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Behaviour of materials under dynamic loading

MSV 780

Prepared for

Universities

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Summary

This document is the master document for the presentation on the fatigue behaviour of materials.

Software applications

The following software applications were used in the execution of this project:

Product Name	Version
PC Matlab	2008R1
Microsoft Excel	2016
Microsoft PowerPoint	2016
Microsoft Word	2016
Acrobat	2020

Calculation files

The following calculation files were used in the analysis:

Description	File Name	Revision
Strain life calculation modelling sequence effect of strain	Strain life example - Sequence effect.xls	0.0
Construction of S-N curve	PlotSNcurve.m	0.0

Models

The following models were used in the analysis:

Model Title	Model Number	Revision
Not applicable		

Input Data Files

The following input files are applicable:

Description	File Name
Notch effects on welds	Investmech - Structural Integrity (Notch effects of welds) R0.0.pptx
Static failure theories	Investmech - Structural Integrity (Static Failure Theories) R0.0.pptx
Variable amplitude loading on steel members	Investmech - Structural Integrity (Variable Amplitude Loading) R0.0.pptx
Cycle counting using the rainflow method	Investmech - Structural Integrity (Cycle counting) R0.0.pptx
Stress life analysis	Investmech - Fatigue (Stress life analysis) R0.0.pptx
Strain life analysis	Investmech - Fatigue (Strain life analysis) R0.0.pptx
Fatigue life improvement techniques for steel and stainless steel	Investmech - Structural Integrity (Fatigue life improvement techniques for steel and stainless steel) R0.0.pptx

Output Data Files

The following output files are applicable:

Description	File Name
MS Word document with problems done in class as well as notes made in class. This document will be submitted by e-mail to class members after completion of the module	Class notes.docx. The same notes document is used for both days to have all in one document.
MS Excel document with calculations done in class	Class calculations.xlsx. The same Excel document is used for both days to have all in one document.

Table of Contents

1. INTRODUCTION

This document presents the class notes to understand the behaviour of materials and welded structures under dynamic loading. This section introduces concepts to students whereas the following module focuses on the design of welded structures under dynamic loading. Therefore, this section is more information and introduces basic principles. Following sections introduce application of the concepts by calculation.

2. STUDY MATERIAL

The student shall arrange access to the following additional documents:

1. Slides presented in class.
2. Text book: Dowling, N.E. 2013. *Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue*. International Edition. 4th Edition. Pearson, Boston.
3. Class notes made in class for download from the website.

3. TERMINOLOGY

This section summarises terminology used in the lectures and come from BS EN 1993-1-9 (2005:7:9):

Fatigue

The process of initiation and propagation of cracks through a structural part or member due to the action of fluctuating stress.

Nominal stress

A stress in the parent material or in a weld adjacent to a potential crack location calculated in accordance with elastic theory excluding all stress concentration effects.j The nominal stress can be a direct stress, shear stress, principal stress or equivalent stress.

Modified nominal stress

A nominal stress multiplied by an appropriate stress concentration factor k_f to allow for a geometric discontinuity that has not been taken into account in the classification of a particular constructional detail.

Geometric stress = hot spot stress

The maximum principal stress in the parent material adjacent to the weld toe, taking into account stress concentration effects due to the overall geometry of a particular constructional detail. Local stress concentration effects e.g. from the weld profile shape (which is already included in the detail categories in Appendix B of BS EN 1993-1-9 for the geometric stress method)) need not to be considered.

Residual stress

Residual stress is a permanent state of stress in a structure that is in equilibrium and is independent of the applied actions. Residual stresses can arise from rolling stresses, cutting processes, welding shrinkage or lack of fit between members or from any loading event that causes yielding of part of the structure.

Loading event

A defined loading sequence applied to the structure and giving rise to a stress history, which is normally repeated a defined number of times in the life of the structure.

Stress history

A record or a calculation of the stress variation at a particular point in a structure during a loading event.

Rainflow method

Particular cycle counting method of producing a stress-range spectrum from a given stress history.

Reservoir method

Particular cycle counting method of producing a stress-range spectrum from a given stress history.

Stress range

Algebraic difference between the two extremes of a particular stress cycle derived from a stress history.

Stress-range spectrum

Histogram of the number of occurrences for all stress ranges of different magnitudes recorded or calculated for a particular loading event.

Design spectrum

The total of all stress-range spectra in the design life of a structure relevant to the fatigue assessment.

Design life

The reference period of time for which a structure is required to perform safely with an acceptable probability that failure by fatigue cracking will occur.

Fatigue life

The predicted period of time to cause fatigue failure under the application of the design spectrum.

Miner's summation

A linear cumulative damage calculation based on the Palmgren-Miner rule.

Equivalent constant amplitude stress range

The constant amplitude stress range that would result in the same fatigue life as for the design spectrum, when the comparison is based on a Miner's summation.

Fatigue loading

A set of action parameters based on typical loading events described by the positions of loads, their magnitudes, frequencies of occurrence, sequence and relative phasing.

Equivalent constant amplitude fatigue loading

Simplified constant amplitude loading causing the same fatigue damage effects as a series of actual variable amplitude loading events.

Fatigue strength curve

The quantitative relationships between the stress range and number of stress cycles to fatigue failure, used for the fatigue assessment of a particular category of structural detail. The fatigue strengths given in BS EN 1993-1-9 are lower bound values based on the evaluation of fatigue tests with large scale test specimens in accordance with EN 1990 – Annex D.

Detail category

The numerical designation given to a particular detail for a given direction of stress fluctuation, in order to indicate which fatigue strength curve is applicable for the fatigue assessment (The detail category number indicates the reference fatigue strength $\Delta\sigma_C$ in MPa).

Constant amplitude fatigue limit

The limiting direct or shear stress range value below which no fatigue damage will occur in tests under constant amplitude stress conditions. Under variable amplitude conditions all stress range have to be below this limit for no fatigue damage to occur.

Cut-off limit

The limit below which stress ranges of the design spectrum do not contribute to the calculated cumulative damage.

Endurance

The life to failure expressed in cycles, under the action of a constant amplitude stress history.

Nominal stress

Nominal, also called average stress, S, is the stress calculated from force and/or moment combinations.

Point stress

Notches produce stress concentrations with the elastic theoretical stress concentration factor, k_t. The point is the stress of interest for fatigue which, in many instances, is equal to the nominal stress, but in others is equal to: $\sigma = k_t S$.

Reference fatigue strength

The constant amplitude stress range $\Delta\sigma_C$ for a particular detail category for an endurance of $N = 1 \times 10^6$ cycles

4. BEHAVIOUR OF MATERIALS UNDER VARIABLE AMPLITUDE LOADING

Presentation used in class:	Filename: Investmech - MASTER - Behaviour of materials under dynamic loading R0.0.pptx

4.1. Objective

Understand in detail:

1. The development of fatigue
2. Calculation of load cycles
3. Influence of notches and their avoidance

4.2. Scope

The following material will be covered:

1. Types and variables of cyclic loading
2. Statistical stress analysis on real structures
3. S-N diagrams
4. Stress collective
5. Fatigue strength
6. Effect of mean stress including residual stresses
7. Effect of stress range
8. Stress distribution
9. Influence of notches
10. Influence of weld imperfections
11. Weld fatigue improvement techniques
    a. Surface protection
    b. Needle peening
    c. TIG dressing
    d. Burr grinding
    e. Hammering
    f. Stress relieving
12. Standards, ISO, CEN and National
13. Palmgren-Miner rule
14. Classification of weld joints

4.3. Outcomes

After completion of this section you will be able to:

1. Draw and interpret an S-N diagram.
2. Explain fully the methods of counting load cycles.
3. Calculate the stress ratio.
4. Detail the influence of notches and weld defects.
5. Explain fully the methods for improving fatigue performance.

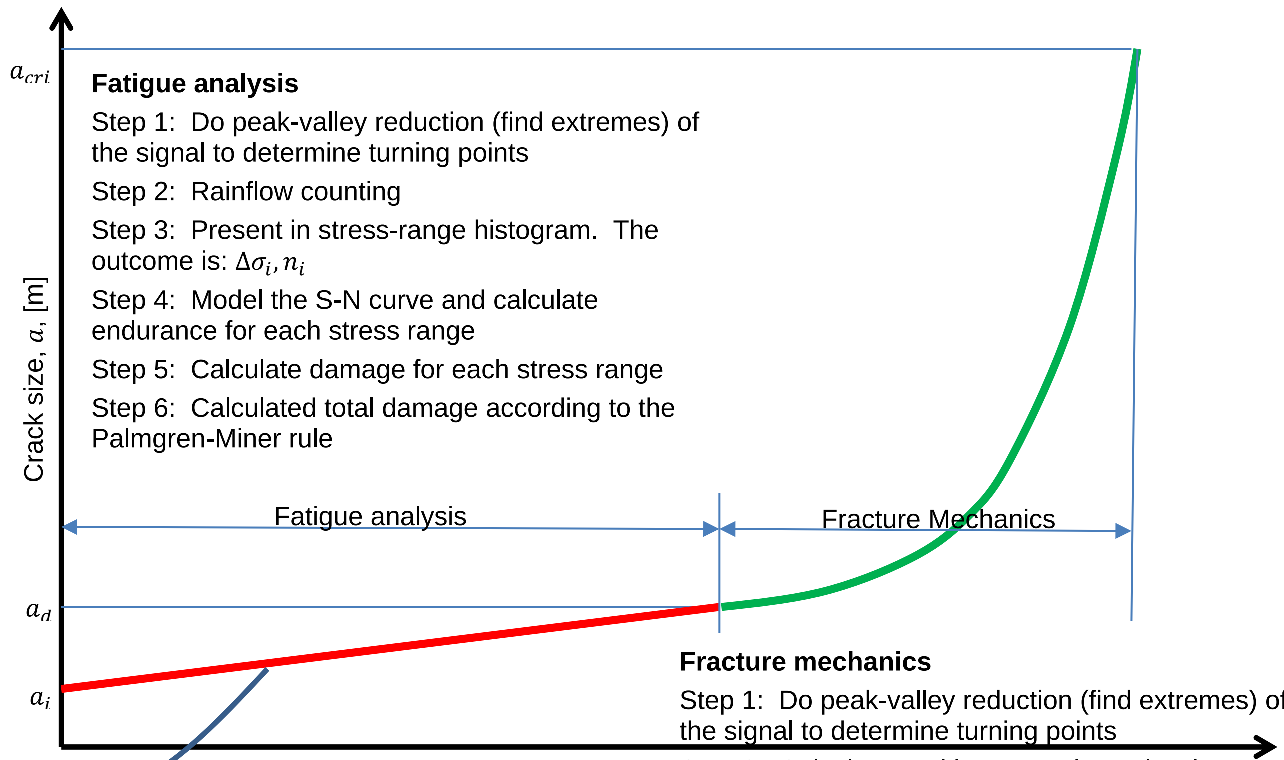
Structural Integrity and Structural Mechanics

Static loading = use static failure criteria:

- Maximum principal stress theory
- Maximum shear stress theory
- Von Mises theory
- Octahedral stress theory
- Mohr's failure theory
- Maximum normal strain theory
- Octahedral strain theory
- Total strain energy theory
- Fracture for cracked members
- Standards

Variable amplitude loading

A defect in the material grows to a detectable size where after it propagates to fracture.



Fatigue analysis

Step 1: Do peak-valley reduction (find extremes) of the signal to determine turning points

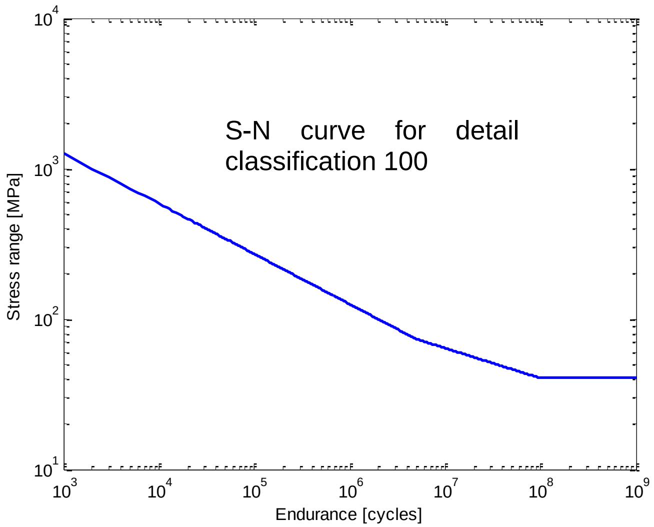
Step 2: Rainflow counting

Step 3: Present in stress-range histogram. The outcome is: $\Delta\sigma_i, n_i$

Step 4: Model the S-N curve and calculate endurance for each stress range

Step 5: Calculate damage for each stress range

Step 6: Calculated total damage according to the Palmgren-Miner rule



The critical crack size is the failure crack size that will result in fracture ($K_I = K_{IC}$) or plastic collapse. The condition that occurs first determines the critical crack size.

Fracture mechanics

Step 1: Do peak-valley reduction (find extremes) of the signal to determine turning points

Step 2: Calculate positive stress intensity change for each reversal (half cycle) $\Delta K = \Delta\sigma\beta\sqrt{\pi a}$. Only use positive values

Step 3: Calculate crack propagation from the Paris rule: $\frac{da}{dN} = C\Delta K^m$ for each cycle

Step 4: Accumulate crack size to the critical crack size, a_{cri}

5. FORCES, MOMENTS AND STRESS DISTRIBUTIONS

The objective of this section is to refresh the students background on the calculation of membrane (bending + normal) stresses and includes a section on the distribution of shear force and bending moments on structural members.

Presentation used in class:	Forces, moments and stress distributions Filename: Investmech (Force moment and stress distributions) P R0.0

The slides presented in the class will be made available for download on the website. In summary, the following principles apply:

- Shear force diagram.
- Bending moment diagram.
- Coordinate systems and the expressions for normal stress.
- Euler-Bernoulli beam theory to link moments with change of slope: $E(x)I(x)\left(\frac{dw^2}{dx^2}\right) = M(x)$.
- Modelling boundary conditions.
- Normal stress distribution:

$$\sigma = \frac{F}{A} + \frac{M_y I_{zz} + M_z I_{yz}}{I_{zz} I_{yy} - I_{yz}^2} z - \frac{M_z I_{yy} + M_y I_{yz}}{I_{zz} I_{yy} - I_{yz}^2} y$$

- Shear stress distribution:
  - The first moment of area around the x- and y-axes are:

$$Q_x = \int y dA \, Q_y = \int x dA$$

  - From which the shear stress is:

$$\tau_y = \frac{V_y Q_x}{I_{xx} W}$$

$$\tau_x = \frac{V_x Q_y}{I_{yy} H}$$

$$\tau = \tau_x + \tau_y$$

  - Average shear stress:

$$\tau_{ave} = \frac{V}{A}$$

- The shear stress for unsymmetrical sections is:

$$\tau = \frac{I_{zz} Q_y - I_{yz} Q_z}{b\left(I_{zz} I_{yy} - I_{yz}^2\right)} V_z + \frac{I_{zz} Q_z - I_{yz} Q_y}{b\left(I_{zz} I_{yy} - I_{yz}^2\right)} V_y$$

6. FATIGUE S-N CURVES AND DAMAGE

The objective of this section is to introduce S-N fatigue curves to the student and enable the student to carry out fatigue calculations for constant amplitude loads.

Presentation used in class:	S-N curves and damage Filename: Investmech – Fatigue (S-N curves and damage) R0.0

Fatigue is the process of damage and failure due to cyclic loading due to microscopic cracks that develop into macroscopic damage or failure as shown in Figure 1.

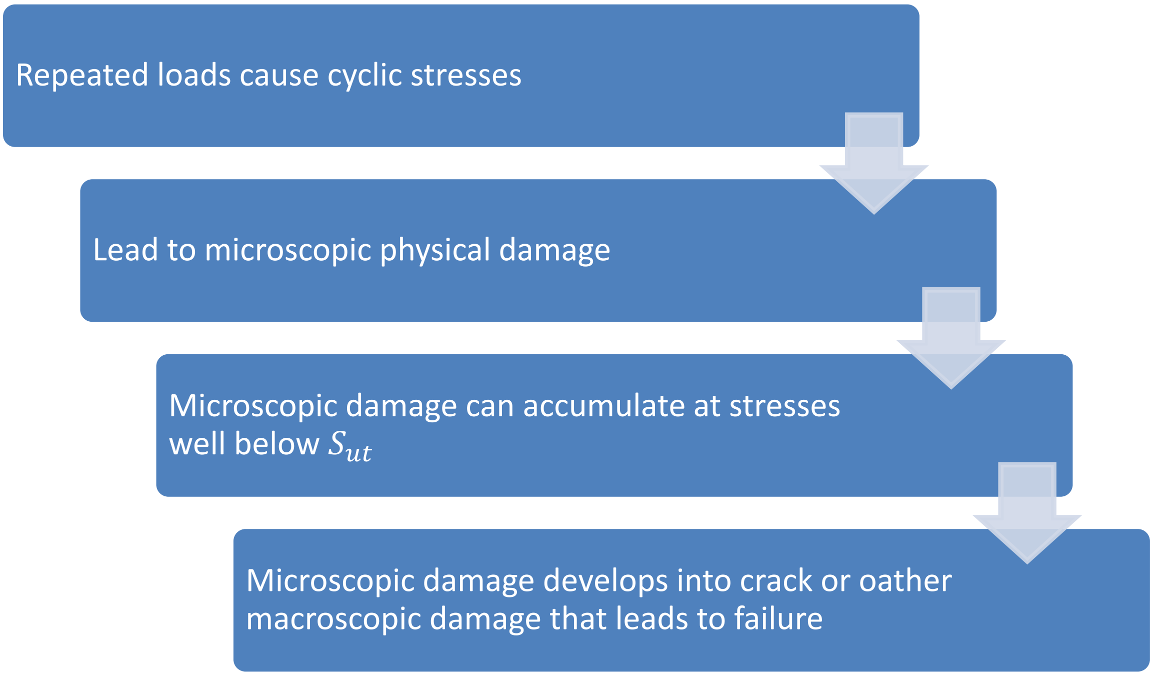


Figure 1: Fatigue is the process of damage and failure due to cyclic loading

6.1. Constant amplitude alternating stress parameters

Fatigue failure occurs when a material is subject to repeated cyclic loading as in the case of a train wheel, ball mill, shafts, etc. Fatigue failure involves crack initiation/formation, growth and final fracture. Fatigue failure can occur at stresses well below the yield or ultimate tensile strength of the material and is affected by alternating stresses, any mean stresses, surface finish, environmental conditions, and presence of notches and flaws. An example of an alternating stress signal with the applicable statistical stresses can be seen in Figure 2.

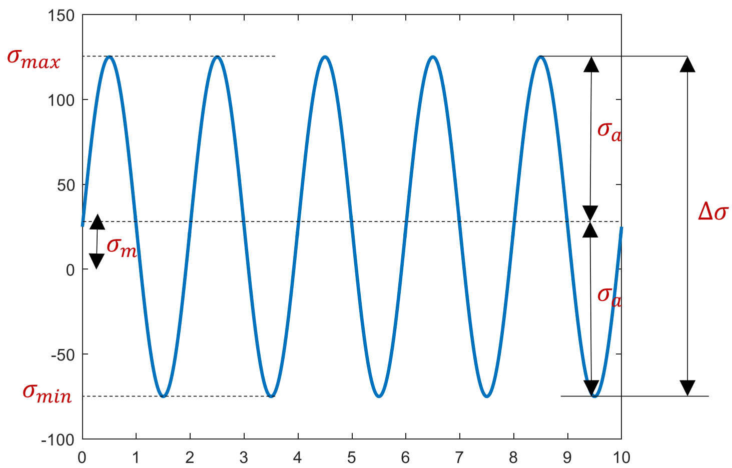


Figure 2: An example of a constant amplitude alternating stress signal

The stress range is calculated using the following equation:

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \quad (1)$$

The stress amplitude is calculated using the following equation:

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} \quad (2)$$

The mean stress is calculated using the following equation:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \quad (3)$$

The stress ratio is:

$$R = \frac{\sigma_{min}}{\sigma_{max}} \quad (4)$$

The amplitude ratio is:

$$A = \frac{\sigma_a}{\sigma_m} \quad (5)$$

Where:

$\Delta\sigma$	Stress range [MPa]
σ_a	Stress amplitude [MPa]
σ_m	Mean stress [MPa]
σ_{max}	Maximum stress [MPa]
σ_{min}	Minimum stress [MPa]

From these it is clear to see that for:

- Completely reversed cycling:
  - $R = -1; A = \infty$
- Cycling from zero to a maximum, also called *zero-to-tension* cycling:
  - $R = 0; A = 1$
- Signal from zero to a minimum:
  - $R = \infty; A = -1.$

In the equations above, symbol S is be used to indicate nominal stress away from the stress concentration.

The following relationships are derived from the equations above. Stress amplitude in terms of maximum stress and stress ratio:

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max}}{2}(1 - R) \quad (6)$$

Mean stress vs. maximum stress:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{\sigma_{max} + R\sigma_{max}}{2} = \frac{\sigma_{max}}{2}(1 + R) \quad (7)$$

Stress ratio vs. amplitude ratio:

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{\sigma_m - \sigma_a}{\sigma_m + \sigma_a} = \frac{1 - \frac{\sigma_a}{\sigma_m}}{1 + \frac{\sigma_a}{\sigma_m}} = \frac{1 - A}{1 + A} \quad (8)$$

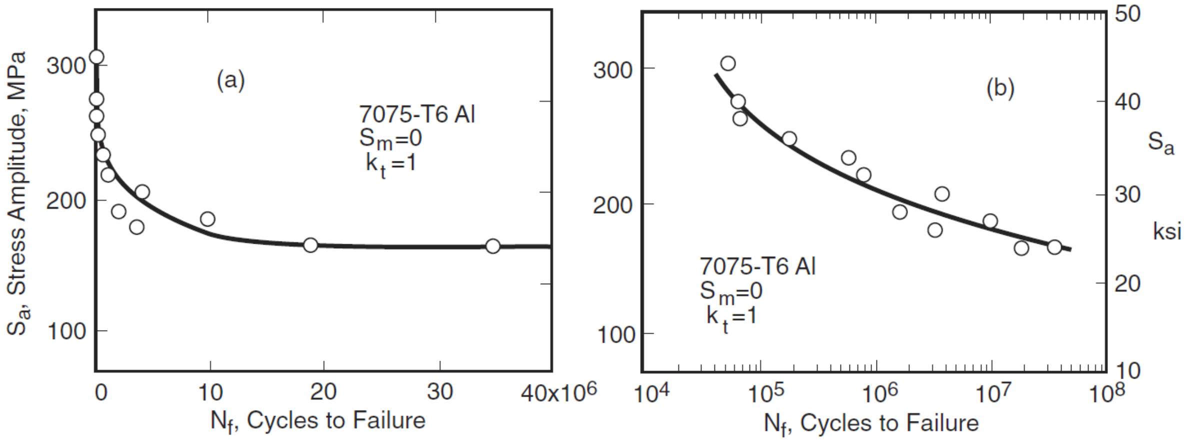
And, in the same way, the amplitude ratio vs stress ratio:

$$A = \frac{1 - R}{1 + R} \quad (9)$$

6.2. Stress vs Life

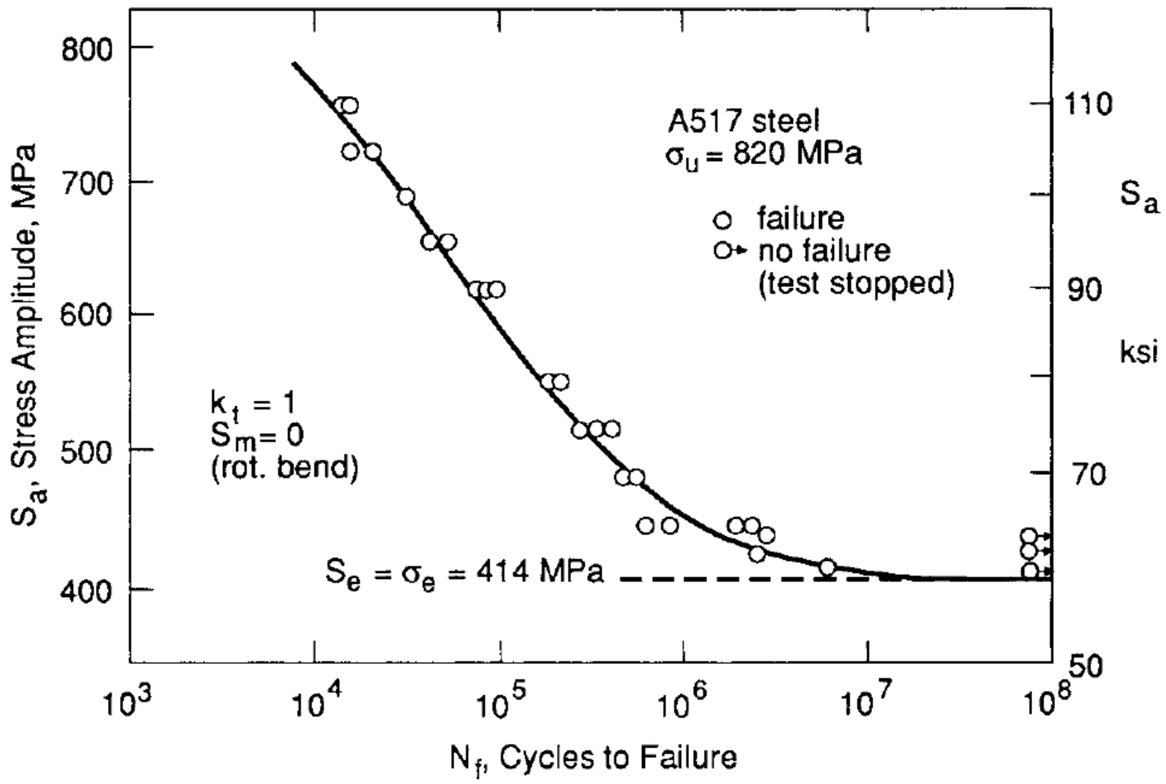
Figures 3 and 4 respectively shows the stress vs life (ordinate vs abscissa) test results and curves for unnotched 7075-T6 Al and A517 steel specimens under rotating bending test, from which it is clear that the aluminium alloy does not have a distinct endurance limit (also called fatigue limit in some text books) compared to the steel that shows a clear endurance limit. The ordinate could be either stress amplitude, σ_a or nominal stress amplitude, S_a. Symbol S is used to indicate nominal stress away from a stress concentration on the specimen. Later more on this.

Stress range, $\Delta\sigma$, or even maximum stress, σ_{max} may be used as the ordinate. The equations given in the previous section can be used to present the stress vs life curve in any form.



