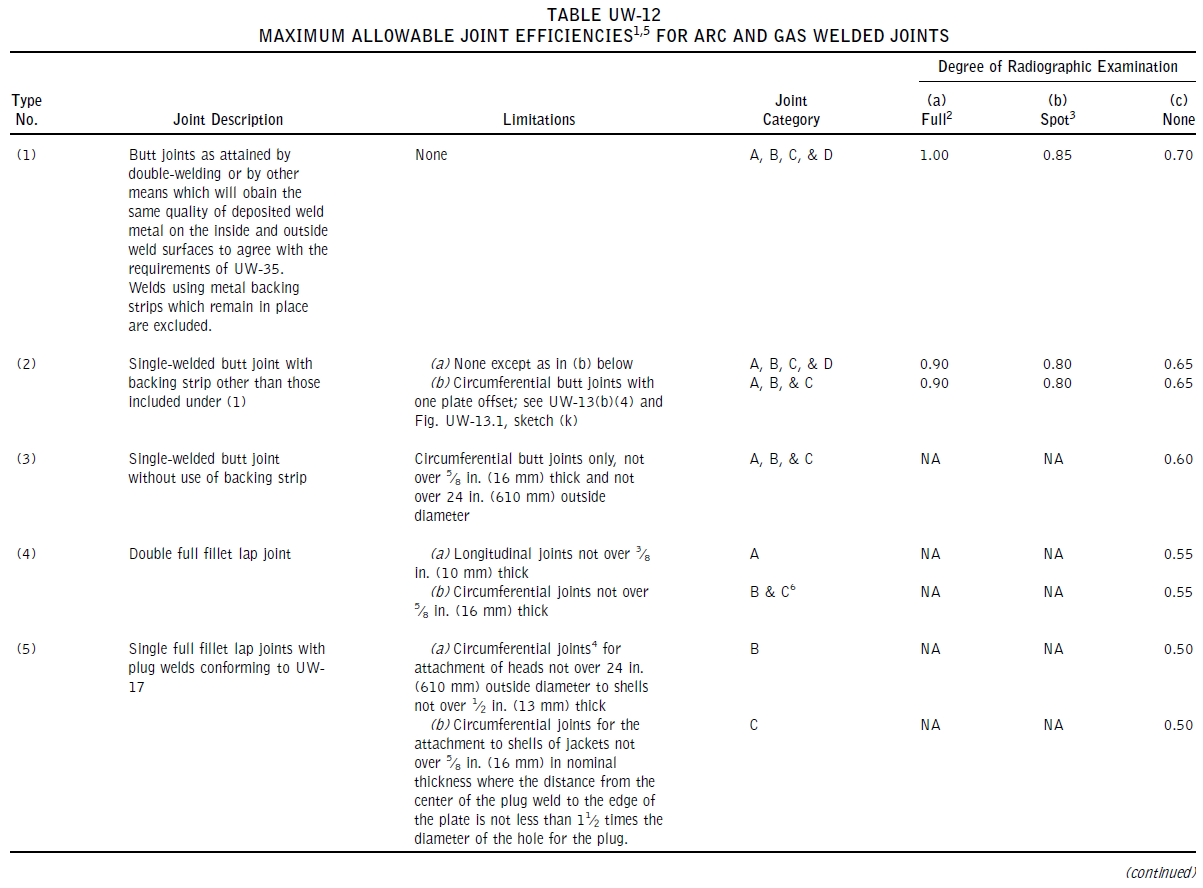


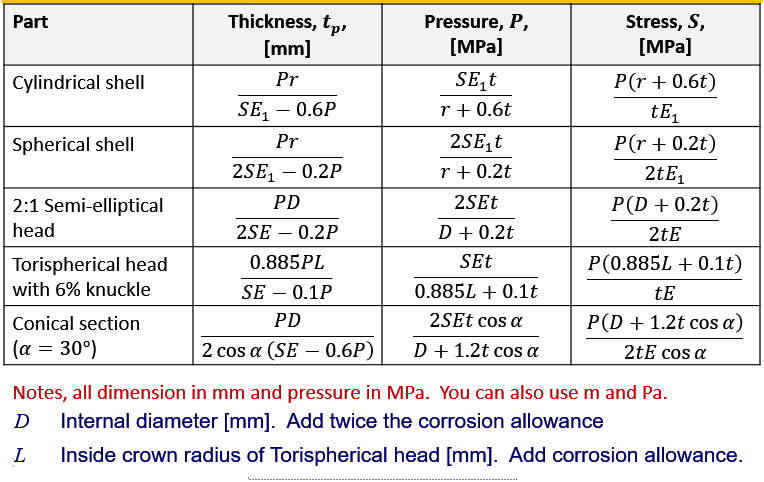
### Circumferential stress

#### Joint efficiency and thickness

The joint efficiency for the longitudinal welds is .85. The thickness required is:

Confirm that this thickness is available for the material.