Source: (Dowling, 2013, p. 422)

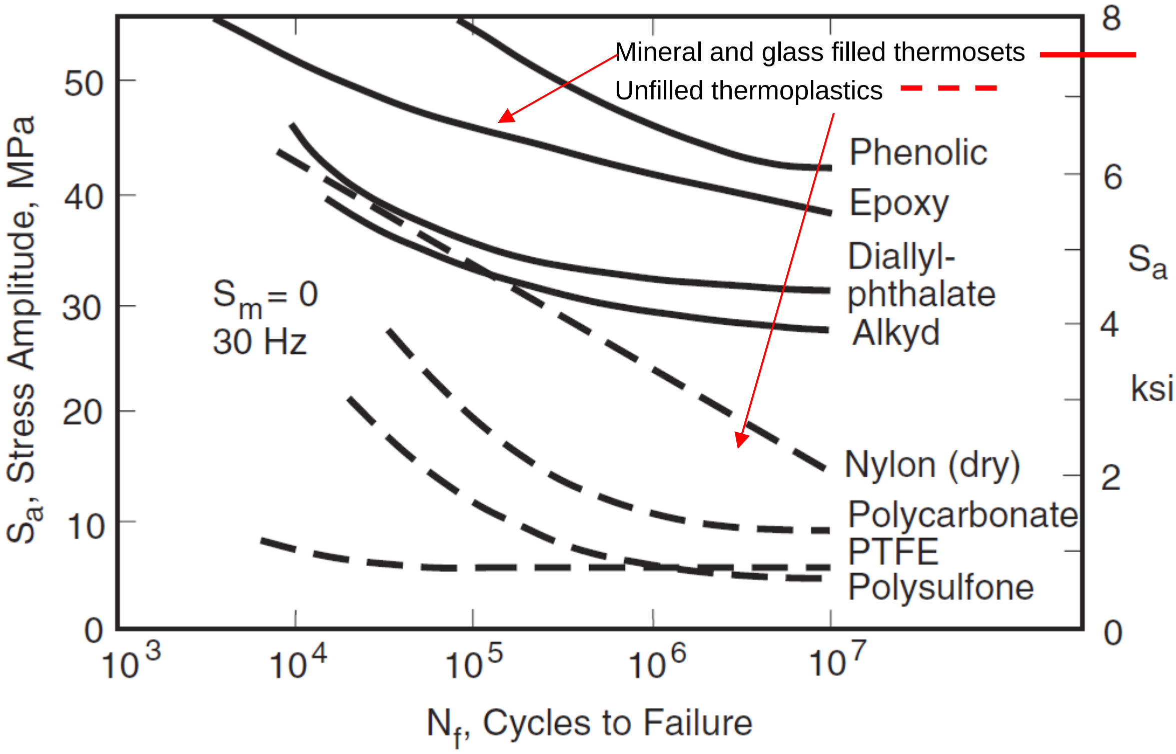
Figure 3: Stress versus life (S-N) curves from rotating bending tests of unnotched specimens of an aluminium alloy



Source: (Dowling, 2013)

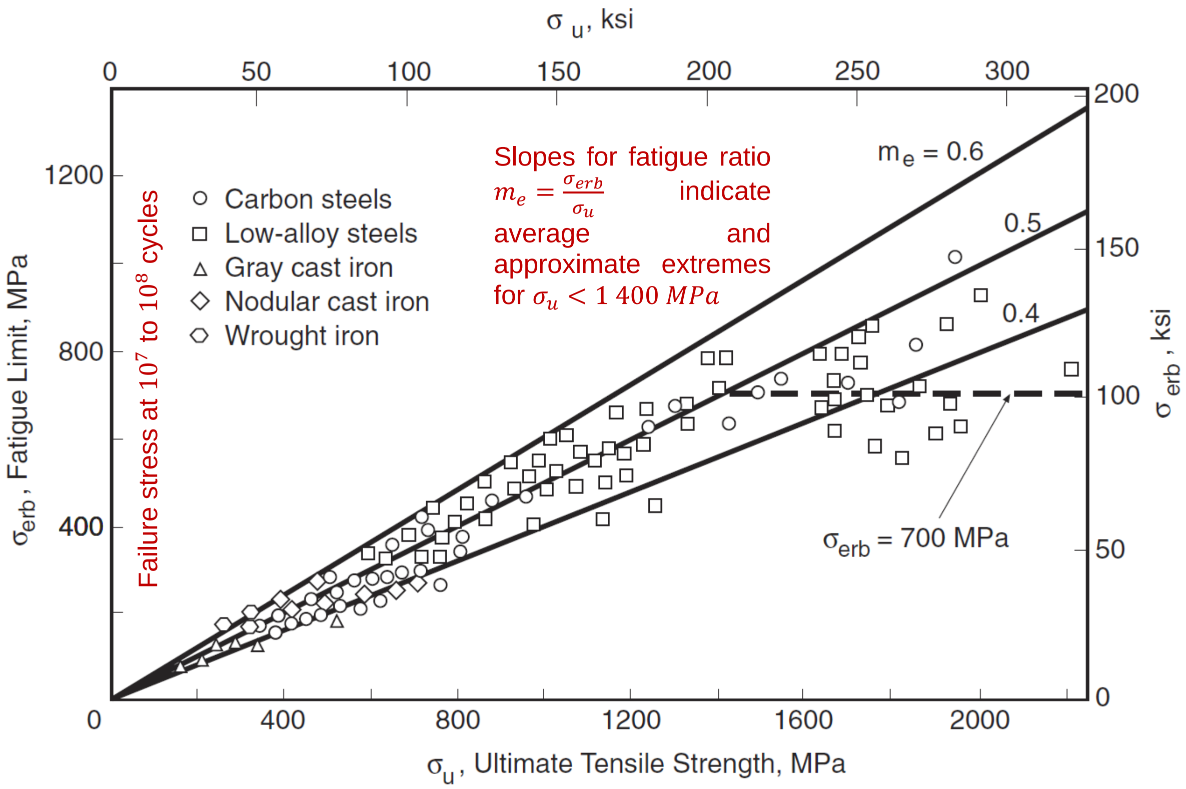
Figure 4: Rotating bending S-N curve for unnotched A517 steel specimens

6.2.1. S-N curves for metals



Source: (Dowling, 2013, pp. 442, Figure 9.23 - note permissions)

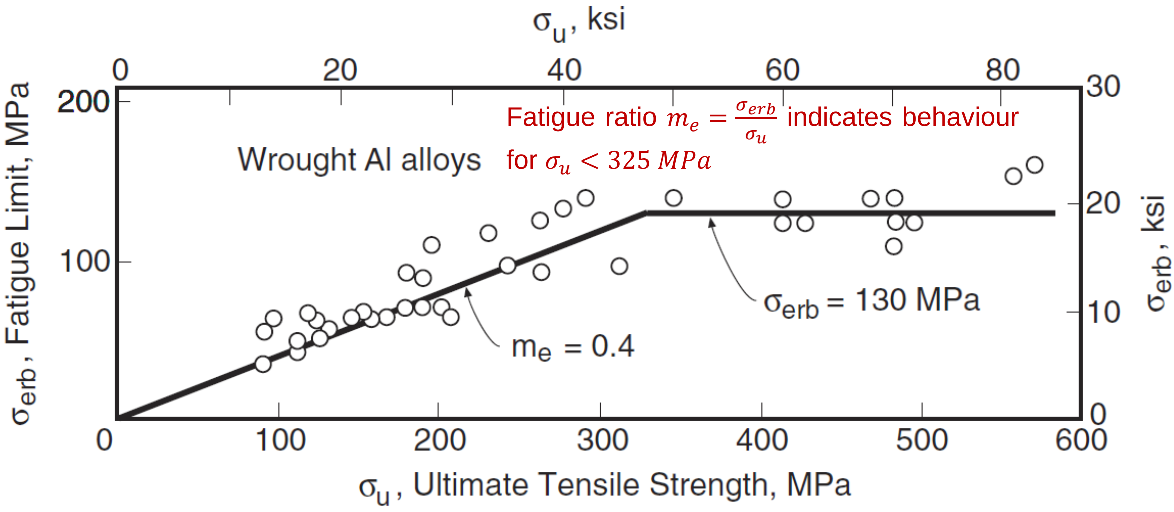
Figure 5: Stress-life curves from cantilever bending of mineral and glass thermosets and unfilled thermoplastics



Slopes $m_e = \frac{\sigma_{erb}}{\sigma_u}$ indicated average and approximate extremes for $\sigma_u < 1\,400\ MPa$

Source: (Dowling, 2013, pp. 442, Figure 9.24)

Figure 6: Rotating bending fatigue limits, or failure stresses from polished specimens



Note: Fatigue strength at 5×10^8 cycles for alloys including 1100, 2014, 2024, 3003, 5052, 6061, 6063, and 7075.

Source: (Dowling, 2013, pp. 443, note permission applicable)

Figure 7: Fatigue strengths in rotating bending for tempers of common wrought aluminium alloys

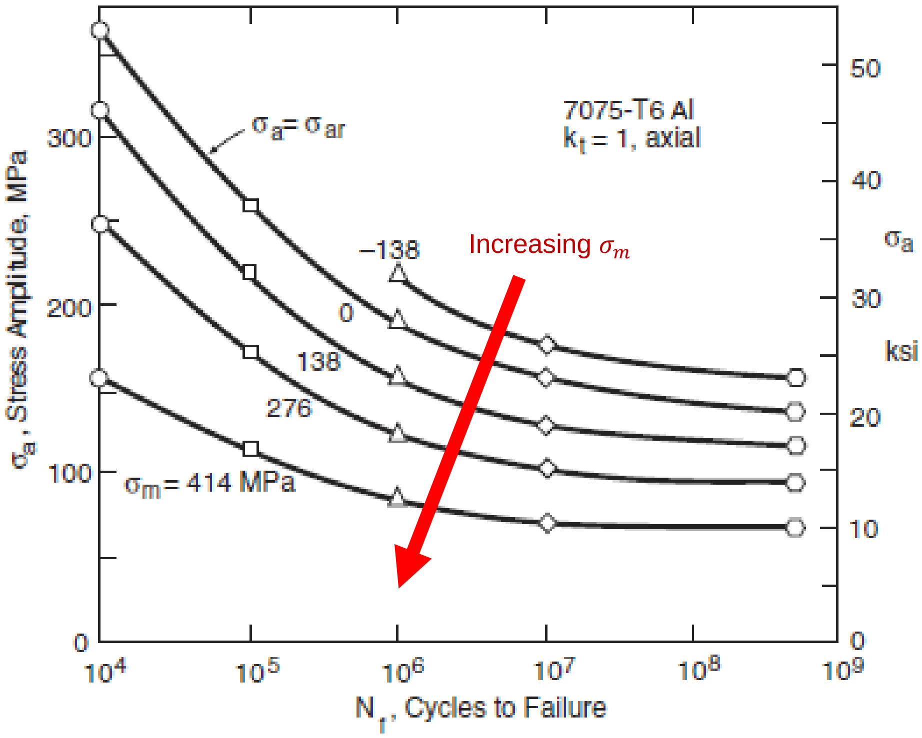
6.2.2. Glass-fibre reinforced thermoplastics

Typically tested under zero-to-maximum tension or bending. In this case the maximum stress is used.

$$\sigma_{max} = \sigma_u(1 - 0.1 \log_{10} N_f) \tag{10}$$

6.2.3. Mean stress

On metals, increasing mean stress reduce fatigue strength.



(Dowling, 2013, p. 444)

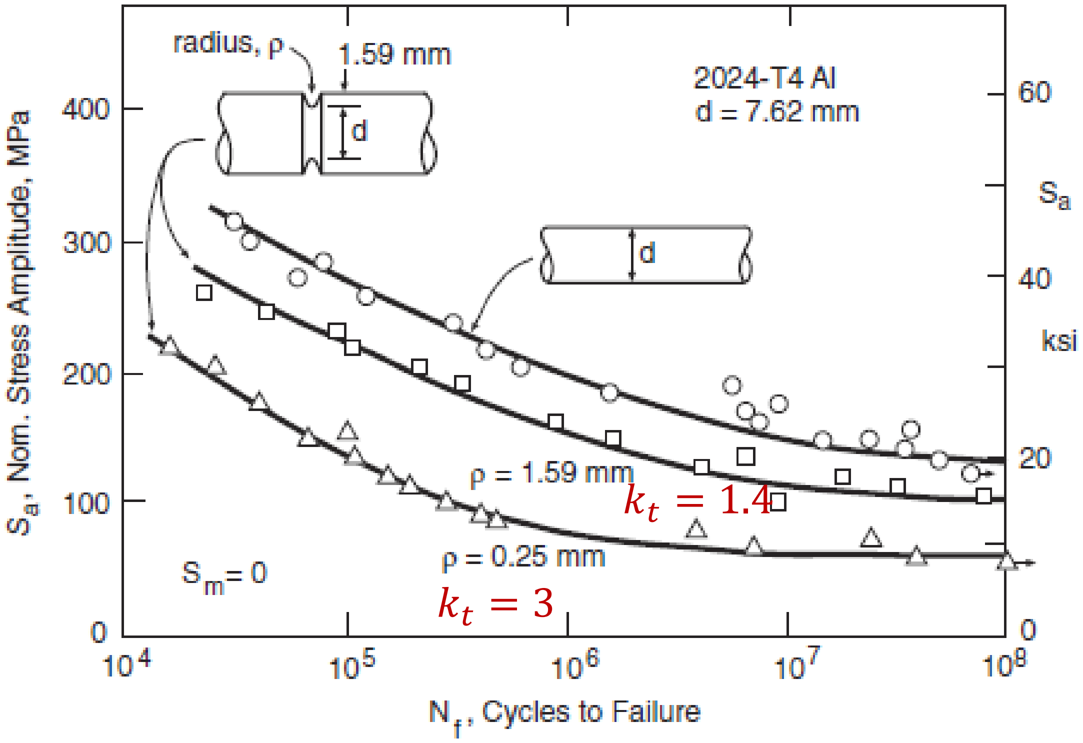
Figure 8: Axial loading S-N curves for unnotched specimens of aluminium alloy

6.2.4. Notched specimens

Notched specimens have a lower fatigue life then unnotched specimens of the same material. High-strength, limited ductility materials are more notch sensitive.

Reason – notches are stress concentrations – causing higher stress.

Note that the nominal stress amplitude is used on the ordinate of Figure 9. If the stress in the notch was used as ordinate, the fatigue curves will be closely spaced. For manual as well as feasible FEA-based fatigue calculations, it is for most geometries efficient to base fatigue assessments on nominal stress.



Source: (Dowling, 2013, p. 444)

Figure 9: Effect of notches on rotating bending S-N curves of an aluminium alloy

6.2.5. Environmental effects and frequency of cycling

Read Dowling Chapter 9.6.2 and take special note of the following:

1. Chemical environments, temperature and the time effect thereof that result in reduced strength under low frequency testing, not because of the strain rate dependence, but, because of delayed affects at the crack tip and elsewhere.
2. Temperature effects and hysteresis in polymers under cyclic loading.

6.2.6. Effects of microstructure

Read Dowling Chapter 9.6.3. and take note of the following:

1. Reducing size of inclusions and voids – enhance fatigue resistance.
2. Reducing grain size – enhance fatigue resistance.
    a. Thorough annealing result in larger grains size that lowers fatigue resistance.
3. Presence of a dense network of dislocations – enhance fatigue resistance.
    a. Effect of cold work by drawing.
4. Lower fatigue resistance normal to the long direction after rolling.
5. In laminated structures – higher fatigue resistance where larger numbers of fibers are parallel to the applied stress, and lower for stresses normal to the plane of laminated structures.

6.2.7. Residual stress

Read Dowling Chapter 9.6.4. and take note of the following:

1. Compressive residual stresses are beneficial.
2. Shot peening: bombarding the surface with small steel or glass shot, producing a compressive surface layer.
3. Presetting: Bending to cause elongation at an outer point on a beam causing a compressive residual stress on release. Note the opposite effect on the side subject to compressive stress.
4. Machining: Smoother surfaces improve fatigue resistance. Some machining procedures can be harmful due to tensile residual stresses.
5. Surface treatments:
    a. May alter the microstructure, chemical composition, and/or residual stress of the surface.
    b. Plating (nickel, chromium)
        i. Introduce tensile residual stress – lowers fatigue strength.
        ii. Deposited material may have poorer resistance to fatigue than base metal.
        iii. Shot peen after plating if required.
6. Welding:
    a. Introduce notches and other stress raisers.
    b. Haver tensile residual stress.
    c. Unusual microstructure distributions.

6.3. S-N curve in the form $S = AN^b$

The following equation can be used to model the situation where S-N data are found to approximate a straight line:

$$\begin{aligned}\sigma_a &= C + D\log N_f \\ &= AN_f^b\end{aligned} \tag{11}$$

The following then applies for any points on the S-N curve:

$$\begin{aligned}\sigma_1 &= AN_1^b \\ \sigma_2 &= AN_2^b \\ \sigma_i &= AN_i^b \\ \frac{\sigma_1}{\sigma_2} &= \left(\frac{N_1}{N_2}\right)^b \\ N_1 &= \left(\frac{\sigma_2}{\sigma_1}\right)^{\frac{1}{b}} N_2\end{aligned} \tag{12}$$

The equation can be written as:

$$\begin{aligned}\sigma_a &= \sigma_f'\left(2N_f\right)^b \\ A &= 2^b\sigma_f'\end{aligned} \tag{13}$$

Table 1 shows the coefficients and exponents for the equation above. The coefficient, σ_f', is the failure strength after one reversal (half cycle) $N_f = 0.5$, to the far left of the S-N curve. Note, this point is extrapolated to the ordinate at $N_f = 0.5$ and then Equation (13) becomes:

$$\begin{aligned}\sigma_a &= \sigma_f'(2 \times 0.5)^b \\ &= \sigma_f'\end{aligned} \tag{14}$$

For short fatigue life under high stress:

- High stresses are involved.
- Plastic strains could also be present.
- In this case:
    - Amplitudes of true stress are needed for quite large strains.
    - Then use, $\sigma_f' = \tilde{\sigma}_f$, the true fracture strength from tension test. The true fracture strength for ductile materials is more than the ultimate tensile strength.

Table 1: Constants for stress-life curves for ductile engineering metals on unnotched axial specimens

Material	Yield strength [MPa]	Ultimate strength [MPa]	True fracture strength [MPa]	$\sigma_a = \sigma'_f(2N_f)^b = AN_f^b$		
	σ_o	σ_u	$\tilde{\sigma}_{fB}$	σ'_f	A	b
Steels:						
SAE 1015 (normalized)	228	415	726	1 020	927	-0.138
Man-Ten (hot-rolled)	322	357	990	1 089	1 006	-0.115
RQC-100 (roller, Q &T)	683	758	1 186	938	897	-0.0648
SAE 4142 (Q& T, 450 HB)	1 584	1 757	1 998	1 937	1 837	-0.0762
AISI 4340 (aircraft quality)	1 103	1 172	1 634	1 758	1 643	-0.0977
Other Metals						
2024-T4 Al	303	476	631	900	839	-0.102
Ti-6Al-4V (solution treated and aged)	1 185	1 233	1 717	2 030	1 889	-0.104
Notes: 1. Units are in MPa except for the dimensionless exponent b. 2. Parameters obtained by fitting test data for unnotched axial specimens tested under completely reversed **axial loading**.						
Source: (Dowling, 2013, pp. 424, Table 9.1)						

6.4. S-N curve in the form $C = S^m N$

The S-N curve for 300W steel shown in Figure 10 (for 50 % probability of failure) is modelled by the equation:

$$S_a^m N = C \qquad (15)$$

Where

S_a is the completely reversed stress amplitude, in MPa

For steel, the limits of the S-N curve are:

- a life of 1 000 cycles at completely reversed stress amplitude $0.9\sigma_u$, and,
- endurance limit at 10^6 cycles at completely reversed stress amplitude $0.5\sigma_u$.

These ratios are available for other materials as well and will be discussed in the applicable parts of the course.

The S-N curve constructed from these parameters predicts an endurance, N, with a 70% confidence level of a 50% probability of survival.

With the exponent m and endurance limit S_e, or any other stress amplitude endurance combination known, the S-N curve can be constructed for endurance as follows:

$$N_R = \begin{cases} \left(\frac{S_1}{S_a}\right)^m N_1 & 0.9f_{ut} \geq S_R \geq S_e \\ \infty & S_R < S_e \end{cases}$$

This equation is used with the Palmgren Miner rule to calculate damage for any probability of failure and for any stress spectrum.

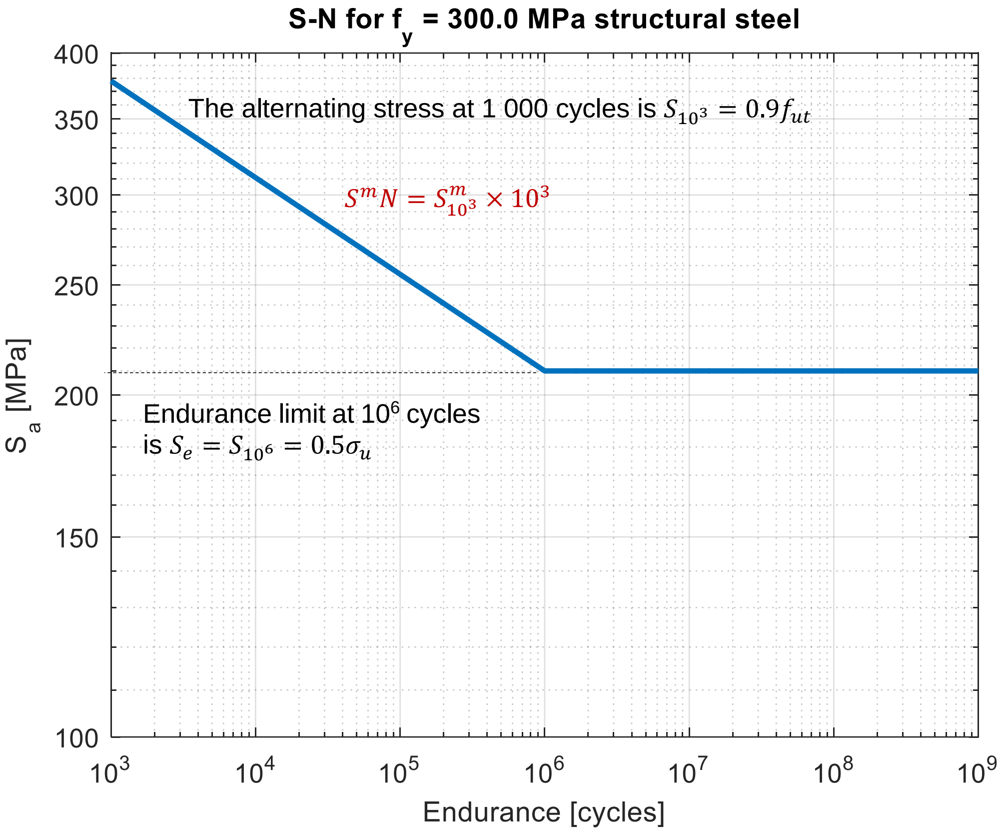


Figure 10: Completely reversed S-N curve for structural steel with yield strength 300 MPa for a confidence level of 70% of 50% probability of survival for N

Using the coefficient of the one reversal strength to the left of the S-N curve, σ_f' at $N_f = 0.5$, the parameters above becomes:

$$\sigma_f'^m \times 0.5 = (0.5\sigma_u)^m \times 10^6$$
$$\sigma_f' = \left(\frac{10^6}{0.5}\right)^{\frac{1}{m}} \cdot 0.5\sigma_u \quad (16)$$

With σ_f' available for the material, the slope of the S-N curve can then be calculated.

6.4.1. Example

Construct the S-N curve for a material with completely reversed stress amplitude $S_1 = 500\ MPa$ at $N_1 = 10^3$ and endurance limit $S_e = 150\ MPa$ at $N_e = 10^6$ cycles.

<u>Solution</u>

Two points are given on the S-N curve, for which the following equation applies:

$$S_1^m N_1 = S_2^m N_2 \quad (17)$$

The exponent, m, can be calculated as follows:

$$\left(\frac{S_1}{S_2}\right)^m = \frac{N_2}{N_1}$$
$$m \log\left(\frac{S_1}{S_2}\right) = \log\left(\frac{N_2}{N_1}\right)$$
$$m = \frac{\log\left(\frac{N_2}{N_1}\right)}{\log\left(\frac{S_1}{S_2}\right)} \quad (18)$$
$$= 5.74$$

Now, the endurance, N_R at any completely reversed stress amplitude, S_a is as follows

$$N_R = \begin{cases} \left(\frac{S_1}{S_a}\right)^m N_1 & 0.9 f_{ut} \geq S_R \geq S_e \\ \infty & S_R < S_e \end{cases} \quad (19)$$

6.5. Non-ferrous alloys

The pseudo-endurance limit is specified as the stress value at 500 million cycles.

6.6. Fatigue ratio

The fatigue ratio is the ratio of endurance limit to ultimate tensile strength:

$$f_r = m_e = \frac{S_e}{\sigma_u}$$

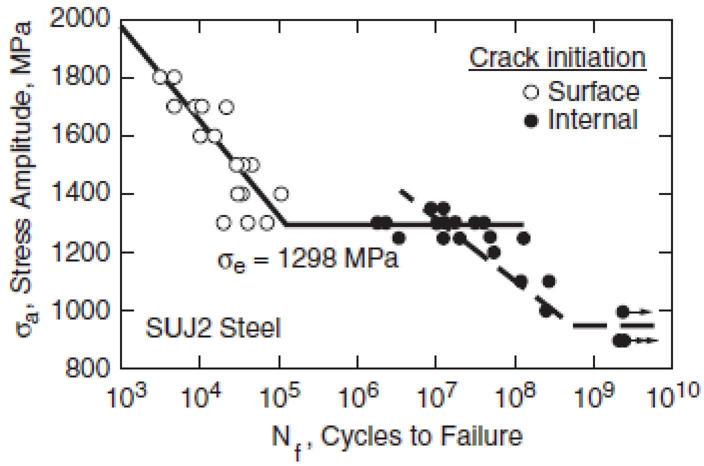
For steel f_r varies between 0.35 and 0.6. Most steels with $\sigma_u < 1\,400\ MPa$ have a fatigue ratio of 0.5.

6.6.1. Fatigue limit behaviour and latest research on endurance (or fatigue) limit

Refer to Dowling Chapter 9.6.5 (Dowling, 2013, p. 448).

For materials with a distinct fatigue limit, recent research showed that:

1. Test data extending 10^9 cycles and beyond showed a surprising drop in the stress-life curve beyond the flat (fatigue or endurance limit) region in the 10^6 to 10^7 cycles range.
2. Two computing methods for fatigue failure exists:
   a. Failure that begins from surface defects.
   b. Failure that begins from internal non-metallic inclusions.
3. Problems with the fatigue limit:
   a. Fatigue limit occurs because the progressive damage process is difficult to initiate below this level.
   b. Fatigue process CAN somehow start & proceed below the fatigue limit.
      i. Corrosion cause small pits – stress raisers – cracks initiate.
      ii. Small number of severe cycles can cause damage that then propagate to failure by stresses below the fatigue limit.
      iii. This occurs for all steels with a distinct fatigue limit, and similar behaviour likely for any metal with a distinct fatigue limit.
      iv. Sequence effects: where prior loading at one stress level affects the behaviour at the second stress level.



Bearing steel with hardness 778 Vickers and $\sigma_u = 2\,350\ MPa$, containing 1% C & 1.45% Cr

Tested in air at 52.5 Hz using cantilever-type rotating bending apparatus

Source: (Dowling, 2013, p. 449)

Figure 11: Stress-life curve for bearing steel

The remainder of the section provides estimates of endurance limits that can be used in calculations.

6.6.2. Endurance limit and surface hardness

As function of **surface hardness** of material:

$$S_e = \begin{cases} 0.25BHN\ ksi & for\ BHN \le 400 \\ 100\ ksi & for\ BHN > 400 \end{cases}$$

6.6.3. Endurance limit and ultimate tensile strength of steel and cast steel

Steel

$$S_e = \begin{cases} 0.5\sigma_u & for\ \sigma_u \le 200\ ksi\ (1\ 400\ MPa) \\ 100\ ksi\ (700\ MPa) & for\ \sigma_u > 200\ ksi\ (1\ 400\ MPa) \end{cases}$$

Cast Iron + Cast Steels:

$$S_e = \begin{cases} 0.45\sigma_u & for\ \sigma_{ut} \le 600\ MPa \\ 275\ MPa & for\ \sigma_{ut} > 600\ MPa \end{cases}$$

Table 2 summarises more fatigue limits for materials according to Dowling.

Table 2: Estimates of smooth specimen fatigue limit

Material	m_e	N_f [cycles]
Aluminium alloys	0.40	5×10^8
Low- and intermediate-strength steels	0.50	10^6
Cast irons	0.40	10^7
Wrought magnesium alloys	0.35	10^8
Titanium alloys	0.5	10^7
Source: (Dowling, 2013, p. 502)		

6.6.4. Pure shear versus bending

For pure shear:

$$\tau_{er} = \frac{\sigma_{erb}}{\sqrt{3}} = 0.577\sigma_{erb} \tag{20}$$

6.6.5. Other materials

Deriving constants for other materials:

- Not so simple because of potential non-linear S-N characteristics
- Linear approximation used in most applications due to empirical data used in analysis – statistical errors exists already

6.7. Typical EN 1993-1-9 $S_r - N$ curve

The fatigue curves used in EN 1993-1-9 standard has the form shown in Figure 12. Please populate the curve with information supplied in the slides.

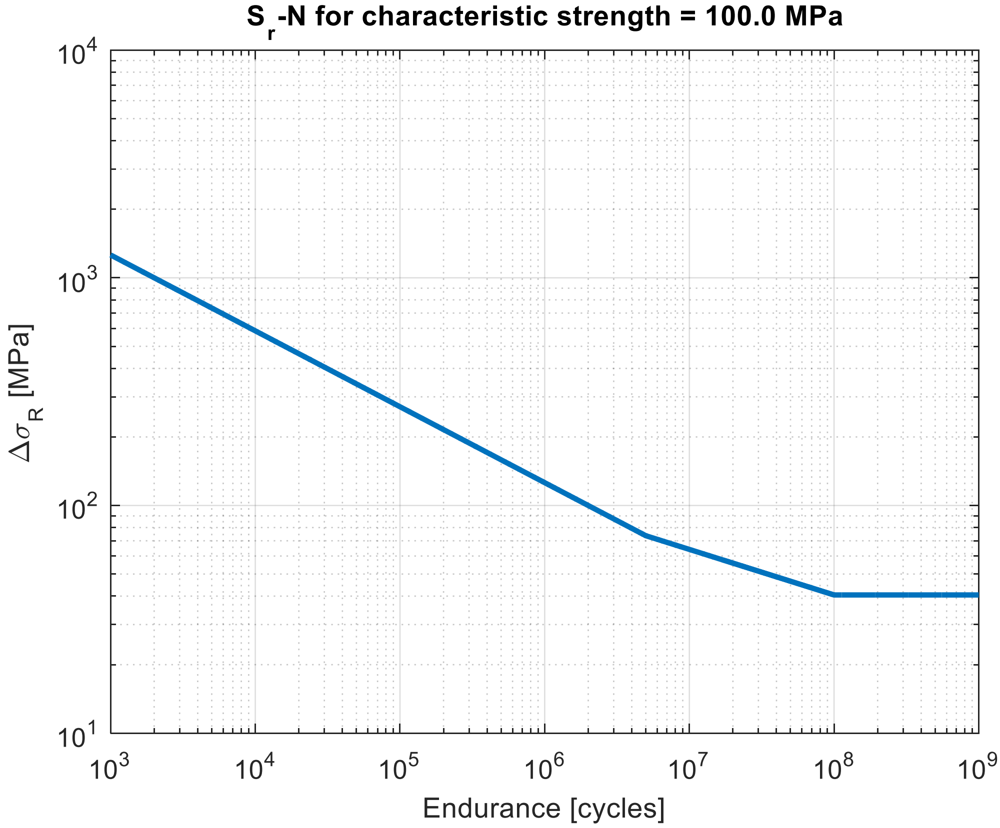


Figure 12: Typical EN 1993-1-9 $S_r - N$ curve for characteristic strength $\Delta\sigma_C = 100\ MPa$

6.7.1. Using the idea

Calculate the constant amplitude fatigue limit and the cut-off limit for a detail category 100, i.e., the characteristic strength is $\Delta\sigma_C = 100$ MPa.

Solution

Make your own notes from information supplied in class.

6.8. Damage modelling and summing

6.8.1. Stress spectrum from cycle counting

After cycle counting, a typical stress spectrum has the following form

Table 3: Example stress spectrum obtained by rainflow counting of the stress signal over a period of 2 weeks

Stress amplitude σ_a [MPa]	Stress mean σ_m [MPa]	Number of cycles n	Stress range $\Delta\sigma = 2\sigma_a$	Goodman correction $\sigma_{ar} = \frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_{ut}}}$	Endurance, for given $P_{survival}$ N	Damage d
100	0	1 000				
200	-100	5 000				
50	50	100 000				
300	100	50 000				
					Total damage, $D = \sum d_i$	
					Number of repetitions, $B_f = \frac{1}{D}$	
					Fatigue life, $L = Period \times B_f$	

6.8.2. Linear damage rule – Palmgren-Miner's rule_

The linear Palmgren-Miner's damage rule states that the damage due to a stress amplitude is equal to the number of cycles that the stress amplitude is applied divided by the endurance (number of cycles to crack initiation) at that stress amplitude.

$$D_i = \frac{n_i}{N_i} \quad (21)$$

If there are more than one stress amplitude applied to the component, then the total damage is the sum of the damages due to the individual stress conditions, as shown in Figure 13.

$$D_T = \sum_{i=1}^{k} \frac{n_i}{N_i} \quad (22)$$

Test results showed that crack initiation occurs when the total damage range between 0.5 and 2.0. For calculation purposes, failure will be assumed when $D_T = 1.0$.

$$0.5 \leq D_T \leq 2.0$$

It is convenient to calculate the number of repetitions, B_f, that a stress spectrum is applied until fatigue damage is expected. In this case:

$$B_f \left[\sum_{i=1}^{k} \frac{n_i}{N_i}\right] = 1$$

$$B_f = \frac{1}{\sum_{i=1}^{k} \frac{n_i}{N_i}} \quad (23)$$

Some textbooks use the term 'blocks' to refer to the repetitions.

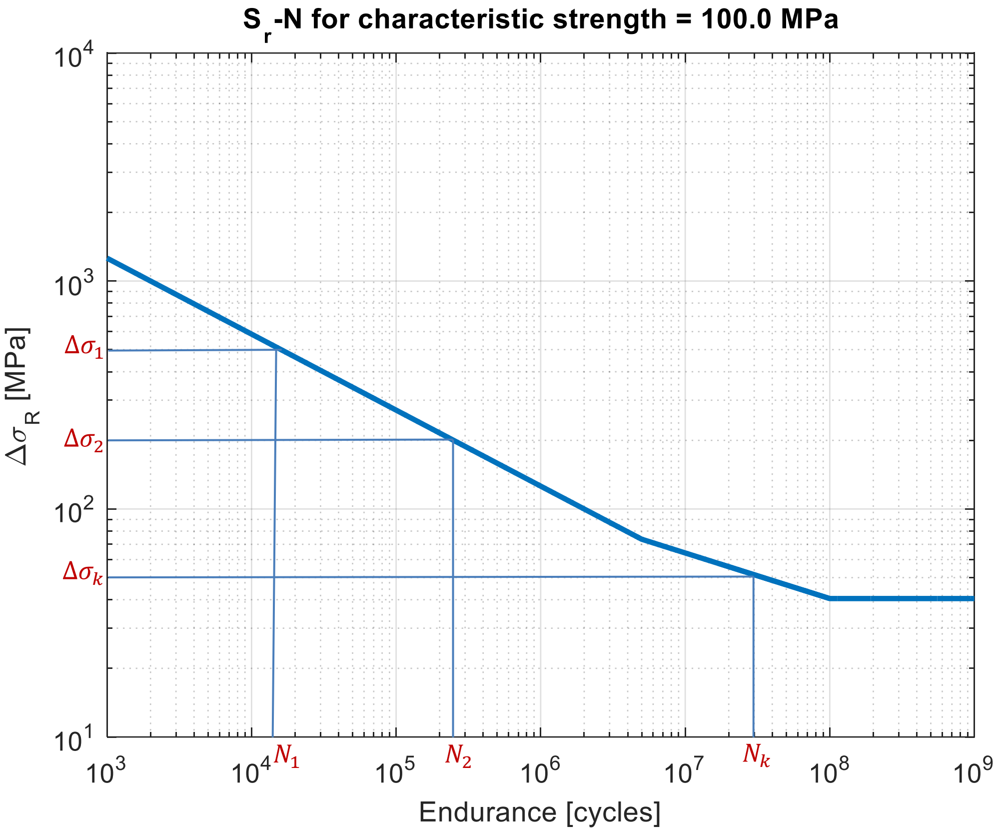


Figure 13: Linear cumulative damage - Palmgren-Miner's rule on an $S_r - N$ curve

6.8.3. Shortcomings of the linear damage rule

- It does not consider sequence effects
- It is amplitude independent
  - Predicts that the rate of damage accumulation is independent of stress level. Observed behaviour show that at high strain amplitudes cracks will initiate in a few cycles, whereas at low strain amplitudes almost all the life is spent initiating a crack.

Non-linear damage theories

- Practical problems
  - Require material and shaping constants which must be determined experimentally
  - Sequence effects must be tested for
- Cannot be guaranteed that these methods will be more accurate than Miner's rule

6.8.4. Summary

- Use the Palmgren-Miner rule
- Non-linear techniques not significantly more accurate
- Damage summation techniques must account for load sequence effects (mean stress, residual stress) ⇨ use strain life to account for initially high stresses

6.9. Safety factors for S-N curves

Two factors of safety are used:

- Safety factor in stress

  In this case the fatigue strength at the actual number of cycles is divided by the actual stress at the same endurance ($N_f = N_{actual}$):

$$X_S = \frac{\sigma_{a1}}{\sigma_{actual}} \tag{24}$$

  Should be similar in magnitude to other stress-based safety factors: 1.5 to 3.
- Safety factor in life

Compare the endurance (fatigue life), N_{f2}, with the actual life, N_{actual}, at the applied stress: $\sigma_a = \sigma_{actual}$:

$$X_N = \frac{N_{f2}}{N_{actual}} \tag{25}$$

Much larger than safety factor ins tress. In the range $X_N = 5\ to\ 20$.

Investmech uses predominantly the safety factor in stress, and calculate, if needed, the associated safety factor in life for the relative section on the S-N curve. Probabilistic analysis using the mean and standard deviation is used in design to calculate safety factor for a given probability of failure (that is, a shift of the mean-line S-N curve) and then comparing the fatigue strength on the shifted S-N curve with that on the 50 % probability of failure (or mean-line) S-N curve.

A typical coefficient of variation for fatigue strength is 10%. This implies a standard deviation of 10% x mean. For a 0.1% failure rate (0.1% probability of crack initiation – or 99.9% probability of survival), the S-N curve should be calculated at $(1 - 3.72 \times 0.1 = 0.691 = 69.1\%)$ the original S-N curve. As shown the new S-N curve is 3.72 standard deviations below the original curve. This gives a safety factor in stress of $X_S = \frac{1}{0.691} = 1.45$.

7. NOTCH STRESS ASSESSMENT OF WELD DETAIL

This section presents an introduction to explain the stress concentrations caused by notches. The notches at weld toes and other weld imperfections are the predominant reason for reduced fatigue strength of weld detail. The focus of this section is on the stress distribution at weld toes and roots.

Presentation used in class:	Notch stress assessment of weld detail Filename: Investmech - Structural Integrity (Notch effects of welds) R0.0

7.1. Principle of the notch stress approach

Weld toes, geometric changes, cuts, holes, etc result in an increase in stress. The notch at a weld toe result in higher stress concentrations where cracks can initiate. In the notch stress approach, the stress in the vicinity of the weld toe is calculated for fatigue purposes. When the stress is predicted with finite element analysis, no plastic constitutive material models may be used. Linear elastic constitutive models shall be used to enable usage of the fatigue curves given by the EN 1993-1-9 and BS 7608 and other standards. The following hypotheses were applicable for notch stress analysis:

- Stress gradient approach (Siebel & Stieler, 1955)
- Stress averaging approach (Nieuber, 1973, 1964 & 1968)
- Critical distance approach (Peterson, 1959)
- Highly stressed volume approach (Kuguel, 1961; Sonsino, 1994 & 1995)

The last three methods ae used for weld notch stress analysis.

7.2. Fictitious notch rounding – effective notch stress approach

It is not possible to model an infinitely sharp notch by finite element analysis methods, because of the requirement to have a certain number of nodes on a curve to ensure proper reconstruction thereof. For finite element analysis, infinitely sharp notches are fictitiously rounded as shown in Figure 14. This method is known as the effective notch stress approach.

According to Neuber (Fricke, 2010):

$$\rho_f = \rho + s\rho^* \qquad (26)$$

Where:

ρ is the actual notch radius [m]

ρ^* is the substitute micro-structural length [m]

s is a factor for stress multiaxiality & strength criterion

For welded joints, $s = 2.5$ for plane strain conditions at the roots of sharp notches, combined with the von Mises strength criterion.

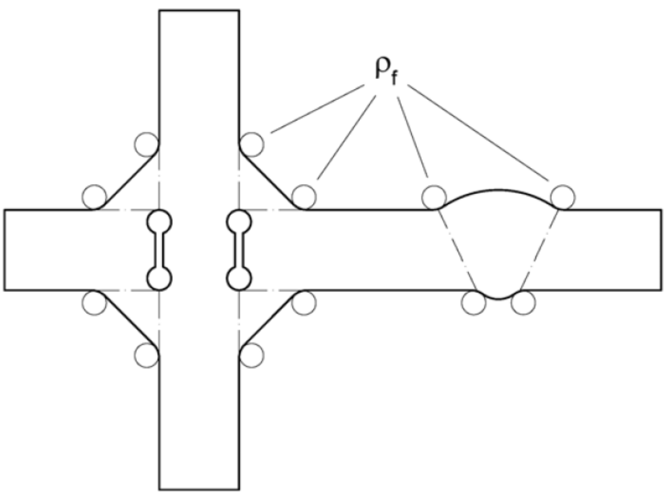
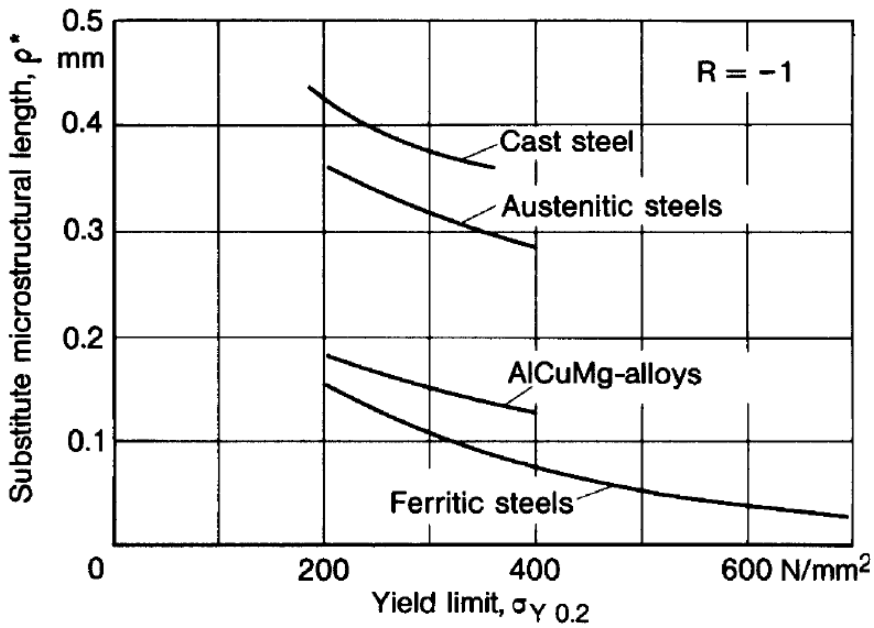


Figure 14: Notch stress application

The substitute micro-structural length for different materials is shown in Figure 15 from which the following can be concluded:

1. For low strength steel, $\rho^* = 0.4$.
    a. This results in an increase of $2.5\rho^* = 1\ mm$ increase in radius.
    b. For the worst case of a notch with actual radius 0, the fictitious radius is: $\rho_f = 0 + 2.5 \times 4.0 = 1.0\ mm$.

Some standards use notch stress fatigue curves for different materials based on one specified notch radius. Use accordingly. Some standards require a notch radius of $50\ \mu m$ on aluminium alloys. Always ensure that you model the notch radius with the minimum specified elements as required by the applicable standard.



Source: Neuber, 1968

Figure 15: Substitute micro-structural length

7.2.1. Critical distance approach

The critical distance approach employs material constants and notch radius to reduce the elastic stress concentration factor K_t to the fatigue notch factor K_f. This will be discussed in detail as part of stress life analysis later in the course.

7.3. Simple design S-N curve according to IIW Bulletin 520

The slide below summarises the fatigue curve with the applicable characteristic values in the table. This enables efficient estimation of fatigue life when notches are modelled.

7.4. Design S-N curve

Both BS 7608 and IIW Bulletin 520 has fatigue curves for use with stress calculated from a notch stress approach with a specified effective notch radius.

Table 4: Characteristic fatigue strength for welds of different materials based on effective notch stress with $r_{ref} = 1\ mm$ (maximum principal stress)

Material	Characteristics strength ($p_s = 97.7\%, N = 2 \times 10^6$)	Reference
Steel	FAT 225	Olivier et al (1989 & 1994) and Hobbacher (2008)
Aluminium alloys	FAT 71	Morgenstern et al. (2004)
Magnesium	FAT 28	Karakas et al. (2007)
Source: (Fricke, 2010, p. 18)		

The equation for the S-N curve above the constant amplitude fatigue limit of steel is given as:

$$\begin{aligned} C &= \Delta\sigma^m N \\ C &= FAT^m \cdot 2 \times 10^6 \\ m &= 3 \end{aligned} \quad (27)$$

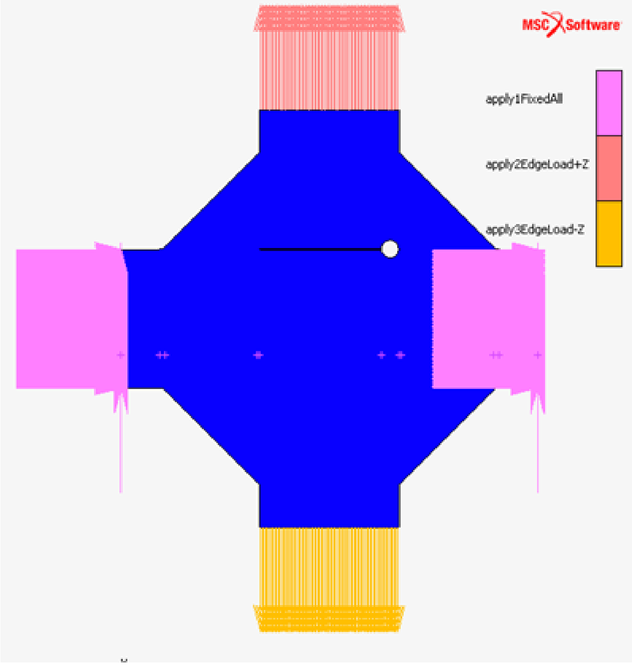
Later in the course more on this.

7.5. Applied to a cruciform joint

The slide below shows the finite element model with boundary conditions of a cruciform joint with:

1. The top weld left end sharp.
2. The top weld right end with notch radius 1.0 mm.
3. Complete joint penetration for the bottom weld.

Remember to always consider the direction of principal stresses in calculations.



Cruciform joint

Material is steel with:

$E = 210\ GPa$

$\nu = 0.3$

$\rho = 7850\ kg/m^3$

Fillet weld at top
Complete joint penetration weld at bottom

Weld toes have radius of 1 mm
Weld root has hole with diameter 2 mm

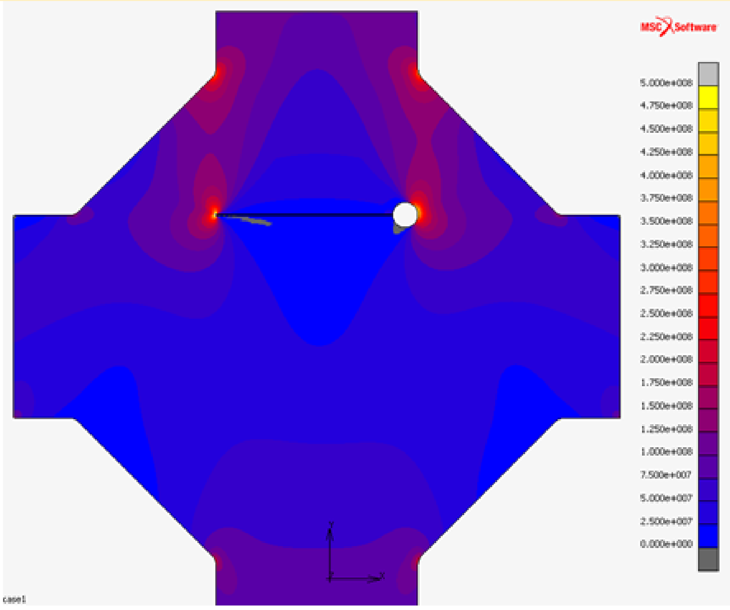
The edges on the horizontal section was constrained for translation in all directions
Edge loads of 100 MPa pressure were applied to the top and bottom sections

For the fillet weld, the weld throat sizes are equal to the cross-section of the 16 mm plate
This give weld leg size 11.2 mm

10

INVESTMECH

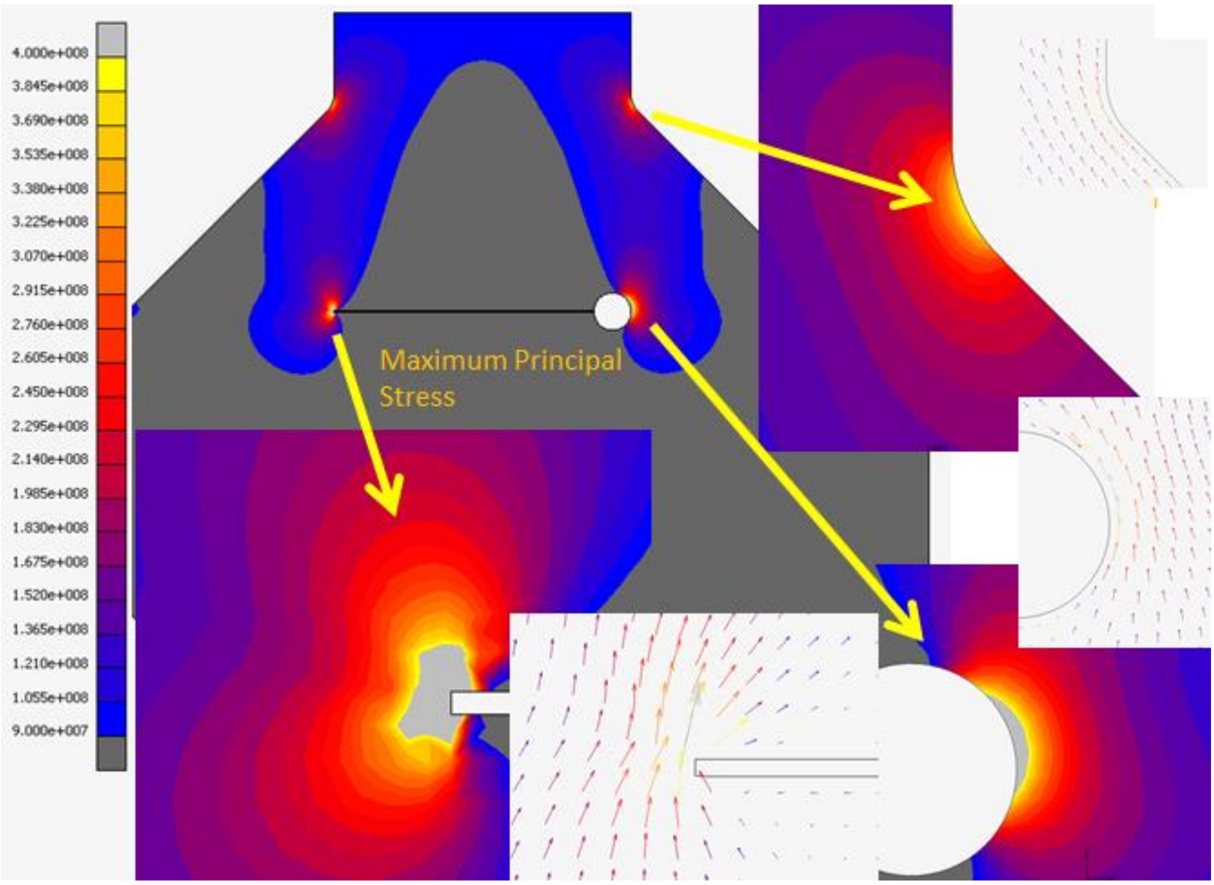
16 mm cruciform joint with 11.2 mm weld



Maximum principal stress in the 16 mm cruciform joint made with 11.2 mm fillet and complete joint penetration welds

Note the stress concentration caused by the geometrical changes at the weld toes and root

11



7.6. References

Fricke, W. 2010. Guideline for the Fatigue Assessment by Notch Stress Analysis for Welded Structures. *International Institute of Welding, IIW-Doc. XIII-2240r2-08/XV-1289r2-08*

8. STATIC FAILURE THEORIES

The objective of this section is to provide revision of static failure theories used as acceptance criteria in design. Investmech assumes that these sections have been discussed in detail in applications by other lecturers and will not be repeated in detail.

This section covers the following:

- Calculation of principal stresses
- Static failure theories
- Buckling
- Variable loading

Always remember to include the design class of the structure in failure theories. For example, multiple load paths classify a structure as fail-safe, and other structures are designed for safe-life. In safe-life applications, damage tolerant design is essential.

Presentation used in class:	Static Failure Criteria Filename: Investmech - Structural Integrity (Static Failure Theories) R0.0

Other references:

1. Dowling, 2014 Chapter 7.

8.1. Principal stress

Stress in any direction $l, m \; and \; n$ can be calculate for a multi-axial stress state using the following equation:

$$\sigma_n = l^2\sigma_x + m^2\sigma_y + n^2\sigma_z + 2\left[lm\tau_{xy} + nl\tau_{xz} + mn\tau_{yz}\right] \tag{28}$$

Where, $l, m \; and \; n$ are the direction cosines of the unit vector directing in the direction in which the stress is required.

The stress tensor transformation is done as follows:

$$\begin{aligned}\sigma_n &= \begin{bmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \begin{bmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \\ &= \boldsymbol{r}^T \boldsymbol{\sigma} \boldsymbol{r}\end{aligned} \tag{29}$$

Where $\boldsymbol{r}$ is the transformation matrix with direction cosines indicated above.

The stress matrix is given as follows:

$$\overline{\boldsymbol{\sigma}} = \begin{bmatrix} \sigma_x & -\sigma_{xy} & -\sigma_{xz} \\ -\sigma_{yx} & \sigma_y & -\sigma_{yz} \\ -\sigma_{zx} & -\sigma_{zy} & \sigma_z \end{bmatrix} \tag{30}$$

Note that $\tau_{lk} = \sigma_{lk}$.

There exists a direction where the shear stress components disappear, and only normal stress remain. These directions and resulting normal stress are the principal axes and principal stresses respectively. The principal stresses are calculated from the eigenvalues of the stress matrix as follows:

$$\begin{gathered}\begin{vmatrix} \sigma - \sigma_x & -\sigma_{xy} & -\sigma_{xz} \\ -\sigma_{yx} & \sigma - \sigma_y & -\sigma_{yz} \\ -\sigma_{zx} & -\sigma_{zy} & \sigma - \sigma_z \end{vmatrix} = 0 \\ \sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \sigma_{xy}^2 - \sigma_{xz}^2 - \sigma_{yz}^2) \\ - (\sigma_x\sigma_y\sigma_z + 2\sigma_{xy}\sigma_{xz}\sigma_{yz} - \sigma_x\sigma_{yz}^2 - \sigma_y\sigma_{xz}^2 - \sigma_z\sigma_{xy}^2)\end{gathered} \tag{31}$$

The eigenvalues, or principal stresses can be easily calculated in PC Matlab using the E=eig(X) command, where X is the stress matrix containing the numerical stress values in the sign convection shown in Equation 30.

For example, say the multi-axial stresses are as follows:

$$\begin{aligned}\sigma_x &= 600 \; MPa \\ \sigma_y &= 0 \; MPa \\ \sigma_z &= 0 \; MPa \\ \sigma_{xy} &= 400 \; MPa\end{aligned}$$

$$\sigma_{xz} = 0\ MPa$$
$$\sigma_{yz} = 0\ MPa$$

Then the matrix X is:

$$X = [600 - 400\ 0;\ -400\ 0\ 0; 0\ 0\ 0]$$

Performing this calculation in Matlab gives:

$$E = \begin{bmatrix} -200 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 800 \end{bmatrix} \quad (32)$$

For a two-dimensional stress problem (e.g. on an unpressurized surface), the maximum (σ_1) and minimum (σ_2) principal stresses can be calculated using the following equations:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} \quad (33)$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2} \quad (34)$$

The principal stresses are then ordered from large to small with $\sigma_1, \sigma_2\ and\ \sigma_3$ the maximum, intermediary and minimum principal stress.