### Longitudinal stress at the top

The longitudinal stress due to pressure and bending is given by the following equation:

Ag nee, this is tto small to demonstrate buckling. Say it is:

The maximum bending moment is at the centre of the pressure vessel:

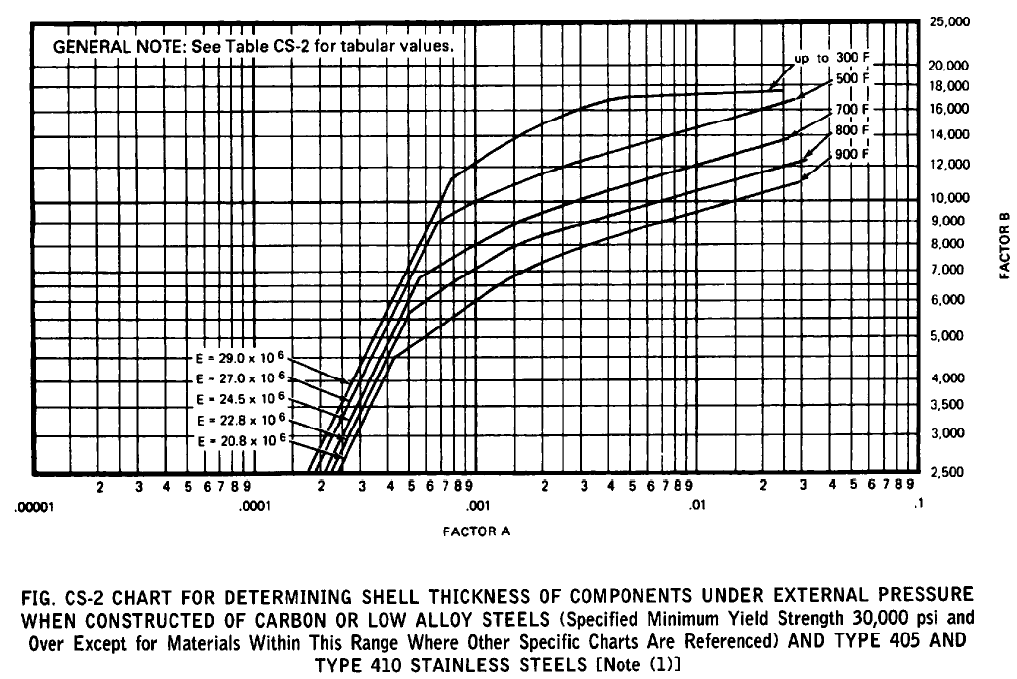
There is compressive stress. Confirm buckling.

### Confirm buckling

Step 1: Calculate A

Step 2: Factor B from graph

From the graph below, the factor is . Therefore, the thickness is adequate to prevent buckling because is more than the compressive stress of 80 MPa.



### Check longitudinal stress at bottom

This is small contribution and will be neglected.

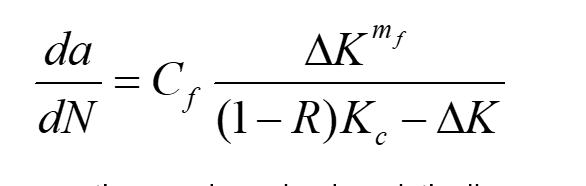
# PLANE STRESS AND STRAIN

The constitutive model for isotropic materials:

Say we have stress state:

Under plane strain conditions, the strains . Then stresses will be generated because:

# FRACTURE MECHANICS





# FRACTURE MECHANICS CLASS PROBLEM

## Problem statement

You have a rectangular cross section steel part with yield strength 500 MPa and Charpy test energy 35 J that has a double edge crack measured 2 mm on each side. The part is subject to a axial stress amplitude 100 MPa, mean 0 MPa, for 50 cycles per hour. The width of the part is 250 mm and thickness is 30 mm. The part is used for 8 hours per day. For how long can the part be used before failure? What can be done to increase the fatigue life.

## Solution

Steps:

1. Determine the plane strain fracture toughness of the material.
2. Calculate the crack size where failure occur.
   1. Fracture
   2. Plastic collapse
3. Get a model for the geometric stress concentration factor.
4. Set up the Paris rule for crack propagation.
5. Solve

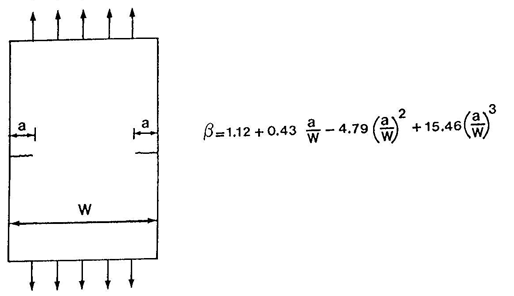
### Plane strain fracture toughness

A lower bound for the plane strain fracture toughness will be used, given by:

### Fracture

Fracture occurs when the crack size reaches a value so that the stress intensity factor equals the plane strain fracture toughness at the maximum tensile stress:

The geometric stress concentration factor is given by the following figure:



### Plastic collapse

Plastic collapse occurs when the crack size reaches a value so that the stress in the remaining section becomes equal to the yield strength:

### Crack propagation

The crack propagation will be modelled as follows:

where

## Calculation sheet

For this load application the stress ration is -1. Therefore the threshold stress intensity level is 5.4 MPam0.5. The applied stress intensity range is 8.9 MPam0.5, which is higher than threshold value, and, crack propagation will occur. From the table below, the crack will propagate to 36 mm in 450 cycles where the fracture toughness is reached. The component fails in fracture. For fracture collapse, a material with plane strain facture toughness of 70 MPam0.5 or more is required to allow the crack to grow to 100 mm in length.

Life can be extended by:

1. Repairing the crack.
2. Drill-stop the crack.
3. Retard the crack by tensile overloading.