8.2. Principal axes

The principal axes are easily determined by PC Matlab using the command: [V,D]=eig(x). The matrix D contains the eigenvalues and the columns of matrix V (similar to **r** in Equation 28) represent the eigenvectors in the coordinate system used to compile the stress matrix. For the previous values for X is used, this calculation gives:

```
>> [V,D]=eig(X)
V =
  -0.4472        0   -0.8944
  -0.8944        0    0.4472
        0   1.0000        0
D =
 -200    0    0
    0    0    0
    0    0  800
```

In this case the maximum principal stress is $\sigma_1 = 800\ MPa$ in the direction $\overline{r_1} = -0.8944\bar{\imath} + 0.4472\bar{\jmath} + 0\bar{k}$. The minimum principal stress is $\sigma_3 = -200\ MPa$ in the direction $\overline{r_3} = -0.4472\bar{\imath} - 0.8944\bar{\jmath} + 0\bar{k}$.

In the following example the calculation is repeated for an obvious direction of the principal axes:

```
>> X=[600 0 0; 0 200 0; 0 0 100];
>> [V,D]=eig(X)
V =
    0    0    1
    0    1    0
    1    0    0
D =
  100    0    0
    0  200    0
    0    0  600
```

In this case the maximum principal stress is $\sigma_1 = 600\ MPa$ and the minimum stress is $\sigma_3 = 100\ MPa$. The direction of the principal axles is clear in this case. For example, $\overline{r_1} = 1\bar{\imath} + 0\bar{\jmath} + 0\bar{k}$ for the stress $\sigma_1 = 600\ MPa$. That is the, the third column of V aligns with the third column (and row) where D = 600 MPa.

This mathematics are carried out on the multi-axial stress state at a point to find the principal stresses and their associated directions relative to the coordinate system used to construct the stress matrix.

8.3. Static failure theories

8.3.1. Maximum normal stress

In this case the maximum principal stresses are compared against the material resistances. The maximum and minimum principal stress is σ_1 and σ_3 respectively, where σ_1 and σ_3 are not necessarily in tension or compression. Consider the unique situation of hydrostatic stress where $\sigma_1 = \sigma_2 = \sigma_3$. Acceptance criteria are normally the material's yield or ultimate strength or a factor thereof. For example, the partial factor for strength in SANS 10162-1 is $\phi = 0.9$ for structural steel sections. Where both tensile and compressive stresses are present, tensile and compressive material resistance must be used.

In mathematical terms:

$$\sigma_N = \max\left(|\sigma_1|, |\sigma_2|, |\sigma_3|\right) \quad (35)$$

Failure when:

$$\sigma_N = \sigma_u, or\ \sigma_o$$

Safety factor against failure:

$$X = \frac{\sigma_u}{\sigma_N}$$

8.3.2. Maximum shear stress (Tresca)

The maximum shear stress is:

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \quad (36)$$

Failure occurs when the maximum shear stress exceeds the fracture stress, σ_f:

$$\begin{aligned} &\tau_{max} \geq \frac{\sigma_f}{2} \\ &or \\ &|\sigma_1 - \sigma_2| \geq \sigma_f \\ &|\sigma_1 - \sigma_3| \geq \sigma_f \\ &|\sigma_3 - \sigma_2| \geq \sigma_f \end{aligned} \quad (37)$$

Normally the yield strength, ultimate strength or factors thereof are used for the fracture strength.

Under hydrostatic compression testing ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma_h$), test results indicate infinite strength. Under hydrostatic tension testing (difficult to do), use the maximum principal stress criterion.

8.3.3. Von Mises failure theory

The Von Mises theory is used to predict yielding of materials under multi-axial stress state. Failure occurs when:

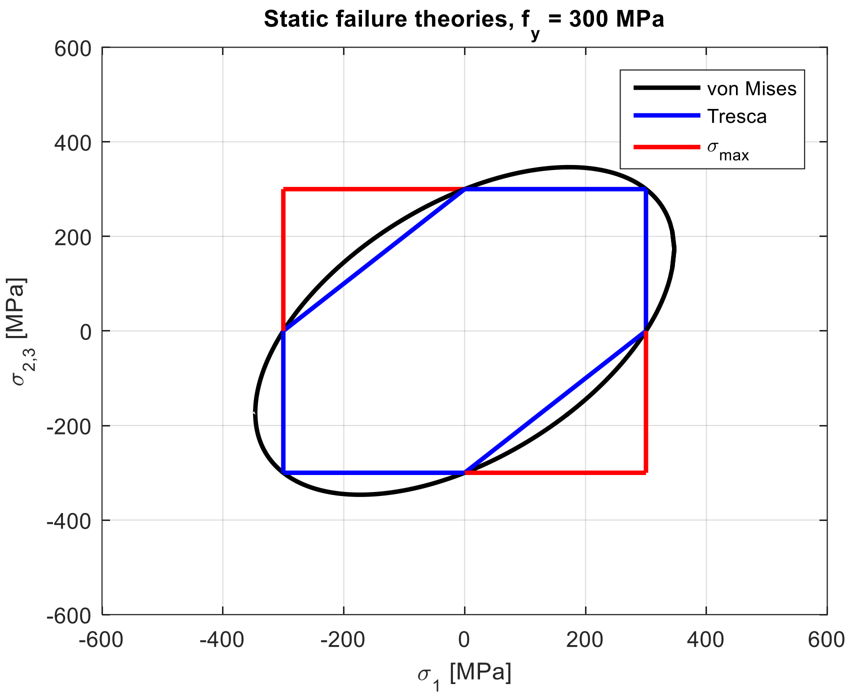
$$\begin{aligned} (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 &\geq 2\sigma_{vm}^2 \\ \sigma_{vm} &= f_y \end{aligned} \quad (38)$$

Where:

$\sigma_{1,2.3}$	Principal stresses [MPa]
f_y	Yield strength of the applicable material [MPa]

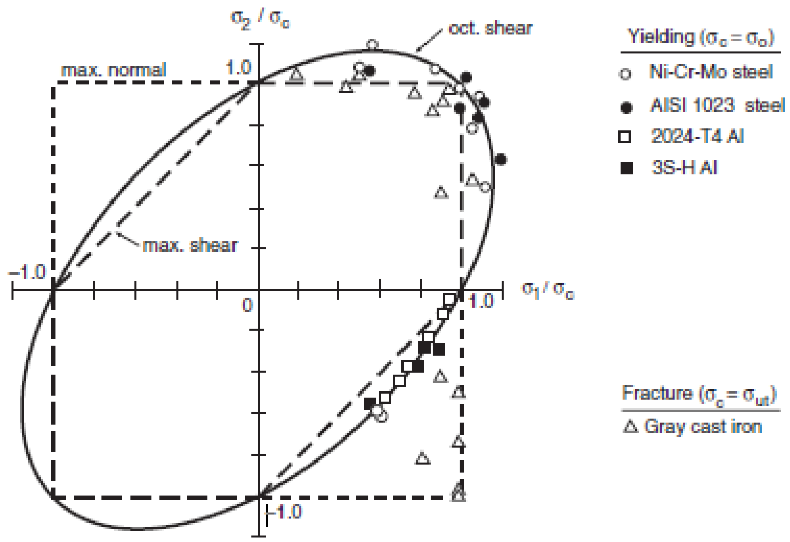
The finite element analysis programs automatically calculate the principal stresses as well as the equivalent Von Mises stress at each node on the mesh.

Figure 16 shows a graph of the static failure theories and Figure 17 show the loci of failures.



Source: Investmech algorithm staticfailuretheories.m

Figure 16: Comparison of failure theories



Source: (Dowling, 2013, p. 296)

Figure 17: Plane stress failure loci for the maximum normal stress, maximum shear stress and octahedral stress criteria

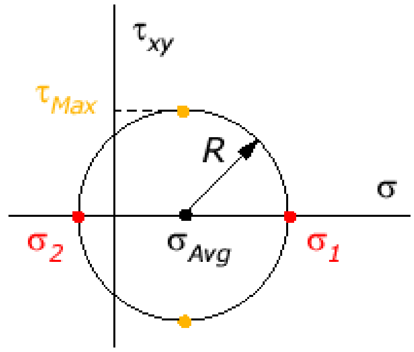
8.3.4. Octahedral shear stress theory

The octahedral shear stress predicts failure exactly as the von Mises failure theory given above.

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} \geq \frac{\sqrt{2}}{3} f_y \quad (39)$$

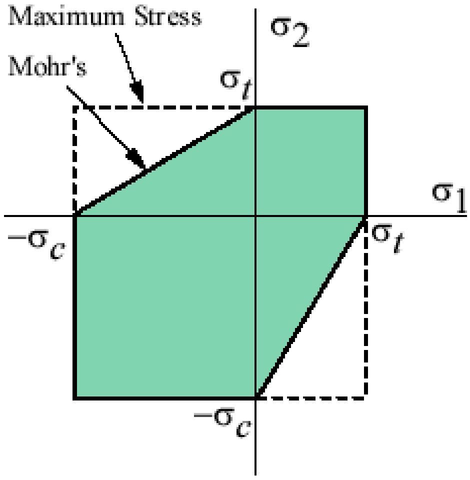
8.3.5. Mohr's failure theory

The Mohr's failure theory is also known as the Coulomb-Mohr criterion or the internal friction theory, and, is based on the Mohr circle shown below. The angle of radius R with the horizontal is 2θ and is used to find the plane of maximum stress. This indicates that if only shear stress is applied, the maximum principal stress will be on an angle of 45° because $2\theta = 90°$.



Case	Principal stresses		Acceptance criterion
1	Both in tension	$\sigma_1 > 0$, $\sigma_2 > 0$	$\sigma_1 < \sigma_t, \sigma_2 < \sigma_t$
2	Both in compression	$\sigma_1 < 0$, $\sigma_2 < 0$	$\sigma_1 > -\sigma_c,\ \sigma_2 > -\sigma_c$
3	σ_1 in tension, σ_2 in compression	$\sigma_1 > 0$, $\sigma_2 < 0$	$\frac{\sigma_1}{\sigma_t} + \frac{\sigma_2}{-\sigma_c} < 1$
4	σ_1 in compression, σ_2 in tension	$\sigma_1 < 0$, $\sigma_2 > 0$	$\frac{\sigma_1}{-\sigma_c} + \frac{\sigma_2}{\sigma_t} < 1$

Mohr's versus the maximum stress criterion will be explained in class according to the sketch below. For brittle materials the compressive strength is higher than the tensile strength.



8.3.6. These are not all

Note that there are several other static failure theories applied by Investmech that are not presented here. For example, additional failure theories include:

- Maximum normal strain theory (St. Venant's theory)
- Total strain energy theory (Beltrami theory)
- Etc.

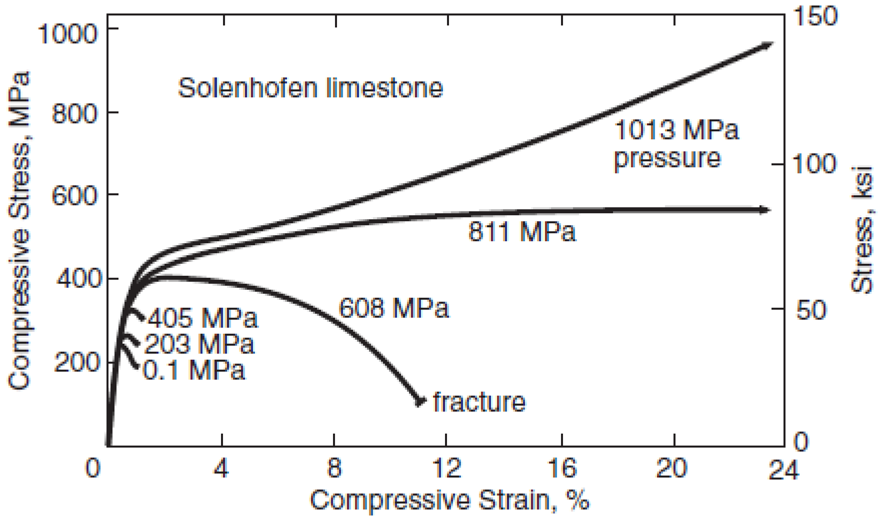
8.3.7. Brittle vs ductile behaviour

- Ductile

Static strength generally limited by yielding
Metals & Polymers

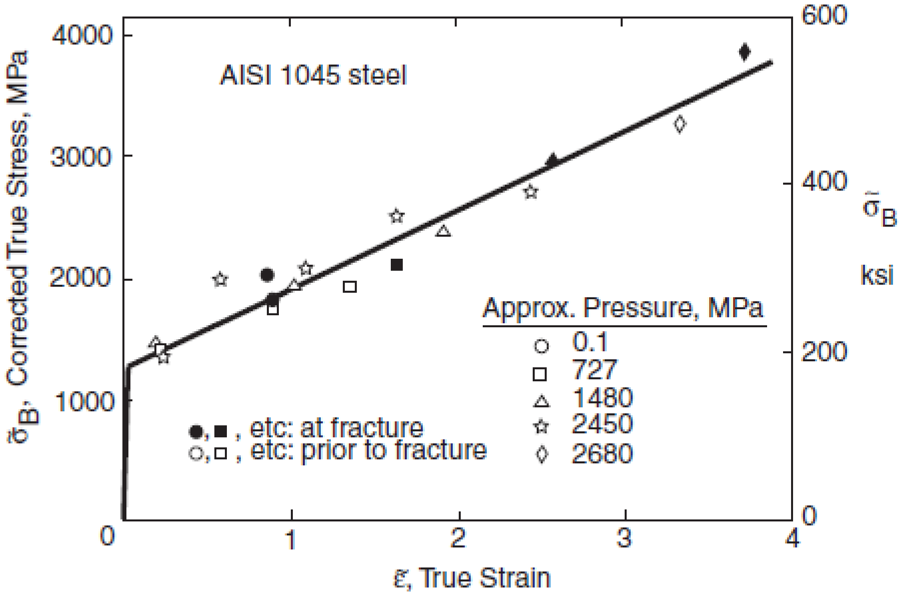
- Brittle

Generally limited by fracture
Gray cast iron & other cast metals
Stone, ceramics and glasses
Concrete
Do not exhibit well-defined yielding behaviour
Fail after small elongation $\leq 5\%$
May exhibit considerable elongation under hydrostatic compressive stress



Source: (Dowling, 2013, p. 319)

Figure 18: Stress-strain data for limestone cylinders tested under axial compression with various hydrostatic pressures



Source: (Dowling, 2013, p. 319)

Figure 19: Effect of pressures ranging from 1 to 26 500 atmospheres on the tensile behaviour of a steel

8.3.8. Which static theory can be used

Research showed that for:

1. Ductile material types (> 5 % elongation at break):
   a. The maximum shear stress criterion and the von Mises criterion are accurate static failure theories
2. Brittle material types (≤ 5 % elongation at break):
   a. The maximum normal stress criterion and the Mohr's theory provide accurate results.

Investmech always applies the maximum principal stress theory and the von Mises theory for static design. The use of the von Mises theory without the maximum normal stress theory could result in wrong answers because of the way in which the von Mises equation eliminates hydrostatic stress conditions.

For crashworthiness, furnace design, or other applications where there is substantial plastic deformation, the maximum engineering strain and true principal strains are used.

8.4. Buckling

Columns under compressive loading have a length dependent critical load where the section will fail under buckling. A vertical column hinged at the bottom has a critical load of:

$$P_{cr} = \frac{\pi^2 EI}{L^2} \qquad (40)$$

Different equations exist for the different boundary conditions and should be applied according to the constraints relevant to the practical problem.

8.5. Non-stress-based criteria

There are several other mission profile driven criteria that also need to be considered in the design of structures. For example:

- Success of parts not necessarily determined by strength
  - Stiffness, vibrational characteristics, fatigue resistance, creep resistance
- Examples
  - Rigidity in automotive vehicles
  - Weight reduction in bicycles
  - Patio deck – stiffness to prevent excessive deformation

8.6. Conclusion

- Always look at all failure criteria, or at least two:
  - The one that prevails first will be the mode in which failure will occur
- For furnaces, the maximum total strain of 20% is used to quantify the number of heat-up cycles
- Fatigue analysis is concerned with the calculation of damage to the structure and is the life until a detectable crack initiate.
- Fracture Mechanics determines failure during the crack growth phase.

9. VARIABLE AMPLITUDE LOADING

The objective of this section is to provide an understanding of the analysis of dynamic loads on structures.

The scope of the work includes:

1. Types of loading.
2. Statistical stress analysis on real structures.
3. Stress collective, S-N curve.
4. Mean stress calculation and its effect

After completion of this section you will be able to:

1. Describe methods of counting load cycles.
2. Calculate stress ration and other statistical parameters.

Presentation used in class:	Variable Amplitude Loading Filename: Investmech - Structural Integrity (Variable Amplitude Loading) R0.0

9.1. Types of loading

Loads can be tensile loads, compressive loads and/or shear loads (torsion) that produce stress and deformation of the component.

Loads can be divided into the following:

- Static load
  - Do not change over time
  - SANS 10160-1 and other standard refer to static loads as dead- or permanent loads or actions.
- Quasi-static load
  - Applied at rate lower than lowest natural period of the structure

  - Investmech uses a load application period of 5 times the lowest natural period as a quasi-static load. The is equal to a frequency of 1/5th the lowest natural frequency.
- Dynamic loads
  - Load application period shorter than 5 × lowest natural period, and typically cause force effects due to natural modes
  - Shock load
    - Impact load period significantly shorter than the natural period

9.2. Causes for loads on structures

There are many causes for variable amplitude loads on structures of which the following is a typical list:

- Thermal
  - Normally quasi-static
  - Can be deterministic
- Process activities
  - Random, but steady-state
- Wind loads
- Cavitation
- Fluid-structural interactions
  - Water-hammer
  - Tidal – normally quasi-static of nature
  - Waves
- Mechanical loads
  - unbalance, misalignment, screen suspension, crusher supports, impact hammers, etc.
- Human interactions
  - Dropping objects, explosions, crushes, etc.
- Cluster events (Storms, process in reactors, accidents, derailing of railway vehicles, etc.)
- Etc.

9.3. Loads and stress-strain

Loads are converted to stress. The stress can be determined in typically the following ways:

- Elementary equations:
  - Normal stress: $\sigma_n = \frac{F}{A}$
  - Bending stress: $\sigma_b = \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$
  - Shear stress (Torsion): $\tau = \frac{VQ}{A} + \frac{Tr}{J}$, note the directions!
- Finite element analysis
- Strain measurement and conversion to stress

9.4. Classification of signal types

A signal is ergodic if its statistical parameters (such as mean and variance) can be determined from a single, sufficiently long sample of the process.

For stationary signals, time and frequency domain analysis is done. Take note of the ergodic and non-ergodic effects.

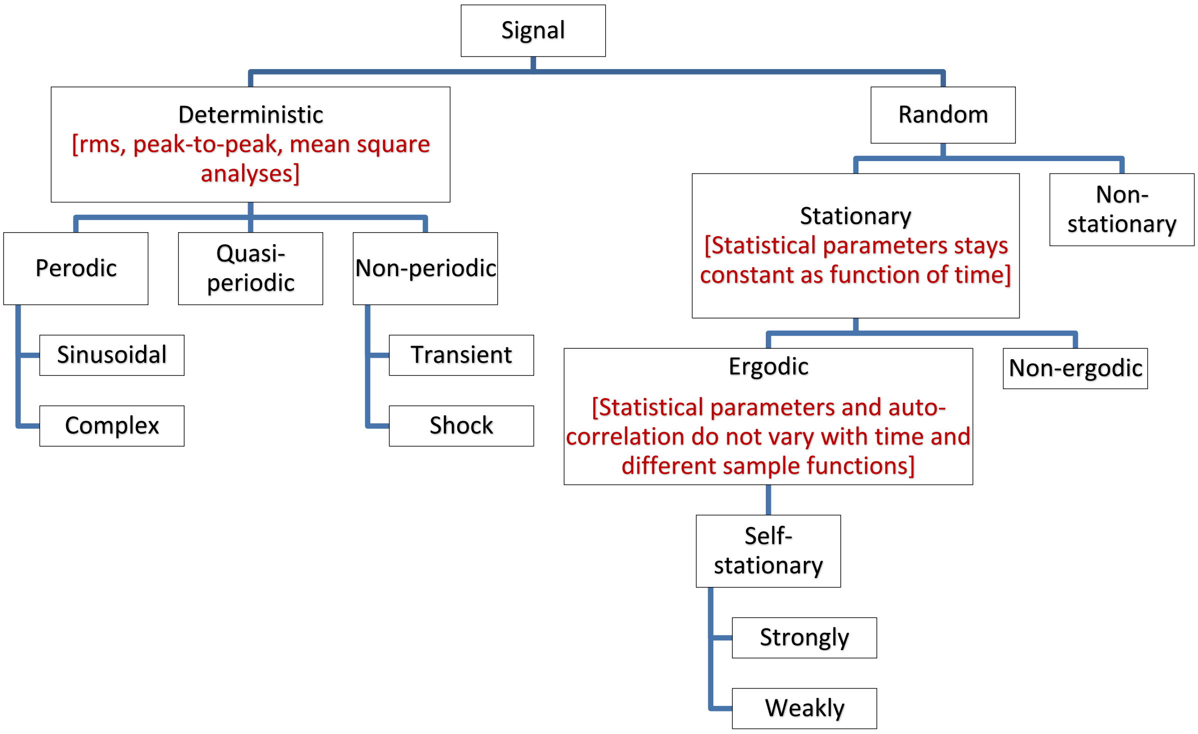


Figure 20: Classification of signal types

9.5. Statistical parameters

9.5.1. *Peak value*

This is the maximum absolute value of the signal $V_P = \max(|signal(t)|)$. In Matlab the command is:

Vp=max(abs(signal));

In most cases the peak value will refer to the maximum value of the signal, and not of the absolute value of the signal. Then it is the highest point on a plot of the signal.

9.5.2. *Peak-to-peak value*

The peak-to-peak value is the same as the range of the signal and is the difference between the maximum and the minimum value:

$$V_{p-p} = \max\big(signal(t)\big) - \min\,(signal(t))$$

9.5.3. *Mean*

$$\mu = \lim_{T\to\infty} \frac{1}{T}\int_0^T f(t)dt$$
$$= \frac{1}{N}\sum_{i=1}^{N} f(i)$$

- For a random signal the mean is zero

9.5.1. *Root-mean-square*

$$RMS = \sqrt{\lim_{T\to\infty} \frac{1}{T}\int_0^T f(t)^2 dt}$$

$$= \sqrt{\frac{1}{N}\sum_{i=1}^{N} f(i)^2}$$

- Gives the intensity of the data which is an indication of the energy
- This is an Overall Value, that is, one value that describes the characteristic of all the values
- With band-pass filter will give narrow-band intensity

9.5.2. Crest factor

$$CF = \frac{Peak\ of\ the\ signal}{RMS}$$

- For a pure sine wave cf = $2^{(1/2)}$
- A cf > 3 indicates on irregularities in the signal
- The cf is not monotome
    - Will not necessarily increase with an increase in RMS
- Used to describe the "peakiness" of a function/signal

9.5.3. Variance and standard deviation

$$\sigma^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T (f(t)-\mu)^2 dt$$

$$= \frac{1}{N}\sum_{i=1}^{N}(f(i)-\mu)^2$$

Variance = (standard deviation)^2 = σ^2
The standard deviation quantifies the distribution of data points around the mean.

9.5.4. Kurtosis

$$KU = \frac{1}{\sigma^4}\lim_{T\to\infty}\frac{1}{T}\int_0^T (f(t)-\mu)^4 dt$$

$$= \frac{1}{\sigma^4}\times\frac{1}{N}\sum_{i=1}^{N}(f(i)-\mu)^4$$

- The kurtosis is not monotome
- Describe the peakiness of a signal
- For sine wave KU=2
- For a random signal KU=1.5

9.5.5. Example

Space allowed for class problem on time domain analysis
Sample record given: -1; -0,5; 0; 0,5; 1; 0,75; 0,5; 0,2; -0,2

Peak-to-peak value

Mean

RMS

Standard deviation

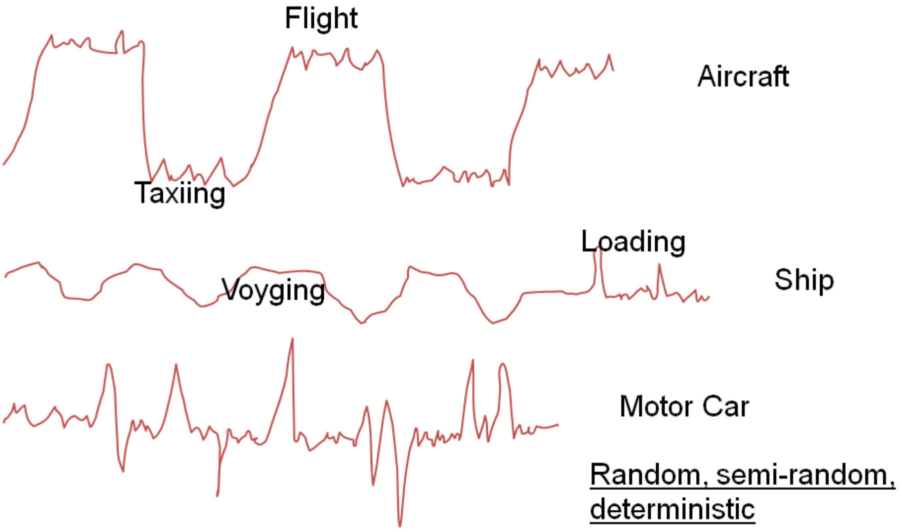
Crest factor

9.6. Presentation of stress data

It is important that stress data is accurate. Stress data for fatigue calculation is normally presented as the stress ranges with the number of occurrences in the signal. That is, a table with the one column containing stress ranges and the next column the number of times that the specific stress range occurred in the signal. Stress data can also be presented as a spectrum using FFT calculations. This is only accurate for steady-state responses. In most cases the actual stress signal is not known exactly, but, can be accurately described by the mean and standard deviation.

9.7. Stress histories

The sketch below provides a list of typical stress histories.



Peak-valley reduction:
To calculate high and low turning points in a signal

9.8. Obtaining stress spectra for fatigue calculations

With computing power today, measured strains converted to stress can be used directly to calculate the stress spectrum for the measurement point.

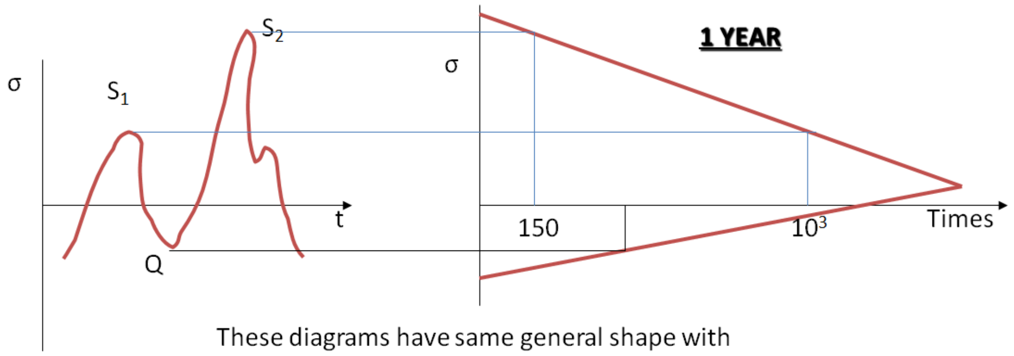
Load spectra for stress measured or from transient analysis:

- PSD or Spectral analysis
  - Note, the results is for a specific period and must be scaled!
- Counting methods
    - Peak counting, Mean-crossing peak count, Range pair count, Range-pair-mean count, Rainflow count, Reservoir counting method
  - The counting method must produce the correct crack initiation and growth result
  - Counting method must detect peak, mean, minimum, and maximum of signal
  - The results are presented in a histogram for $\Delta\sigma_R$ and n_R
  - Note, the results is also only for the duration of the signal measured and must be scaled

9.9. Exceedance diagrams

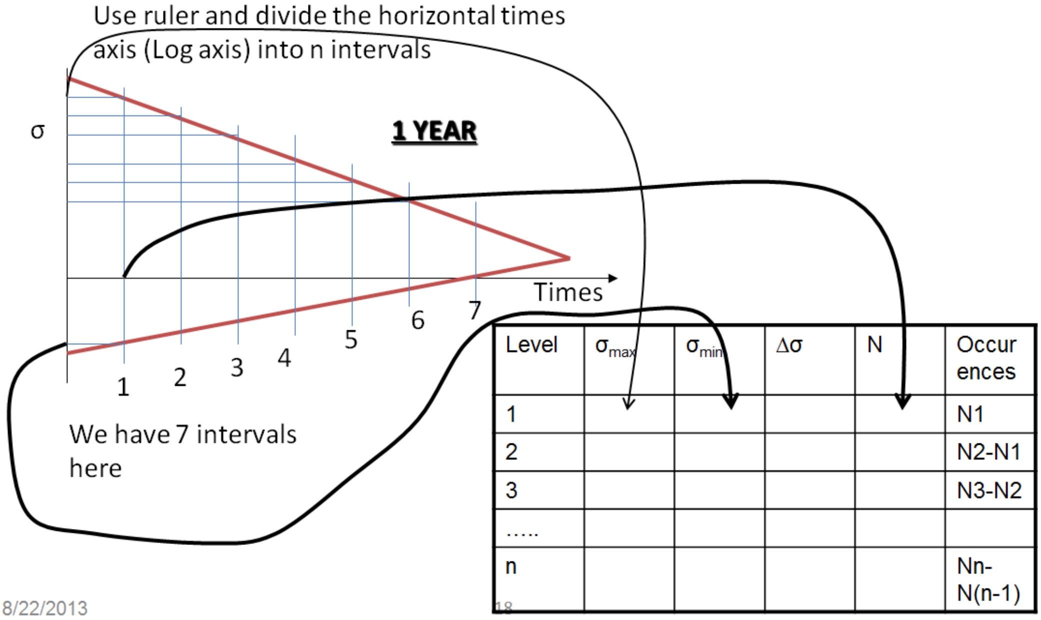
This is beyond the scope of this course and is only presented in class as an example.

The diagram shows how many times a certain stress or load level is exceeded. The method makes use of peaks and troughs.



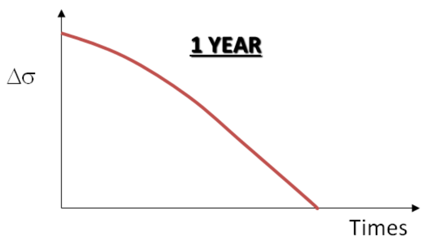
These diagrams have same general shape with deviations on the "straightness" of the lines.

Use ruler and divide the horizontal times axis (Log axis) into n intervals



Level	σ_{max}	σ_{min}	$\Delta\sigma$	N	Occurences
1					N1
2					N2-N1
3					N3-N2
…..					
n					Nn-N(n-1)

- The exceedance diagram could also be constructed in terms of stress ranges – which are typically found from rainflow counting results



<u>Standard spectra:</u>

- Exceedance diagrams from a great number of structures are available and can be used in calculations

9.9.1. Errors and effects

The following is a list of actions carried out on counted cycles that can affect accuracy. However, this is not part of the scope of this course and is only listed for information.

1. Clipping
    a. Counting error because a certain level was not exceeded.
2. Truncation:
    a. Truncation is the process to reduce the number of small cycles to save on computing time
    b. Process reconstructs the lower step
    c. Requires judgement and evaluation of its effect
    d. The larger the number of stress levels, the less the effect of truncation
3. Different periods of severity
    a. For example a vehicle driving on tar and gravel roads. The relative distances are essential to ensure proper design.

9.10. Variable amplitude loading

- Most service loading histories have variable amplitude
- Sometimes stochastic of nature (random probability distribution, may be analysed statistically but cannot be predicted precisely)
- The following aspects need to be addressed:
    - Nature of fatigue damage and how it can be related to load history
    - Damage summation methods
    - Cycle counting techniques to recognise damaging events
    - Crack propagation behaviour under variable amplitude loading
    - How to deal with service load histories
- Fatigue is the tendency of materials to fail due to cracks that initiates and propagates
- Definition of <u>fatigue damage</u>
    - The measurable propagation portion of fatigue
        - Damage is directly related to crack length ⇨ it is observable, measurable
        - Inspection intervals used to monitor crack growth
    - Initiation phase
        - Mechanisms on microscopic level (dislocations, slip bands, micro-cracks, etc.)
        - Only measurable in highly controlled laboratory environment

⇨ **Most damage summing methods during initiation phase empirical of nature**

9.11. Cycle counting methods – discussed

The objective of cycle counting techniques for fatigue analysis is to reduce the data required in analysis. For example, in a strain/stress signal for fatigue analysis, the turning points (extrema) are required, not the data points in between.

9.11.1. Reorder stress history

The first step in the process is to reorder the stress history to start and end at the peak or valley with the maximum absolute value. If there are repetitions of the maximum absolute value, then the reorder the stress history to start and end with the first peak or valley with maximum absolute value.

9.11.2. Level crossing

Number of times that strain or stress of certain value is crossed. Can be done on the original signal or range levels.

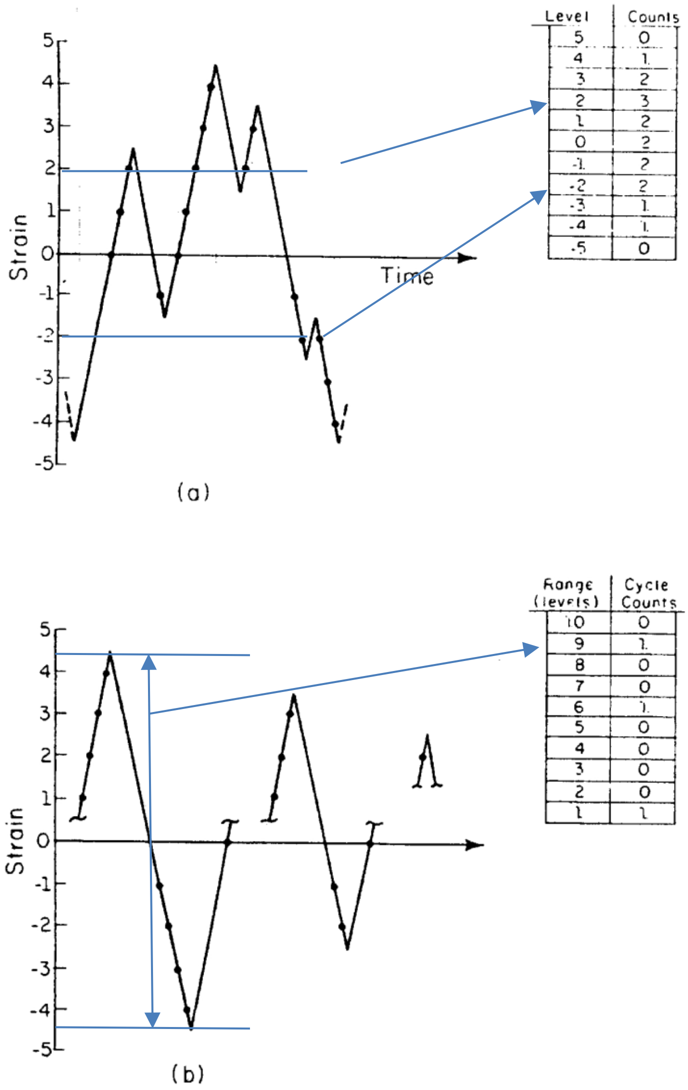
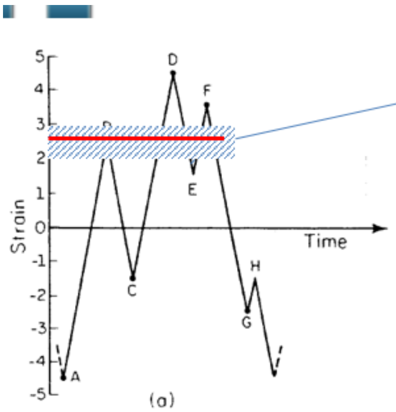


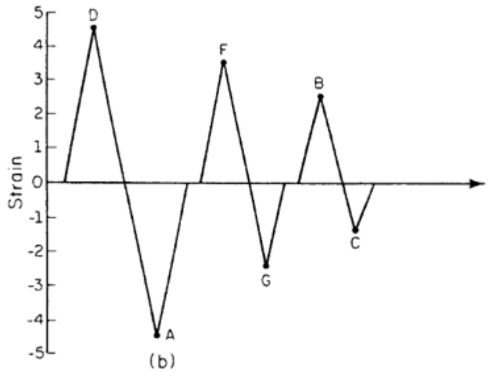
Figure 21: Level-crossing counting

9.11.3. Peak counting

Counts the number of times that a peak is formed in a specified range, 1 in this case. The example below scales down, that is why Point B is indicated to be peak 2. Investmech scales up, so, in this case Peak B would have had value 3. Upscaling is conservative.



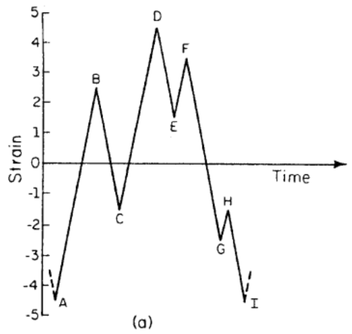
Peak	Counts
5	0
4	1
3	1
2	1
1	0
0	0
-1	1
-2	1
-3	0
-4	1
-5	0

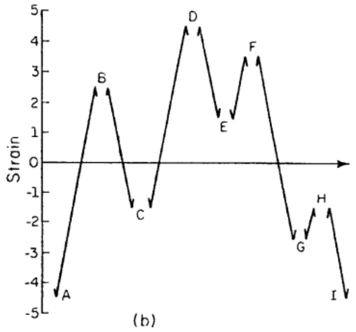


Range (levels)	Cycle Counts
10	0
9	1
8	0
7	0
6	1
5	0
4	1
3	0
2	0
1	0

Figure 22: Peak-counting

9.11.4. Simple-range counting





Range (levels)	Cycle Counts
10	0
9	0
8	0
7	.5
6	1
5	0
4	.5
3	1
2	.5
1	.5

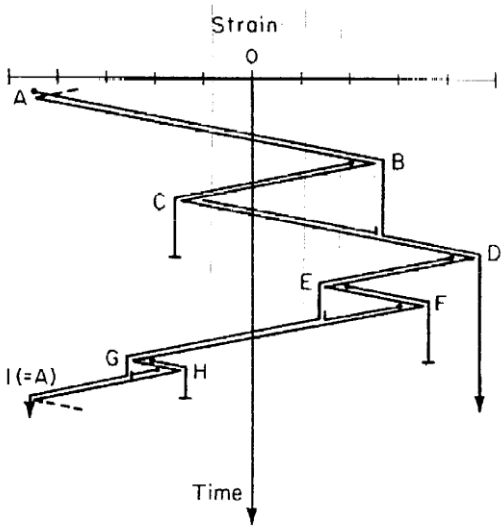
Figure 23: Simple-range counting

9.11.5. Rainflow counting

Rainflow counting, giving similar answers than reservoir counting, is the most widely used counting method for stress life analysis. See the following demonstrations of the method:

1. Demonstration done in class.
2. Videos:

a. https://youtu.be/rIsy4fDXDwM
b. https://youtu.be/Fvk64Qn9K7E



9.11.6. Sequence effects

Strain-time histories may yield very different stress-strain responses, especially in notches where plasticity occurs as shown in Figure 24. Level-crossing, peak-counting and simple range counting do not include sequence effects. If the rainflow counting algorithm is applied in sequence, it does include sequence effects (from-to values in the Markov matrix from which mean, amplitude and range can be calculated), one of the reasons why it is used widely. For these situations, the strain dependent residual strains in the notch need to be modelled, for which strain life principles are used.

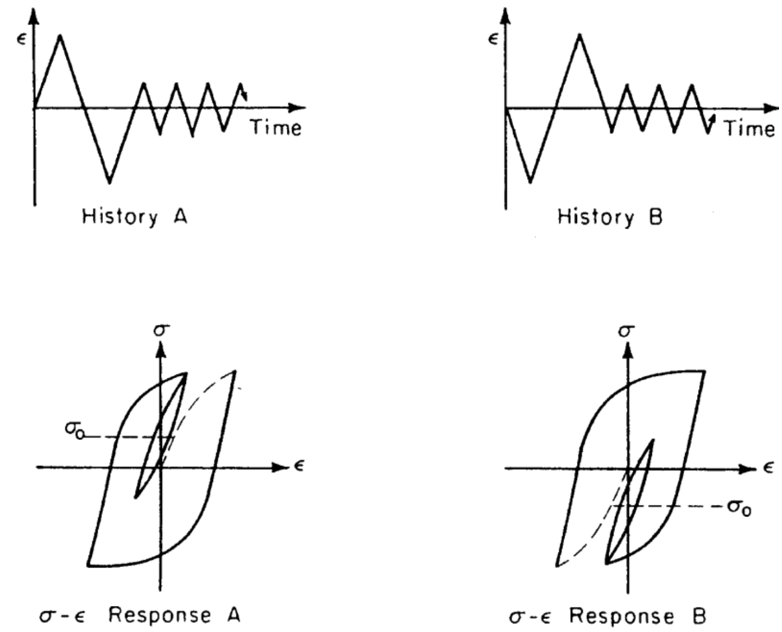


Figure 24: Load sequence effects

9.11.7. More info on Rainflow counting

- A number of rainflow counting techniques are in use
- If the strain-time history being analyzed begins and ends at the strain value having the largest magnitude, whether it occurs at a peak or a valley, all of the rainflow counting techniques yield identical results
- Develop the Markov Matrix and find strain amplitudes and mean stress
- Use the Morrow equation to solve the fatigue life at each strain level:

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f' - \sigma_o}{E}\left[2N_f\right]^b + \epsilon_f'\left[2N_f\right]^c$$

– Calculate cumulative damage from Miner's rule: $D = \sum \frac{0.5}{N_f} \geq 1$, note, reversals = half cycles are used in the Morrow equation
– ASTM standard for Rainflow counting in literature

9.12. Crack growth retardation using load sequence effects

Under variable amplitude loading, and the residual stress caused by preceding load cycles, crack propagation depends on the preceding load cycle that can significantly affect the crack growth rate as shown in Figure 25, from which the following may be concluded:

- Single overload causes a decrease in crack growth rate
    - If overload is large enough, crack arrest can occur
    - Crack growth retardation remains in effect for a period related to the size of the plastic zone
        - The larger the plastic zone, the longer the crack growth retardation remains in effect

If the overload results in tensile residual stress, crack growth acceleration occurs.

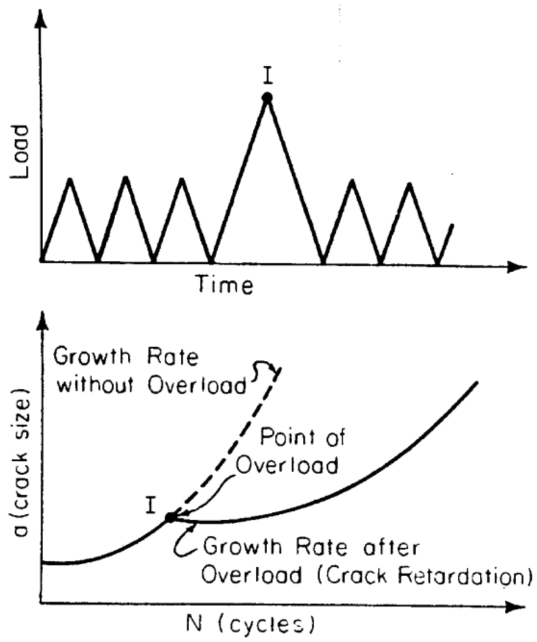
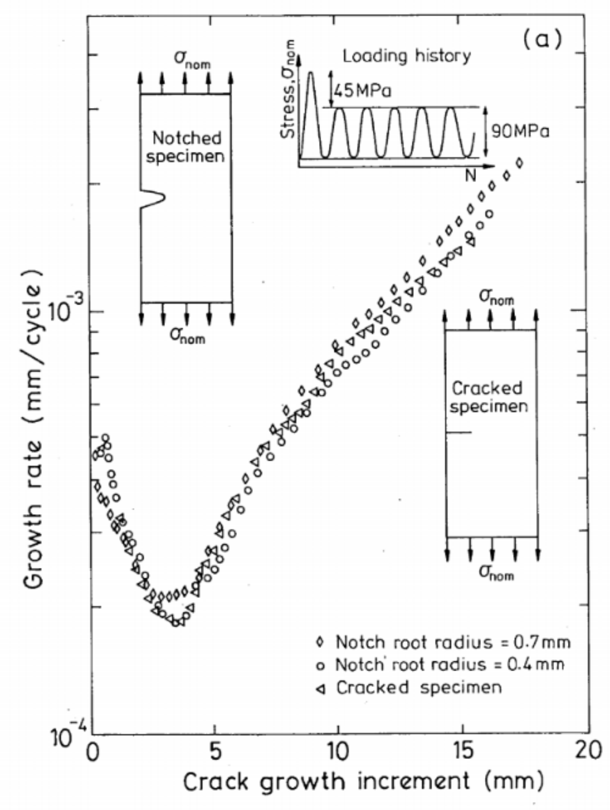


Figure 25: Crack growth retardation due to overloading



Source: (Shin & Fleck, 1987, p. 386)

Figure 26: Crack growth retardation of a structural steel

The Plastic zone size at a crack tip is given by the following equation:

$$r_y = \frac{1}{\beta\pi}\left(\frac{K}{\sigma_y}\right)^2$$

Where $\beta = 2$ for plane stress and $\beta = 6$ for plane stress.

Periodic overloads are not always beneficial. In low-cycle fatigue it may cause crack growth acceleration and need to be modelled applying the correct theories.

Compressive overload (sometimes referred to as underload) generally causes an acceleration in the crack growth rate because of tensile residual stress.

<u>Explanation of the mechanism of crack growth retardation</u>

The mechanism of the load sequence dependent crack growth retardation:

- Crack-tip blunting:
  - The crack tip blunts during overload → stress concentration becomes less severe → slower crack growth rate
  - Is not in line with practise where it was found that crack growth retardation comes in effect after crack has grown a portion through the plastic zone ***(Delayed retardation)***
- Compressive residual stresses
  - Reduce the effective stress → reduction in crack growth rate
  - Does not predict ***delayed retardation***
- Crack closure models
  - Variations in opening stress → stress intensity
  - This model does predict ***delayed retardation***

THE NEXT STEP IS TO DEVELOP MODELS FOR CRACK CLOSURE

9.12.1. Crack growth retardation prediction methods

In tests done on structural steels, it was found that when the baseline stress intensity range, ΔK, is much greater than the threshold stress intensity, ΔK_{th} and plane stress conditions prevail, growth retardation is primarily due to plasticity-induced crack closure (Shin & Fleck, 1987, p. 392). For lower baseline stress intensity, ΔK_b close to the threshold stress intensity and plane strain conditions apply, an overload produces immediate crack arrest at the surface of the specimen, cut, not in the bulk of the specimen. Shin & Fleck postulated that this is due to combination of strain hardening and residual stress at the crack tip.

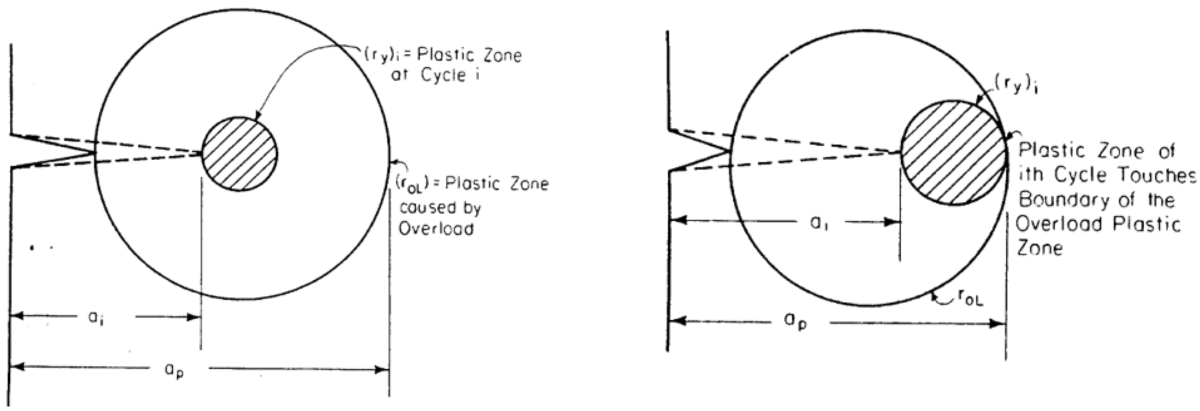


Figure 27: Crack-tip plasticity

The following approaches are typically used:

- Statistical Methods
  - Use the root mean square stress intensity factor
  - Only applicable to short spectra
  - Do not account for load sequence effects
  - Very restricted application
  - Does not predict crack growth retardation
- Crack closure models
  - Does predict crack growth retardation
  - Must estimate the opening stress for variable loading
  - Must be done cycle for cycle
  - Good correlation has been obtained

9.12.2. Further reading

(Carlson, Kardomateas, & Bates, 1991)

(Shin & Fleck, 1987)

9.13. Block (repetition) loading

The application of block (or repetition) loading will be explained in class problems on fatigue. The principle behind block (repetition) loading is to calculated damager per day, month, year, take-off-flight-landing, etc. event, and then calculated the number of times that the modelled event can be repeated. For the automotive industry one block is typically a mission profile driven length of route with known test route severity rating.

Block loading:

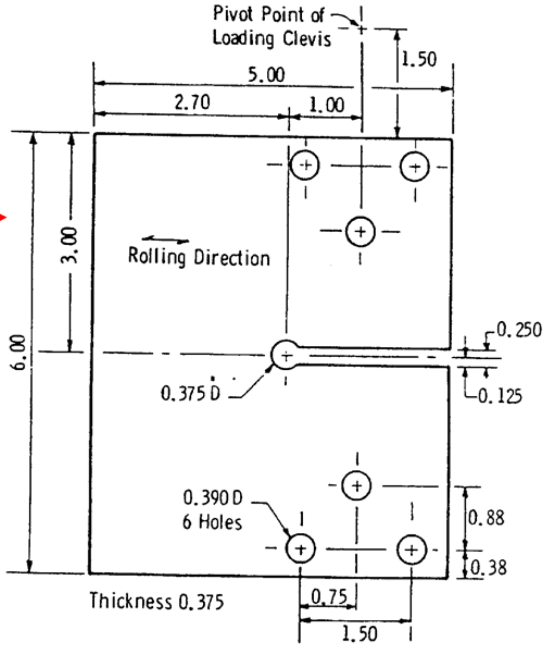
- Use blocks instead of cycle-for-cycle counting
- Considerable savings in time
- Limited to short spectra of loading
- Crack growth per block less than the plastic zone caused by the largest load cycle
- Damage is assumed to occur only when the crack is open
  - The crack opening stress must be determined
  - This means use of the change in positive stress intensity
    - Compressive residual stress beneficial

Dealing with service histories:

- Sometimes the service load history is unknown
- A representative load history or loading block may be determined from field tests
  - Analytical construction of service loads also done
- Fatigue life, or damage may then be calculated from the load blocks

9.14. Cumulative damage test program

- SAE Cumulative Damage Test Program
  - The component used in the study
  - Two steels were used:
    - ManTen
      - Yield strength = 80 ksi = 552 MPa
    - RQC-100
      - Yield strength = 120 ksi = 827 MPa
  - Different loadings were used



A series of tests were done to determine

- Baseline material strain-life and crack growth data
- Constant amplitude component load-life data
- Variable amplitude component data

The following analysis techniques were used to predict lives

- Rainflow counting was used to find ranges
- Miner's rule was used for damage summation
- The life analysis was done using
- Stress-life approach and the fatigue strength reduction factor K_f
- Load-life curves
- Local strain approach
  - Neuber analysis using K_f
  - Finite element analysis results
  - Assumption of elastic strain behaviour
  - Load-strain calibration curves using strain gauge measurements
- Analysis were made ignoring and considering mean stresses
- Techniques were also used to condense load histories
- No analysis was made of crack propagation lives

Results of the program:

- There was not a significant difference in the predictions made by any method that **used a reasonable estimate of notch root stress-strain behaviour**
- Good predictions were made using the Neuber approach that tended to be slightly conservative
- There was not a large difference between predictions which included and excluded mean stresses
- Predictions made using the simple **stress-life approach** showed correlation which was as good as those predicted by more complicated techniques
- Another study showed that the following method predicted very good propagation lives
  - Use FEA to determine crack opening levels
  - **Rainflow counting** + Linear Elastic Fracture Mechanics (LEFM)

9.15. Conclusion

- Miner's linear damage rule provides reasonable life estimates

- Most effective cycle counting procedures relate damaging events to the stress-strain response of the material (Like Rainflow counting)
- Repeated block loading analysis techniques may be applied to save time
- Application of large overloads **may** cause crack growth retardation

10. CYCLE COUNTING WITH THE RAINFLOW METHOD

During this section Investmech will demonstrate to the student how the rainflow counting method works and how Matlab is applied in rainflow counting. For the purpose of this course the student is not expected to perform cycle counting, however, the student must be able to demonstrate the rainflow counting method.

Note, it is expected from the student to make notes of the rainflow counting method as explained by the lecturer in class. Notes will not be issued in class.

Presentation used in class:	Cycle counting at Investmech Filename: Investmech - Structural Integrity (Cycle counting) R0.0

Use the remainder of this page and the following page to make notes during the lecture.

10.1. General process

The general process followed in Matlab is as follows:

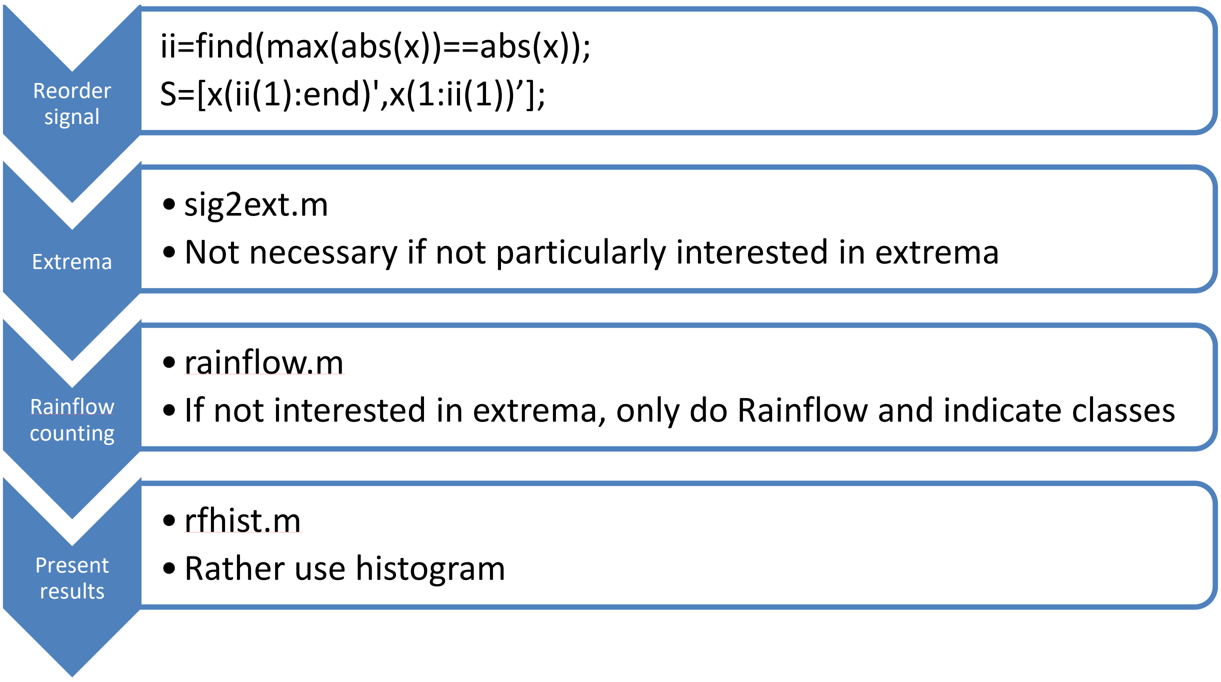


Figure 28: General process followed to do rainflow counting in Matlab

10.2. [ext,exttime]=sig2ext(sig, dt, clsn)

- Options:

[ext]=sig2ext(sig)
[ext,exttime]=sig2ext(sig)
[ext,exttime]=sig2ext(sig,dt)
[ext,exttime]=sig2ext(sig,dt,clsn)

- clsn

a number of classes of SIG (division is performed before searching of extrema)
no CLSN means no division into classes
Therefore, for counting turning points, the range of the signal is first divided by the classes to determine the stress intervals used for determination of turning points
A random signal will demonstrate this effect the best
According to BS7608, $clsn \geq 32$
Use 40 at Investmech

10.3. []=Rainflow()

c = rainflow(x) returns cycle counts for the load time history, x, according to the ASTM E 1049 standard. See Algorithms for more information.

c = rainflow(x,fs) returns cycle counts for x sampled at a rate fs.

c = rainflow(x,t) returns cycle counts for x sampled at the time values stored in t.

c = rainflow(xt) returns cycle counts for the time history stored in the MATLAB® timetable xt.

c = rainflow(___,'ext') specifies the time history as a vector of identified reversals (peaks and valleys). 'ext' can be used with any of the previous syntaxes.

[c,rm,rmr,rmm] = rainflow(___) outputs a rainflow matrix, rm, and two vectors, rmr and rmm, containing histogram bin edges for the rows and columns of rm, respectively.

[c,rm,rmr,rmm,idx] = rainflow(___) also returns the linear indices of the reversals identified in the input.

rainflow(___) with no output arguments plots load reversals and a rainflow matrix histogram in the current figure.

Leave open for notes.

11. STRESS-LIFE ANALYSIS

This section introduces stress life analysis.

Presentation used in class:	Investmech - Fatigue (Stress life analysis) R0.0

Discuss according to slides. The remainder of this section summarises problems done in class.

11.1. Slides not in notes

There are several slides presented in class that are not in the notes. Please download the slides to obtain digital copies of these slides. They are informative of nature, but, important and include:

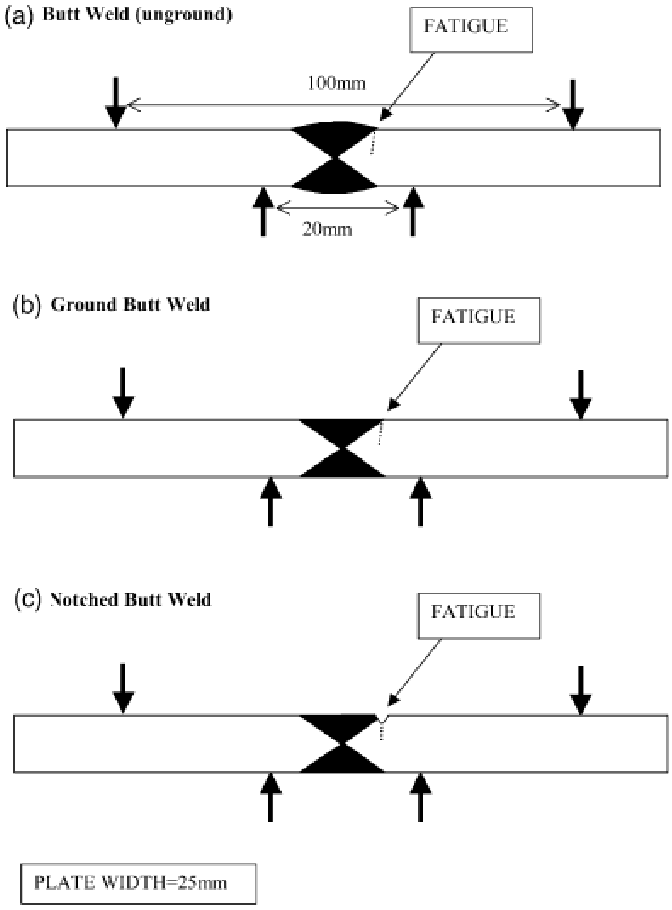
– Total life curve
– Images of fracture surfaces

11.2. Notches

In general you were introduced to static failure criteria, of which maximum principal stress, von Mises and Tresca (maximum shear stress theory) are the most widely used theories. The stress-life approach cannot account for load sequence events, which can be modelled by the strain-life method. Local stress concentrations and residual stress are also accounted for in the strain-life method. Fracture mechanics is applied to modelled crack propagation after crack initiation.

11.2.1. Weld toes

Stress concentrations exist at weld toes. Accurate modelling of these stress concentration under low loads allows the use of the general fatigue curve. It is clear that cracks will initiate at the highest stressed points, which are mostly in the stress concentrations. Weld toes leave geometry that produce stress concentrations, the reason why cracks initiate in weld toes.



Source: (Taylor, Barrett, & Lucano, 2002)

Figure 29: Stress concentrations at weld toes result in crack initiation

11.3. Theoretic stress concentration factor

Stress concentrations (with theoretical stress concentration, K_t) due to geometrical or micro-structural discontinuities result in higher local stress, σ, than the nominal stress away from the stress concentration, σ:

$$\sigma = K_t S$$
$$K_t = k_t = \frac{\sigma}{S} \geq 1 \qquad (41)$$

The ration between the maximum local stress and the nominal stress in a notched material is the theoretical stress concentration factor, K_t. K_t is dependent on the geometry and the model of loading. Theoretical stress concentration factors for various geometries and modes of loading can be seen in Figure 30.

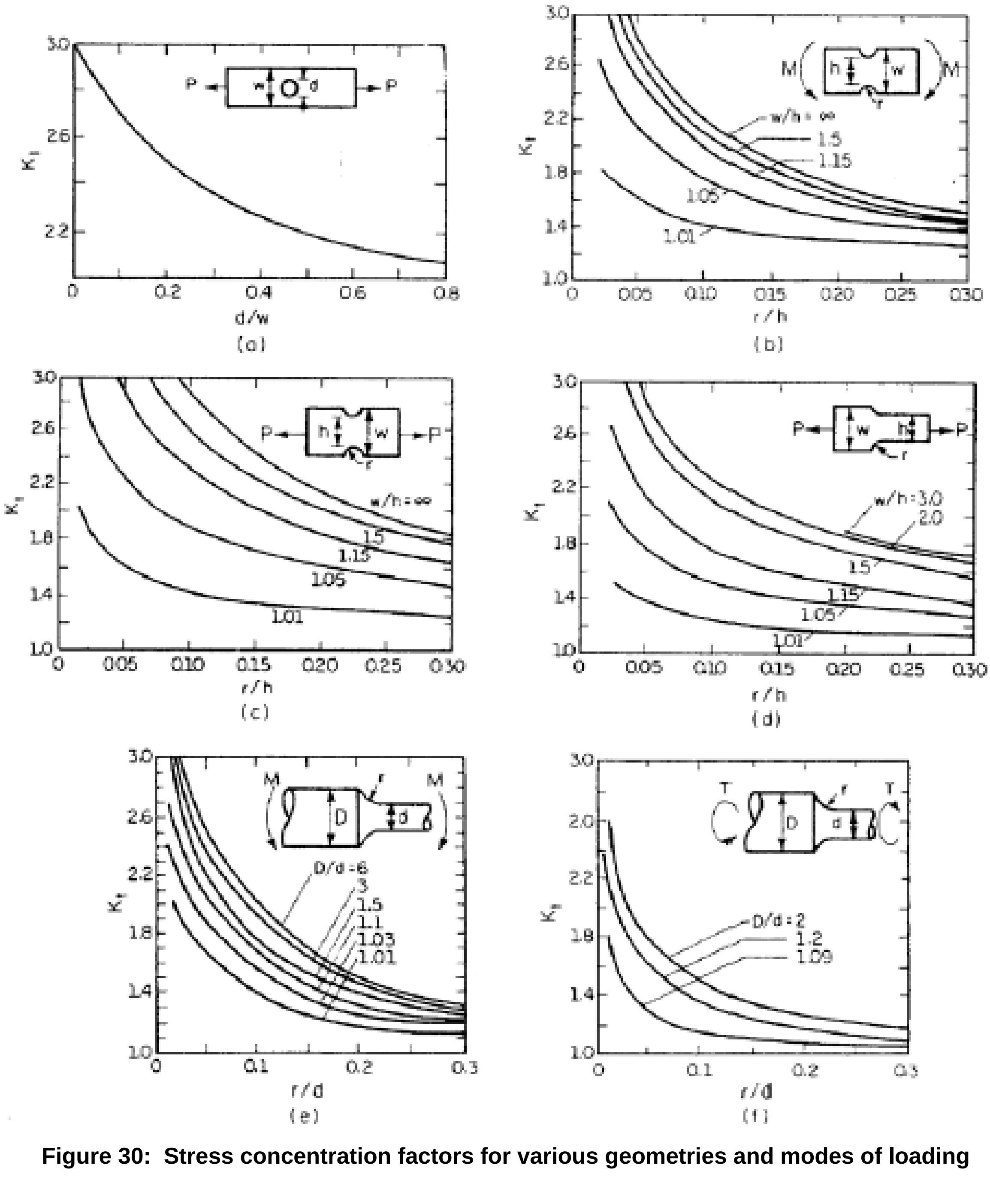
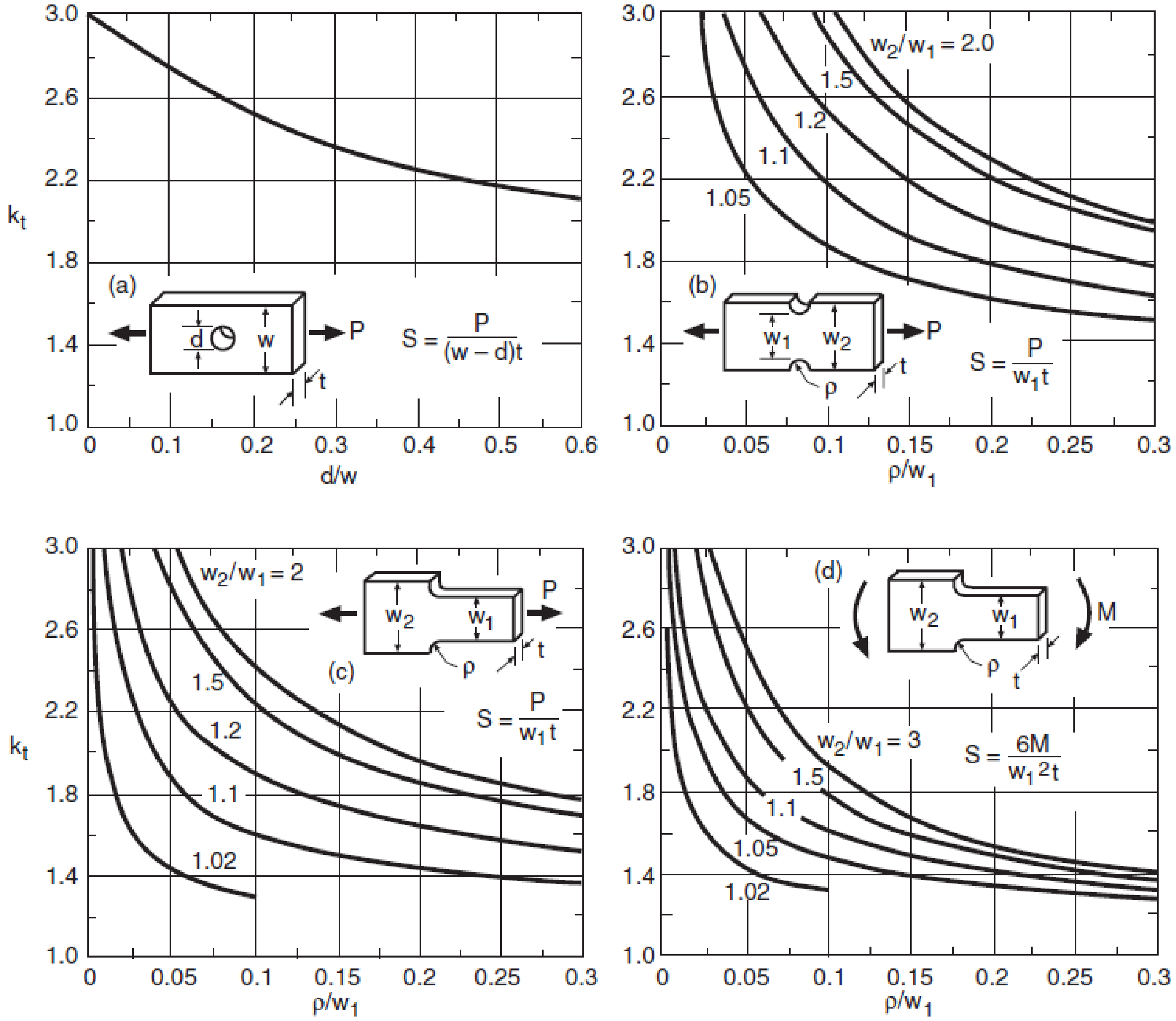
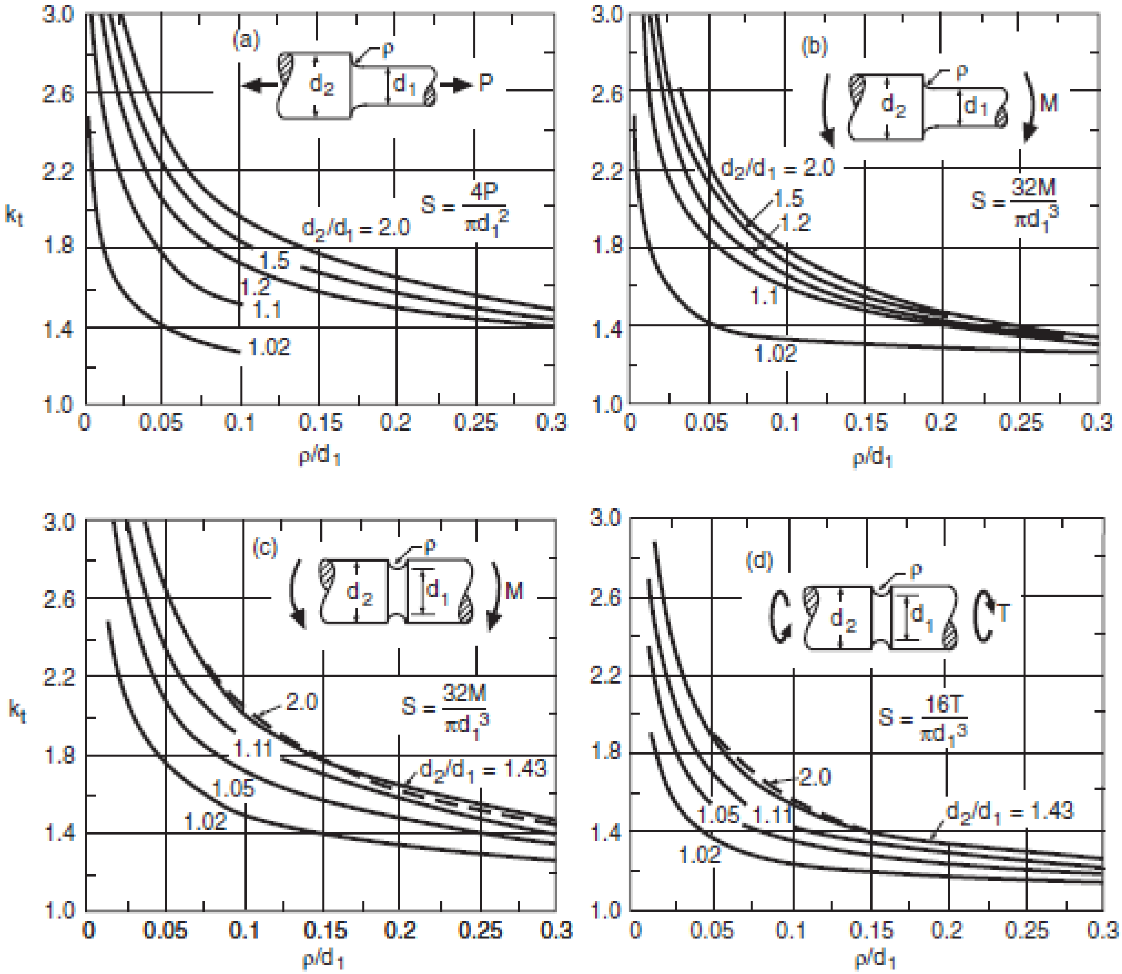


Figure 30: Stress concentration factors for various geometries and modes of loading



Source: (Dowling, 2013, p. 891)

Figure 31: Elastic stress concentration factors for notched plates



Source: (Dowling, 2013, p. 892)

Figure 32: Elastic stress concentration factors for notched circular shafts

11.4. Fatigue notch factor

A fatigue notch factor is used to determine the relation between the endurance limit for a notched and un-notched specimen, because it is used to estimate the long fatigue life of the material. It is given as the ratio of the stress at a local point, σ and the nominal stress in the specimen away from the notch, S.

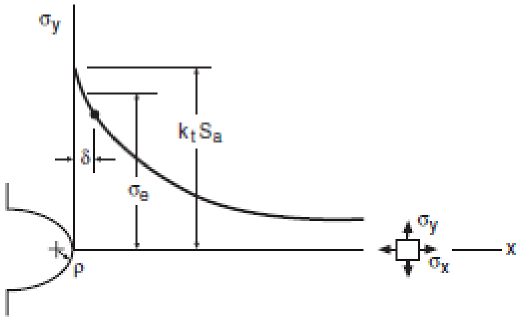
$$k_f = \frac{\sigma_{ar}}{S_{ar}} \tag{42}$$

Where:

k_f	Fatigue notch factor
S_{ar}	Nominal completely reversed stress amplitude for the notched member [Pa]
σ_{ar}	Completely reversed stress amplitude for the smooth member [Pa]

From Figure 33 it is shown that the fatigue notch factor is less than the theoretical stress concentration factor and is the stress at a distance δ from the notch root. Therefore, the endurance limit of the smooth specimen, σ_e, is related to the nominal stress endurance limit of the notched specimen by:

$$k_f = \frac{average\ \sigma_y\ at\ x = \delta}{S_a} = \frac{\sigma_e}{S_e} \tag{43}$$



Source: (Dowling, 2013, p. 495)

Figure 33: Stress distribution at a notch

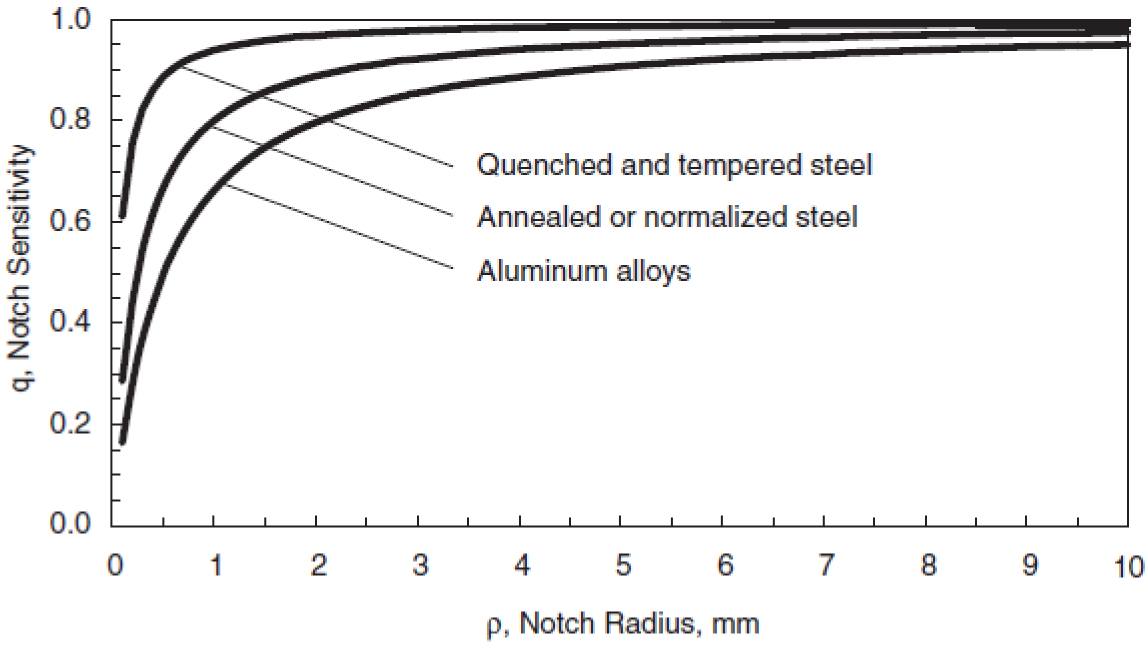
11.4.1. Notch sensitivity

The notch sensitivity q is given as follows and shown in Figure 34:

$$q = \frac{k_f - 1}{k_t - 1} \quad (44)$$

From which the fatigue notch factor is given as:

$$k_f = q(k_t - 1) + 1 \quad (45)$$



Source: (Dowling, 2013, p. 498)

Figure 34: Notch sensitivity vs notch radius

The fatigue notch factor is also dependent on material type and can be calculated using the following equation:

$$q = \frac{1}{1 + \frac{\alpha}{\rho}}$$

$$k_f = 1 + \frac{k_t - 1}{\left(1 + \frac{\alpha}{\rho}\right)} \quad (46)$$

Where:

α Factor depending on the material used [mm]

ρ Notch radius [mm]

Estimates for the material factor, α

$$\alpha = \left[\frac{300}{f_u[ksi]}\right]^{1.8} \times 10^{-3}\ in. \quad (47)$$

Peterson for steels (Dowling, 2013, p. 498):

$$\begin{aligned} \log\alpha &= 2.654 \times 10^{-7}\sigma_u^2 - 1.309 \times 10^{-3}\sigma_u + 0.01103 \\ \alpha\ [in\ mm] &= 10^{\log\alpha} \ \ (345 \leq \sigma_u \leq 2\,070\ MPa) \\ &= 10^{2.654\times10^{-7}\sigma_u^2 - 1.309\times10^{-3}\sigma_u + 0.01103} \end{aligned} \quad (48)$$

Where:

σ_u Ultimate tensile strength [MPa]

Table 5: Material constant, α, for some materials

α [mm]	Material
0.51	Aluminium alloys
0.25	Annealed or normalized low-carbon steels (BHN ~ 170)
0.064	Quenched and tempered steels (BHN ~ 360)
0.0254	Highly hardened steels (BHN ~ 600)

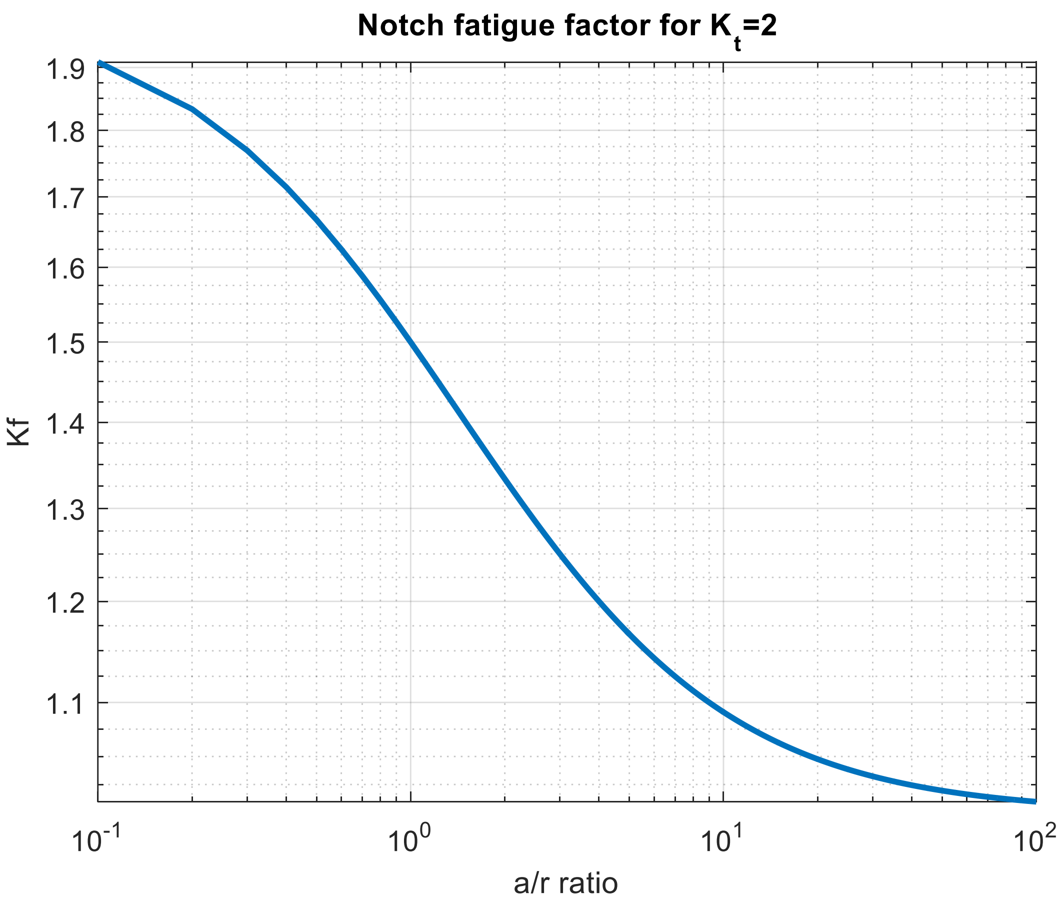


Figure 35: Fatigue notch factor for K_t=2

Another equation for the notch sensitivity is in terms of the Neuber constant, β (Dowling, 2013, p. 499):

$$q = \frac{k_f - 1}{k_t - 1} = \frac{1}{1 + \sqrt{\frac{\beta}{\rho}}} \quad (49)$$

From which the fatigue notch factor is:

$$k_f = 1 + \frac{k_t - 1}{\left(1 + \sqrt{\frac{\beta}{\rho}}\right)} \quad (50)$$

- For steel (Dowling, 2013, p. 499)

$$\begin{gathered}\log\beta = -1.079 \times 10^{-9}\sigma_u^3 + 2.740 \times 10^{-6}\sigma_u^2 - 3.740 \times 10^{-3}\sigma_u + 0.6404 \\ \beta\ [in\ mm] = 10^{\log\beta}\ \ (345 \leq \sigma_u \leq 1\,725\ MPa)\end{gathered} \quad (51)$$

- For aluminium

$$\begin{gathered}\log\beta = -9.402 \times 10^{-9}\sigma_u^3 + 1.422 \times 10^{-5}\sigma_u^2 - 8.249 \times 10^{-3}\sigma_u + 1.451 \\ \beta\ [in\ mm] = 10^{\log\beta}\end{gathered} \quad (52)$$

The fatigue notch factor can be used to adjust the S-N curve as can be seen in Figure 36.

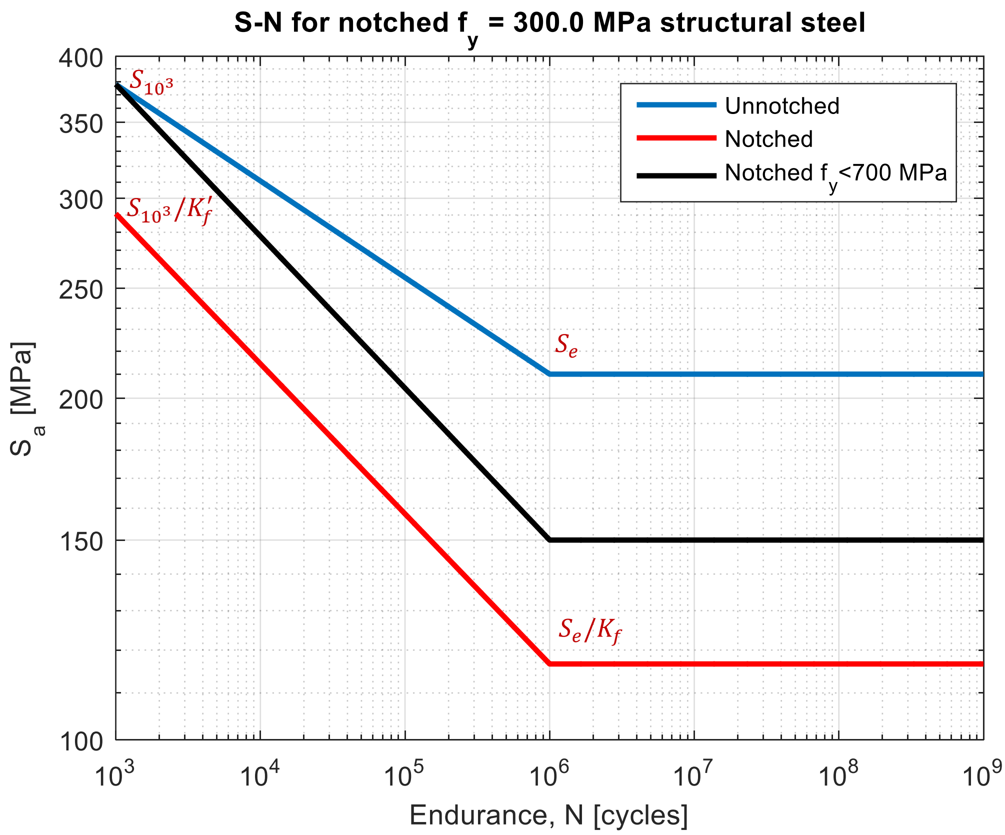


Figure 36: S-N curve adjusted for a notched material

Ultimately, stress concentrations occurring from geometric discontinuities significantly lower than the fatigue life and endurance limit of a material.

An example of a finite element analysis done on a plate with a hole in the middle can be seen in Figure 37.

The applicable dimensions for the plate are:

- Hole diameter (d): 20 mm.
- Plate width (w): 50 mm.

This example demonstrates the stress concentration that exists in the regions next to the hole. The plate was loaded with a nominal stress of 100 MPa, and from Figure 37 it can be seen that the maximum stress is 334 MPa. The nominal stress in that area of the hole is $100 \times \frac{50}{30} = 167\ MPa$ and thus, the stress concentration factor is $\frac{334}{167} = 2$ according to the finite element analysis.

The same problem can be assessed with the stress concentration factors as can be seen in Figure 30. Thus the ratio is equal to $\frac{d}{w} = \frac{20}{50} = 0.4$ and from Figure 38 it can be seen that the stress concentration factor is roughly $K_t = 2.25$. Always make sure to calculate the stress in the area prescribed by the table. In some instances the nominal stress is calculated away from the stress concentrations, and in other cases the stress is calculated as average stress on remaining material at the stress concentration.

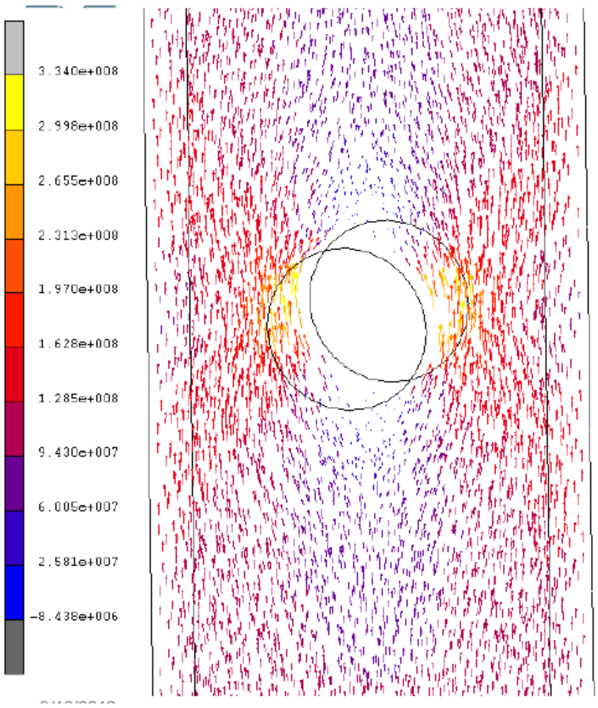


Figure 37: Finite element analysis of a plate with a hole in the middle

Include the questions and answers in the slides here.

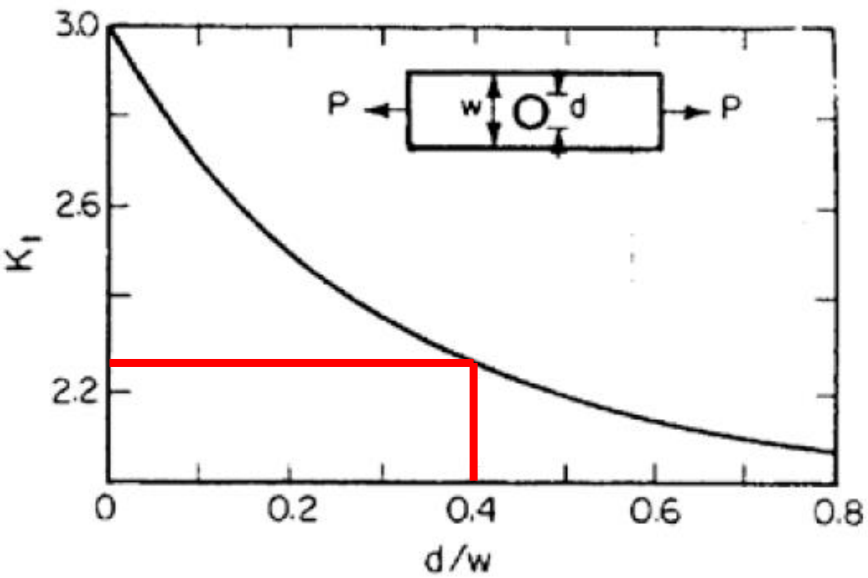


Figure 38: Stress concentration factors for a plate with a hole in the middle

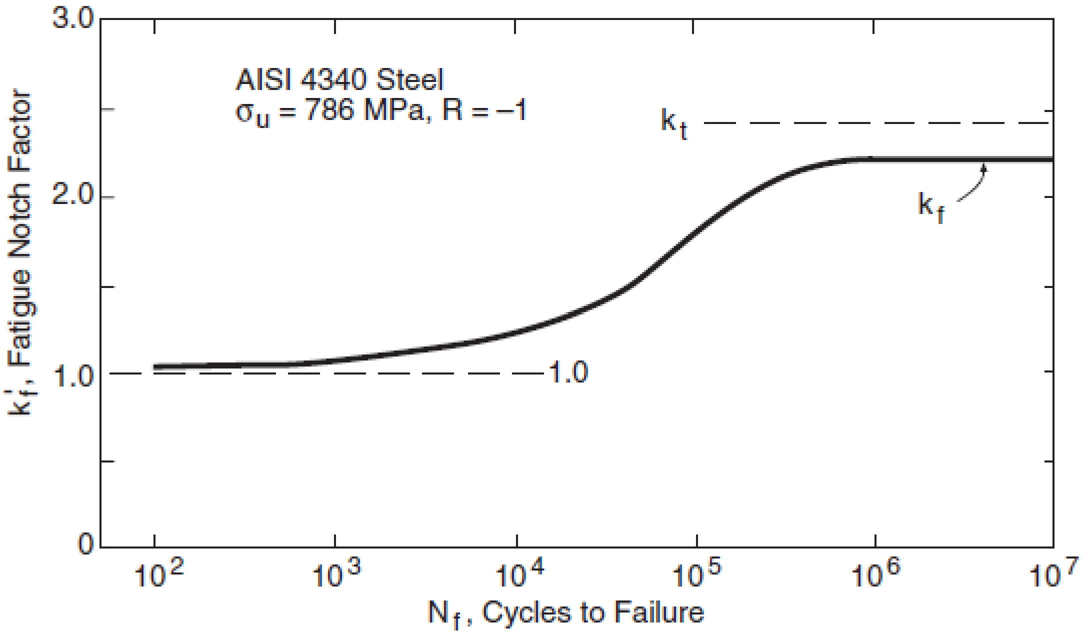
11.4.2. Notch fatigue factor at short lives

At short lives, the reversed yielding effect becomes increasingly important.

Use fatigue notch factor, k_f' at 1 000 cycles as shown in Figure 39. At one reversal, $N_f = 0.5$, the presence of the notch has no effect as there is full yielding.

If there is:

- No yielding at the notch, $k_f' = k_t$
- Local yielding: $k_f' = \frac{\sigma_o}{S_a}$
- Full yielding: $k_f' = 1$



Source: (Dowling, 2013, p. 507)

Figure 39: Variation of the fatigue notch factor with life for a ductile material

The notch fatigue factor at 1 000 cycles and below can be calculated from the curve shown in Figure 40 and the following equation:

$$q = \frac{K_f' - 1}{K_f - 1} = f(f_{ut}) \quad (53)$$

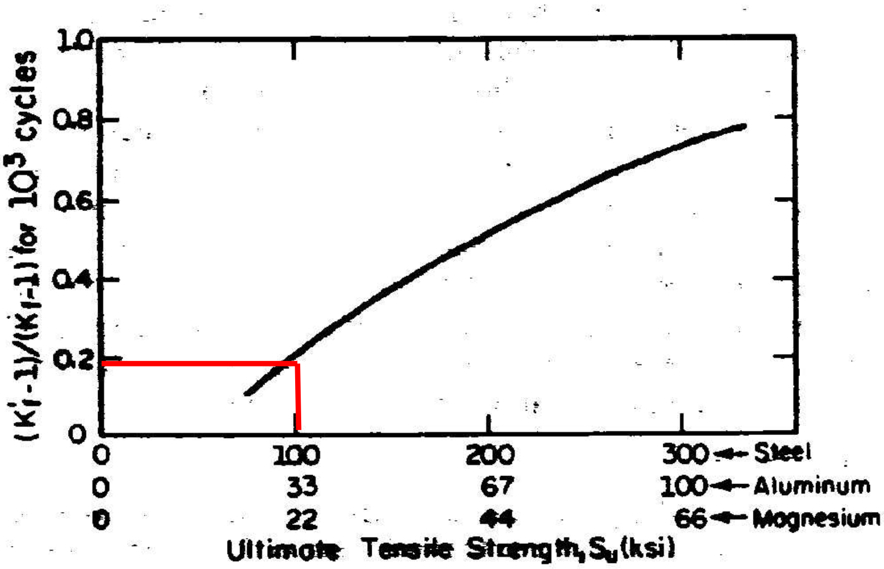


Figure 40: Ration between notch fatigue factors at 1 000 and 1M cycles vs tensile strength

11.5. Mean stresses

11.5.1. Behaviour of materials under mean stress cyclic loading

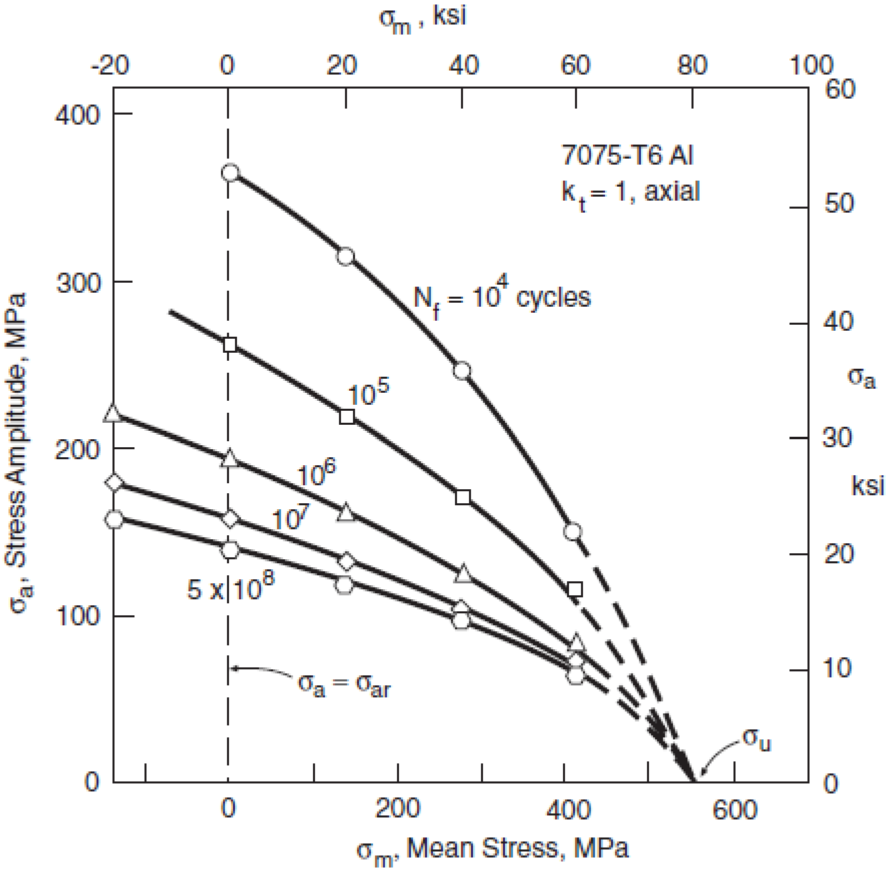
Curves and test data can be used from test specimens subject to fatigue testing at different stress amplitude and mean combinations. However, these are not widely available, and approaches are required to model the effect of mean stress on fatigue strength.

Constant life diagrams (where the same life at different stress amplitude and mean combinations) can also be used to present fatigue curves. These curves are generated for a specific endurance (fatigue life) shown in Figure 41. As shown:

- Increasing the mean stress at any stress amplitude reduce fatigue life.
- Compressive mean stresses are beneficial, tensile mean stresses harmful.

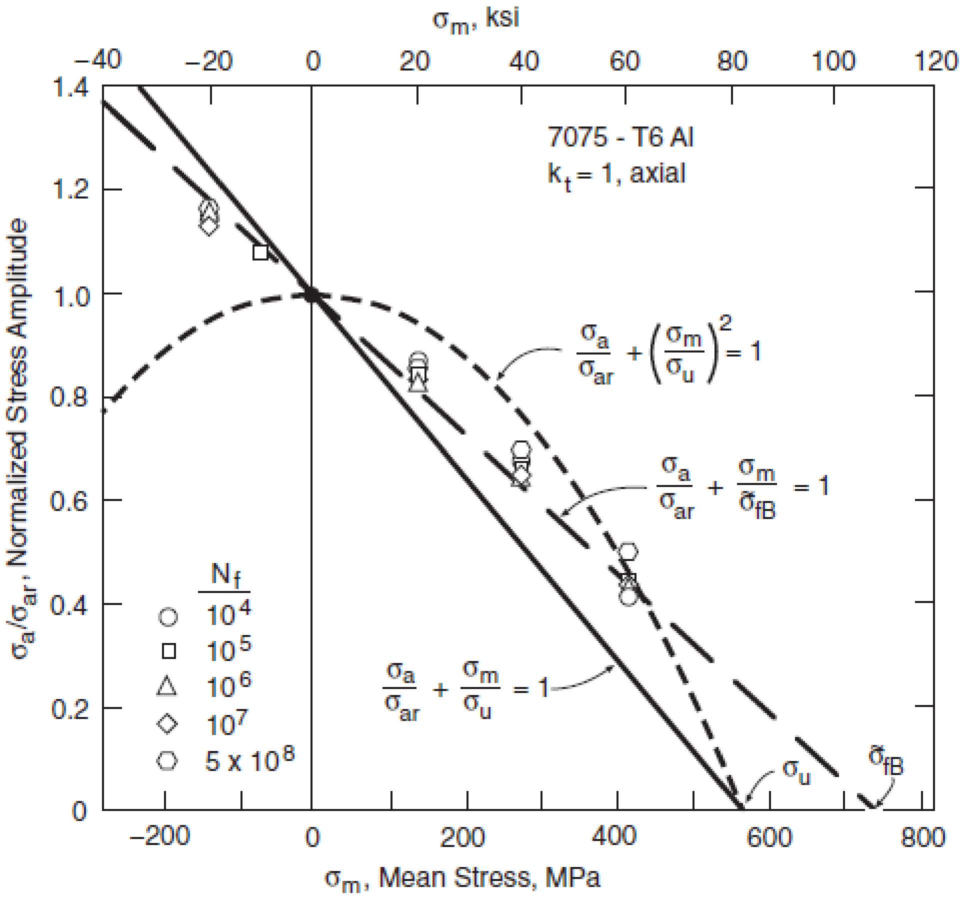
Figure 42 shows the same information with normalized ordinate. Graphs can also be constructed where the abscissa is also normalized with a material property.

A normalized amplitude-mean diagram which yields a straight-line approximation called a Goodman or Haigh diagram can be used to assess life under non-zero mean stress conditions. An example of a Haigh diagram showing the regions of finite and infinite life for a specific alternating and mean stress can be seen in Figure 43. A typical Haigh diagram for various mean and alternating stress conditions is shown in Figure 44.



Source: (Dowling, 2013, p. 452)

Figure 41: Constant-life diagram for 7075-T6 aluminum



Source: (Dowling, 2013, p. 454)

Figure 42: Normalized amplitude-mean diagram for 7075-T6 aluminum

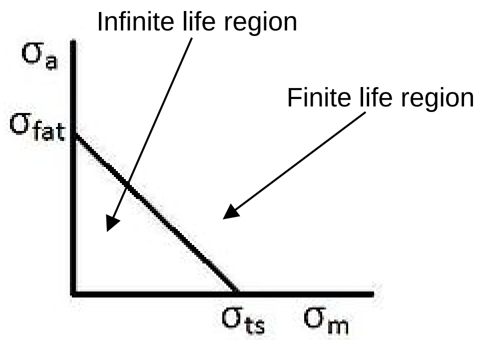


Figure 43: Haigh diagram sketch showing the regions for finite and infinite life

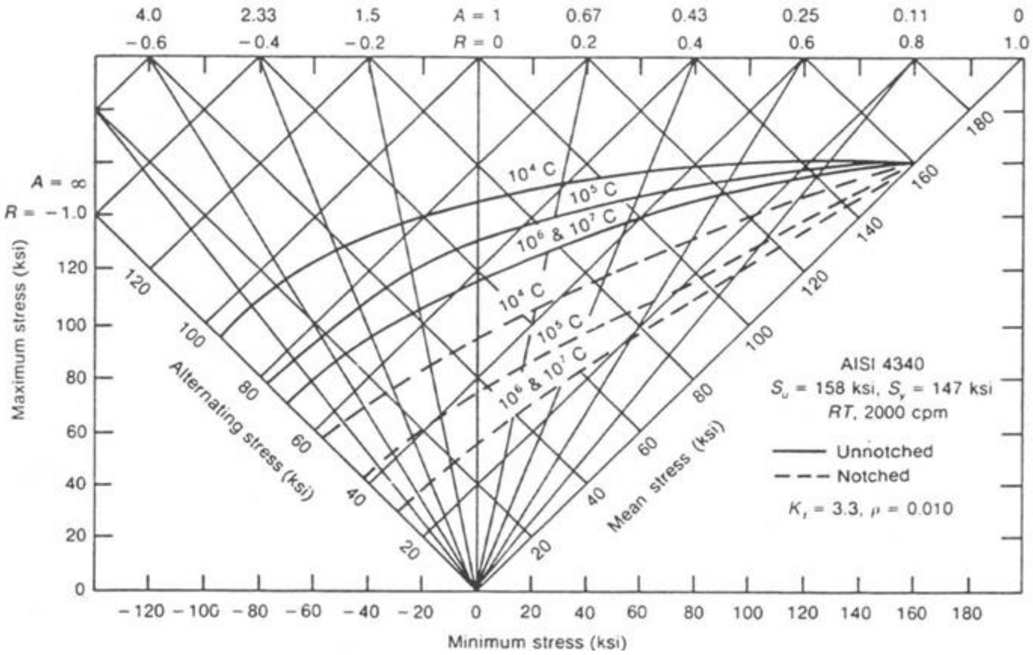


Figure 44: Haigh diagram for AISI 4340 steel (Bannantine *et all,* 1990:7)

11.5.2. Modified Goodman mean stress correction

In instances where dynamic loading with a specific non-zero mean stress occurs, such as with most practical systems, the Goodman equation is used to determine the completely reversed endurance limit of the material. The Goodman equation is given in the following equation:

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$
$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} \qquad (54)$$

Where:

σ_{ar}	Endurance limit, also the equivalent completely reversed stress [MPa]
σ_a	Stress amplitude [MPa]
σ_m	Mean stress [MPa]
σ_{ut}	Ultimate tensile strength [MPa]

11.5.3. Morrow mean stress correction

The Goodman mean stress correction was modified to make provision for ductile materials that show improved fatigue strength.

For ductile materials, the additional fatigue strength can be modelled by replacing the tensile strength with:

- The corrected true fracture strength, $\tilde{\sigma}_{fB}$, from a tension test.

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\tilde{\sigma}_{fB}} = 1$$
$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\tilde{\sigma}_{fB}}} \qquad (55)$$

- The constant, σ_f', from the unnotched axial S-N curve for $\sigma_m = 0$.

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1$$
$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}} \qquad (56)$$

Using σ_f' generally gives reasonable results for steels shown in Equation (56).

For some aluminium alloys $\tilde{\sigma}_{fB}$ and σ_f' may differ significantly, in which cases Equation (55) gives the better agreement using $\tilde{\sigma}_{fB}$.

11.5.4. Gerber mean stress correction

The Gerber mean stress correction is given by the following equation:

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \qquad for\ \sigma_m \geq 0$$
$$\sigma_{ar} = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2}, for\ \sigma_m \geq 0 \tag{57}$$

Where

σ_{ar} is the equivalent completely reversed stress amplitude

σ_u is the material tensile strength

This equation:

- Is limited to tensile mean stress only (it will incorrectly predict harmful effect of compressive mean stress)
- Not used often.

11.5.5. Smith, Watson and Topper (SWT) mean stress correction

The SWT mean stress correction is as follows (Dowling, 2013, p. 455):

$$\begin{aligned}\sigma_{ar} &= \sqrt{\sigma_{max}\sigma_a} \\ &= \sigma_{max}\sqrt{\frac{1-R}{2}}\end{aligned} \tag{58}$$

The SWT relationship has the advantage of not relying on any material constant.

Single curve on a plot of σ_a/σ_{ar} vs σ_m/σ_{ar}.

11.5.6. Walker mean stress correction

The Walker equation depends on material constant, γ:

$$\begin{aligned}\sigma_{ar} &= \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \qquad (\sigma_{max} > 0) \\ &= \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \qquad (\sigma_{max} > 0)\end{aligned} \tag{59}$$

The SWT equivalent to the Walker equation with $\gamma = 0.5$.

Single curve on a plot of σ_a/σ_{ar} vs σ_m/σ_{ar}.

Dowling give the following equation for estimating γ from the ultimate tensile strength:

$$\gamma = -0.000200\sigma_u + 0.8818 \quad (\sigma_u\ in\ MPa) \tag{60}$$

11.5.7. Which mean stress correction to use

Neither Goodman nor Gerber equation are very accurate.

Goodman is overly conservative.

Gerber is non-conservative.

The Morrow relationship is usually reasonably accurate but use the true fracture strength that is not always known.

The Morrow relationship with σ_f' fits data very well for steels but should be avoided for aluminium alloys and perhaps nonferrous alloys (Dowling, 2013, p. 456).

SWT relationship a good choice for use with aluminium alloys.

Walker relationship is the best choice where data exist for fitting the value γ (Dowling, 2013, p. 456).

11.5.8. Safety factors with mean stress

As before, the safety factor in stress is:

$$X_S = \left.\frac{\sigma_{ar1}}{\hat{\sigma}_{ar}}\right|_{N_f=\hat{N}} \tag{61}$$

And the safety factor in life:

$$X_N = \left.\frac{N_{f2}}{\hat{N}}\right|_{\sigma_{ar}=\tilde{\sigma}_{ar1}} \tag{62}$$

Where:

$\tilde{\sigma}_{ar}$ is calculated from the stress amplitude $\tilde{\sigma}_a$ and mean stress $\tilde{\sigma}_m$ expected to occur in service.

Another option is to use a safety factor for the amplitude and mean of Y_a and Y_m respectively to calculate a value of equivalent completely reversed stress σ'_{ar1}.

Applied to the Morrow expression, the equation becomes:

$$\sigma'_{ar1} = \frac{Y_a\tilde{\sigma}_a}{1 - \frac{Y_m\tilde{\sigma}_m}{\sigma'_f}} \quad \sigma'_{ar1} \leq \sigma'_f(2\tilde{N})^b \tag{63}$$

Applied to the SWT equation:

$$\sigma'_{ar1} = \sqrt{(Y_m\tilde{\sigma}_m + Y_a\tilde{\sigma}_a)Y_a\tilde{\sigma}_a} \quad \sigma'_{ar1} \leq \sigma'_f(2\tilde{N})^b \tag{64}$$

The advantage of the two load factors is that they can be applied separately to make provision for uncertainty.

11.6. Modifying factors and estimating S-N curves

The approach is as given in t

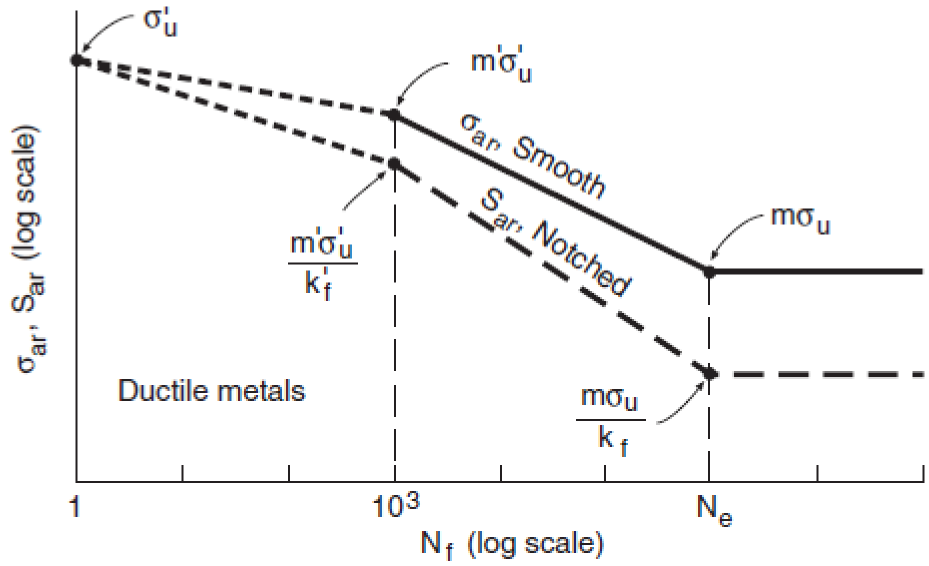


Figure 45: Completely reversed S-N curves for smooth and notched members

The objective of modifying factors is to calculate the endurance limit and the fatigue strength at 1 000 cycles under various conditions. Read Dowling Chapter 10.4.3.

$$\begin{aligned}
\sigma_{er} &= C_{size}C_{load}C_{surf}C_TC_{rel}\sigma_{erb} \\
C_e &= C_{size}C_{load}C_{surf}C_TC_{rel} \\
S_{er} &= \frac{\sigma_{er}}{k_f} \\
&= \frac{C_e \cdot 0.5\sigma_u}{k_f} \\
S_{ar,10^3} &= \sigma_{arb,10^3}C_{load}C_TC_{rel} \\
&= \frac{0.9\sigma_u}{k'_f}C_{load}C_TC_{rel}
\end{aligned} \tag{65}$$

In Dowling, the factors are:

$$\begin{aligned}
Type\ of\ loading: m_t &= C_{load} \\
Size: m_d &= C_{size} \\
Surface\ finish: m_s &= C_{surf}
\end{aligned}$$

$$Other\ effects: m_o = C_T\ for\ temperature, corrosion, etc$$

The general trend of modifying factors is to have less effect at short lives. However, some, like reliability, need to be included. The modifying factors that is expected to affect the fatigue life at low cycle end are C_{load}, C_T and C_{rel}.

11.6.1. Size factor on different shaft sizes

The size factor as function of shaft diameter is as follows:

$$C_{size} = \begin{cases} 1.0, if\ d \leq 8\ mm \\ 1.189d^{-0.097}, if\ 8\ mm < d \leq 250\ mm \end{cases} \quad (66)$$

Table 6 summarises a few values.

Table 6: Shaft diameter dependent size factor

d [mm]	C_size
10	0.95
30	0.85
50	0.81
70	0.79
90	0.77
110	0.75
130	0.74
150	0.73
170	0.72
190	0.71
210	0.71
230	0.70
250	0.70

11.6.2. Size factor on thickness

The size factor, k_b, is given by the following for bending and torsion (Budynas & Nisbett, 2012, p. 280):

$$k_b = 1.51d^{-0.157} \quad (67)$$

Where:

d Section thickness, taken as the trunnion wall thickness [m]

11.6.3. Loading effects

A conservative relationship due to the volume subject to high stress is as follows:

$$S_{e,axial} = 0.70S_{e,bending}$$
$$C_{load} = 0.7\ if\ S - N\ curve\ is\ from\ bending\ tests \quad (68)$$

If the S-N curve was constructed from a bending test, the load factor for a specimen subjected to axial loading will be approximately 0.7. However, if the S-N curve used was constructed from axial tests, the load factor for a specimen loaded in bending will be $\frac{1}{0.7} = 1.43$.

For torsion (see the slides for the derivation) (Dowling, 2013, p. 503):

$$\tau_{er}(torsion) = 0.577S_{e,bending} \quad (69)$$

11.6.4. Surface finish and corrosion

A corrosive environment attacks the surface of a material and produces an oxide film. Cyclic loading causes localized cracking of the layer, exposing the fresh material to the corrosive environment. This causes localized pitting where stress concentration occurs. Corrosion reduces the fatigue life of a material and eliminates the endurance limit. Thus, if an unprotected material operates in a corrosive environment, the material’s endurance limit will fall away and cannot be taken into account.

Surface finish modification factors which can be used to modify the endurance limit of steel is given in Figure 46. Modification factors for steel corroded in water, as well as in salt water, are also given in Figure 46 for steels with different ultimate tensile strengths.

Surface treatment can have a significant effect on fatigue life, because the crack initiates at the free surface. In plating, thermal and mechanical treatment (welding, milling, pressing, etc.), the effect on fatigue life is primarily due to residual stress. Should a residual stress result, pre-stressing or pre-setting (initial overloading of the component, which is only favourable for fatigue loads in the direction of the overload) should be done to produce compressive residual stress at the free surface.

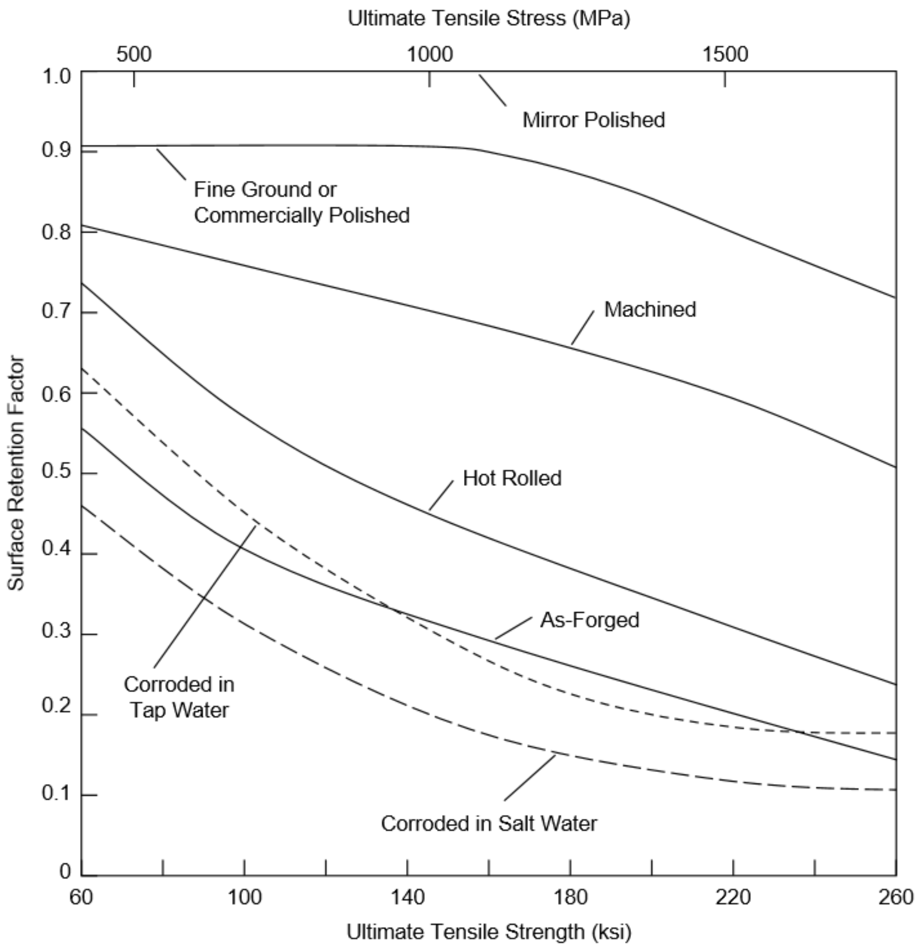


Figure 46: Surface finish modification factors for steel (ASM International, 2008:16)

According to BS 7608 (1993), for unprotected joints exposed to water (specifically sea water) the basic S-N curves should be reduced by a factor of 2 on life for all joint classes. For high strength steels, with yield strengths of higher than 400 MPa, this factor may not be adequate and the use of such materials under corrosion fatigue conditions should be approached with caution.

11.6.4.1. Plating

Chrome and Nickel plating of steels can cause up to 60% reduction in endurance limits. High tensile stresses are generated by plating process. To alleviate residual stress problem:

- Nitride part before plating
- Shot peen part before or after plating (Best to peen after plating)
- Bake or anneal the part after plating

Corrosion resistance offered by plating can more than offset the reduction in fatigue strength seen in non-corrosive environment. Plating with cadmium and zinc appear to have no effect on fatigue strength. Electroplating can cause hydrogen embrittlement.

11.6.5. Temperature

- Diffusion processes such as carburizing and nitriding beneficial for fatigue strength
  - Produces higher strength material on surface
  - Causes volumetric changes that produce residual compressive stresses
- Flame and induction hardening
  - Cause phase transformation which in turn cause volumetric expansion

  - If localized to surface – compressive residual stresses result that is beneficial for fatigue strength
- Hot rolling and forging
  - Cause surface decarburization
  - Loss of carbon atoms from surface causes lower strength and may produce residual stresses
  - Both factors are detrimental to fatigue strength
- Manufacturing processes such as grinding, welding, flame cutting etc.
  - Can set up detrimental residual tensile stresses
  - Shot peening effective to undo damage caused by these processes

- Endurance limits of steels increase at low temperatures (watch out for brittleness)
- Endurance limit for steels disappears at high temperatures due to mobilization of dislocations
- For $T > T_{melt}/2$ creep becomes important
  - Stress-life approach no longer applicable.
- Annealing happens at high temperatures that may remove beneficial residual compressive stresses

According to Roymech:

$$C_T = \begin{matrix} 1.0\ for\ T \leq 450\ ℃ \\ 1 - 5.8^{-3}(T - 450),\ for\ 450 < T \leq 550\ ℃ \end{matrix} \qquad (70)$$

11.6.6. Reliability

The probability of survival, or failure, is statistically driven. BS 7608 provides means to shift various S-N fatigue curves for client specified probability of survival.

Reliability $1 - p_f$	C_r
0.5	1
0.9	0.897
0.95	0.868
0.99	0.814
0.999	0.753
0.9999	0.702
0.99999	0.659
0.999999	0.620

11.6.7. Mechanical modifying factors

- Cold work processes – rolling & shot peening
  - Produce compressive residual stresses
    - Gives the greatest improvement in fatigue life
  - Work-hardens the material
  - Rolling cause deep stress layer (bolts, etc)
  - Shot peening gives (compressive stress = $0.5S_y$) layer of ~1 mm
  - Shot peening:
    - leaves dimpled surface: hone or polish part after shot peening
    - Undo deleterious effects caused by chrome and nickel plating, decarburization, corrosion, grinding, etc.

- Steels with $F_y \leq$ 550MPa seldom cold rolled or shot peened (Easy to introduce plastic strains that wipe out residual stresses)
- Surfaces can be overpeened! Subsurface failures may occur!

- Loading frequency
  - Similar data at various frequency in non-corrosive environment
  - Corrosion-fatigue are greatly influenced by loading frequency

11.7. Derivation of S-N curve for S_{1000} and S_e modified by the fatigue notch factors

From the theory presented in class, the fatigue notch factors modify the S-N curve as follows:

$$S'_{1000} = \frac{S_{1000}}{K'_f}$$
$$S'_e = \frac{S_e}{K_f} \qquad (71)$$

The S-N curve is modelled by equation: $S^m_{10^3} \times 10^3 = S^m_e \times 10^6$. Using this relation, the coefficient and exponent in the notch modified S-N curve equation can be solved as follows:

$$(S'_{1000})^m \times 10^3 = (S'_e)^m \times 10^6$$
$$\left(\frac{S'_{1000}}{S'_e}\right)^m = 10^3$$
$$m = \frac{3}{\log \frac{S'_{1000}}{S'_e}} \qquad (72)$$

11.8. Endurance calculation example

Problem statement:

300WA structural steel has the following material properties:

E = 206 GPa

S_y = 300MPa

S_{ut} =450MPa

Assume a notch fatigue factor of $k_f = 1.7$. What is the endurance limit? How many cycles to failure at:

1. S_a= 200MPa
2. S_a= 300MPa

Assumptions:

Investmech assumed that the fatigue notch factor at 1000 cycles needs to be estimated from the material properties supplied. This information was not provided in the problem statement, but, will be inferred from the class notes.

The S-N curve can be described as: $S = CN^b$.

Solution:

Figure 47 shows the relationship of the following ration to the material ultimate tensile strength of 490 MPa (70 ksi) as approximately 0.15, from which the fatigue notch factor at 1,000 cycles is $K'_f = 1.1$:

$$\frac{K'_f - 1}{K_f - 1} = 0.15$$
$$K'_f = 0.15K_f + 0.85$$
$$= 0.15 \times 1.7 + 0.85$$
$$= 1.1 \qquad (73)$$

From this, the end points (at 1,000 and 1,000,000 cycles) of the S-N curve are as follows:

$$S_{1000} \qquad (74)$$

$$S'_e = \frac{S_e}{K_f}$$
$$= \frac{0.5S_{ut}}{1.7}$$
$$= C10^{6b}$$

Dividing the two equations gives:

$$\frac{[S'_e]}{[S'_{1000}]} = 10^{3b} \qquad (75)$$

The coefficient C is then:

$$C = \frac{[S'_{1000}]}{10^{3b}}$$
$$C = \frac{[S'_e]}{10^{6b}} \qquad (76)$$

The values are as summarised in the Excel sheet below from which the following may be concluded:

1. The endurance limit for the notched specimen is 144 MPa.
2. The number of cycles to failure, which is also called the endurance, for 50% probability of survival at stress amplitude 200 MPa is 108,358 cycles.
3. The number of cycles to failure, also called the endurance, for 50% probability of survival at stress amplitude 300 MPa is 6,929 cycles.

Table 7: Example fatigue calculation

Sy	300	MPa			
Sut	490	MPa			
Kf	1.7				
$\frac{K'_f - 1}{K_f - 1}$	0.15	From the graph		$\frac{\left[\frac{0.5S_{ut}}{1.7}\right]}{\left[\frac{0.9S_{ut}}{1.1}\right]} = 10^{3b}$	
Kf'	1.105				
Calculations					
Endurance limit for the notched specimen	144.1176	MPa			
b	-0.14745				
C=	1105.186		Check:	1105.186	
Endurance for stress limits	**Endurance**				
Stress amplitude					
200	108 358				
300	6 929				

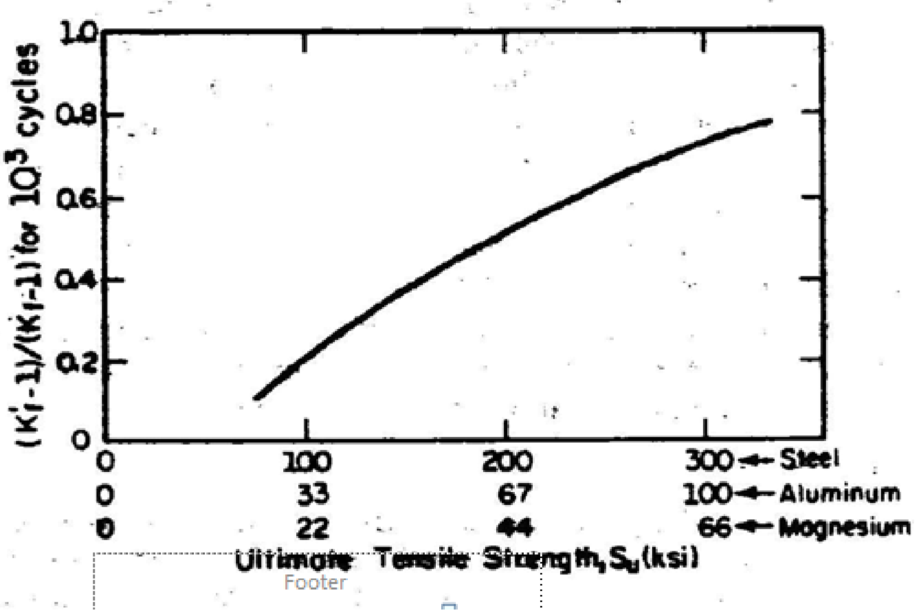


Figure 47: Relationship between fatigue notch factors at endurance and 1000 cycles

11.9. Fatigue life example with mean stress

Problem statement:

Component undergoes cyclic stress as follows:

Stress amplitude [MPa]	Stress mean [MPa]	Number of cycles/block
400	200	10000
500	0	5000
600	100	20000
700	-100	2000

The material is steel with S_{ut} = 1,050MPa and S_e = 420MPa. The fully reversed stress at S_{1000} = 770 MPa.

How many blocks of loading can be loaded on the component until fatigue crack initiation? That is, what is the fatigue life of the component?

Solution:

The solution is presented in the table below. Goodman correction was done. In general, the steps are as follows:

1. Calculate the notch fatigue factor at 1,000 and 1,000,000 cycles.
2. Solve the S-N curve coefficient and exponent.
3. Find the signal mean and amplitude.
4. Do Goodman correction to find the completely reversed stress amplitude.
5. Calculate the endurance for each completely reversed stress amplitude.
6. Calculate damage for each completely reversed stress amplitude.
7. Sum the to find total damage.
8. Failure is when total damage is equal to 1. Calculate the life proportionally.

Sy	??	MPa				
Sut	1050	MPa				
Kf	1					
$\frac{K_f'-1}{K_t-1}$	0.15	From the graph		$\frac{\left[\frac{0.5S_{ut}}{1.7}\right]}{\left[\frac{0.9S_{ut}}{1.1}\right]} = 10^{3b}$		
Kf'	1					
Calculations						
S1000'	945					
Size factor	1					
Surface factor	1					
Temperature factor	1					
Reliability factor	1					
Endurance limit for the notched specimen	525.0	MPa				
b	-0.085					
C=	1701.0		Check:	1701		
Stress amplitude	Mean stress	ni	Compl. Rev. Stress Sa	Ni	Di	
400	200	10000	494	inf	0.00	
500	0	5000	500	inf	0.00	
600	100	20000	663	64 218	0.31	
700	-100	2000	639	99 086	0.02	
				Total Damage	0.33	
				Duration	1	Blocks
				Life	3.02	Blocks

11.10. Example problem

11.10.1. Problem statement

A component undergoes axial cyclic loading as summarized in the table below, which was obtained from Rainflow cycle counting.

Table 8: Stress spectrum for one repetition

σ_a [MPa]	σ_m [MPa]	n_i [cycles/block]
100	200	10 000
50	0	5 000
100	100	20 000
200	-100	2 000

Material is steel with S_{ut} = 1,050 MPa with hardness 350 BHN. The theoretical stress concentration factor at a notch on the part is $K_t = 2$. The notch radius is $r = 4\ mm$. The surface finish is machined. The shaft has a radius of 100 mm and operates at temperature $T = 200\ °C$.

How many blocks/repetitions of loading can be loaded on the component for a 1 % probability of fatigue crack initiation? That is, what is the fatigue life of the component for a probability of survival of 99%?

11.10.2. Solution

The steps that I will follow:

1. Calculate notch fatigue factors:
2. Calculate the influencing factors and the fatigue strength at 1 000 and 1 000 000 cycles as shown below:

$$S_e = S_e' C_{size} C_{load} C_{surf} C_T C_{rel}$$
$$= \frac{0.5 S_{ut}}{Kf} C_{size} C_{load} C_{surf} C_T C_{rel}$$

$$S_{10^3} = S'_{10^3} C_{load} C_T C_{rel}$$
$$= \frac{0.9 S_{ut}}{K_f{}'} C_{load} C_T C_{rel}$$

3. Mean stress correction to calculate the equivalent completely reversed stress amplitude. Use Goodman:

$$S_a = \frac{\sigma_a}{1 - \frac{\sigma_m}{S_{ut}}}$$

4. Calculate damage using the Palmgren-Miner rule

$$D = \sum \frac{n_i}{N_i}$$

5. Calculate the number of repetitions:

$$B_f = \frac{1}{D}$$

6. Calculate life which is (not applicable in this case):

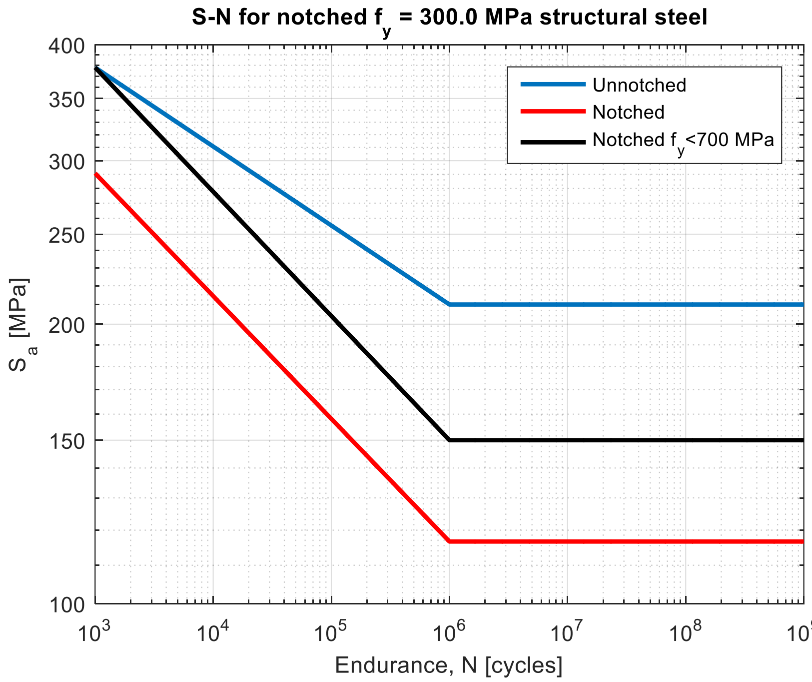
$$Life = B_f \times (Period\ of\ stress\ spectrum)$$

11.10.2.1. Calculate S-N curve for the notched specimen

The theoretical stress concentration factor is $K_t = 2$.

The ultimate tensile strength: $\sigma_u = 1\,050\ MPa$.

Notch root radius: $r = 0.004\ m = 4\ mm$



For this we have the follow equations:

At endurance limit at 1 million cycles:

$$K_f = \frac{\sigma_{erb}^{(un-notched)}}{\sigma_{erb}^{(nothced)}}$$
$$K_f = 1 + \frac{K_t - 1}{\left(1 + \frac{\alpha}{r}\right)}$$

- Approximations for α:

$$\alpha = \left[\frac{300}{\sigma_u[ksi]}\right]^{1.8} \times 10^{-3}\ in.$$
$$= \left[\frac{300 \times 6.89}{\sigma_u[MPa]}\right]^{1.8} \times 10^{-3} \times 25.4\ mm$$

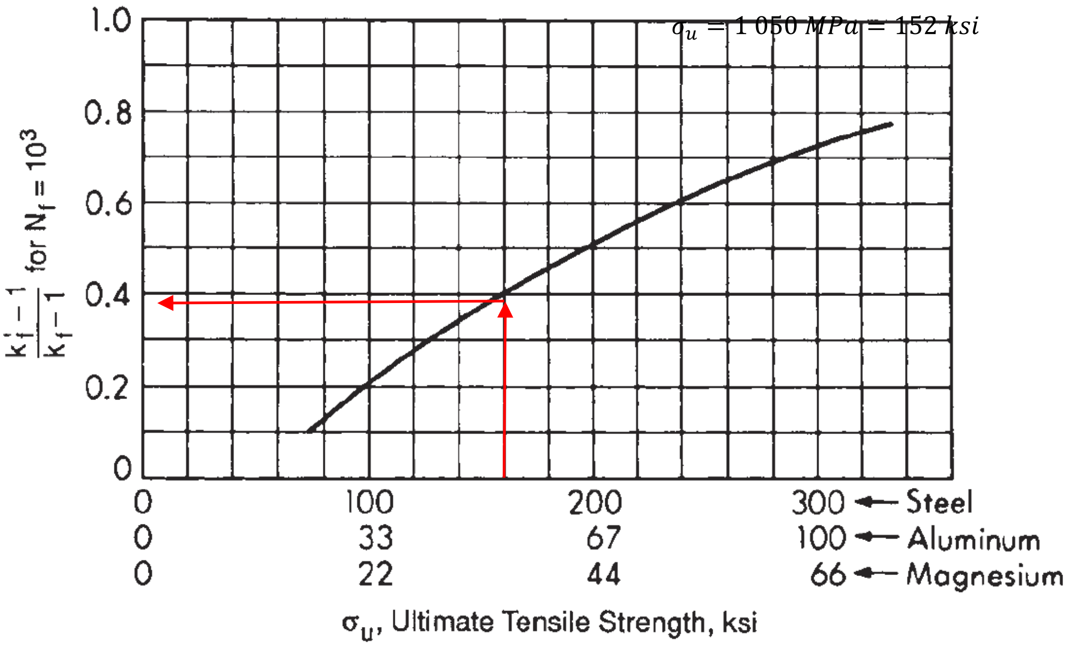
$$= \left[\frac{300 \times 6.89}{1\,050[MPa]}\right]^{1.8} \times 10^{-3} \times 25.4$$
$$= 0.086\ mm$$

Therefore:

$$K_f = 1 + \frac{2-1}{1+\frac{0.086}{4}}$$
$$= 1.98$$

Fatigue notch factor at 1 000 cycles:

$$\frac{K_f' - 1}{K_f - 1} = f(\sigma_u) = q$$
$$\sigma'_{arb,1000} = \frac{\sigma_{arb,1000}}{K_f'}$$
$$\sigma'_{erb} = \frac{\sigma_{erb}}{K_f}$$



From the figure above, we have:

$$\frac{K_f' - 1}{K_f - 1} = f(\sigma_u) = q = 0.38$$
$$K_f' = 0.38(K_f - 1) + 1$$
$$= 0.38 \times 0.98 + 1$$
$$= 1.37$$

11.10.2.2. Modification factors

$$\sigma_{er} = \sigma'_{\mathrm{erb}} C_{size} C_{load} C_{surf} C_T C_{rel}$$
$$\sigma'_{erb} = \frac{\sigma_{erb}}{K_f}$$
$$\sigma_{ar,10^3} = \sigma'_{arb,10^3} C_{load} C_T C_{rel}$$
$$\sigma'_{arb,10^3} = \frac{0.9\sigma_u}{K_f'}$$

Size modification factor

The recommended modification factor for size is:

$$C_{size} = \begin{cases} 1.0, if\ d \le 8\ mm \\ 1.189 d^{-0.097}, if\ 8\ mm < d \le 250\ mm \end{cases}$$

In this case, the component diameter is $D = 2 \times 100 = 200\ mm$. The modification factor for size is then:

$$C_{size} = 1.189d^{-0.097}$$
$$= 1.189(200)^{-0.097}$$
$$= 0.71$$

Load modification factor

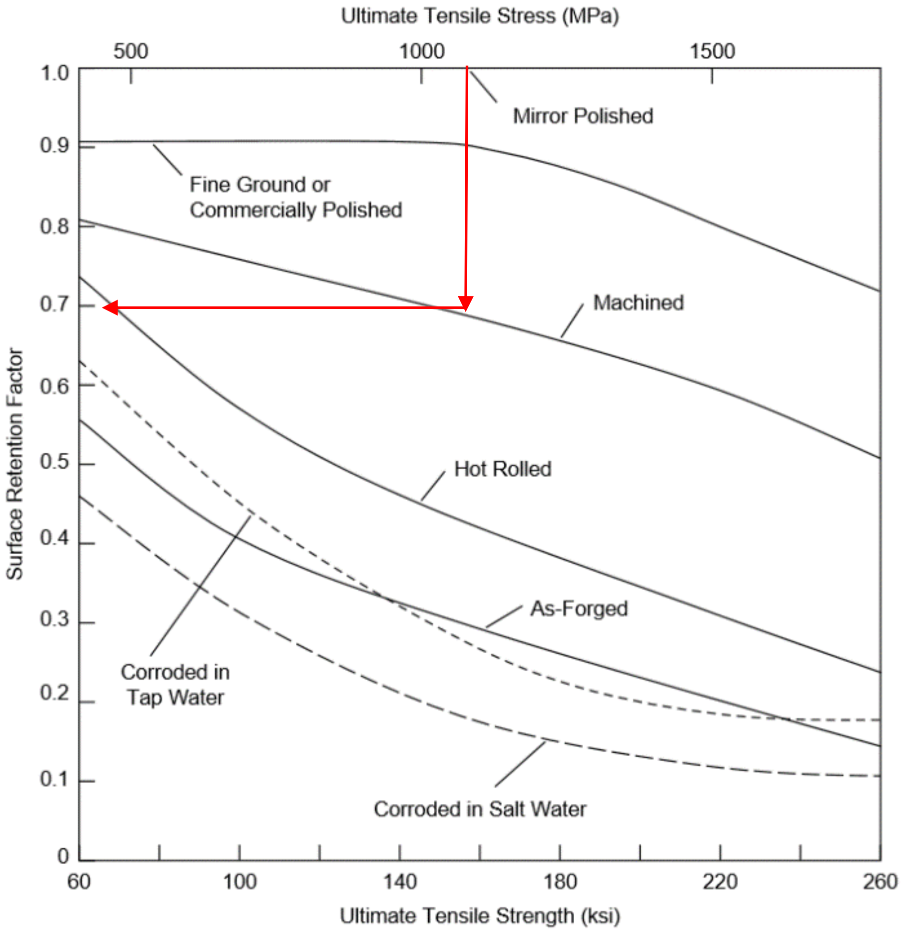
In this problem, the specimen is subject to axial loading. The S-N curve were estimated using equations for rotating bending test. Therefore, a modification must be made for load in this case.

$$\sigma_{er,axial} = 0.70\sigma_{erb}$$

In this case, $C_{load} = 0.70$.

Surface finish modification factor

From the figure below, $C_{surface} = 0.7$.



Temperature modification factor

From the equation below, $C_T = 1.0$ ($T = 200\ °C$).

$$C_T = \begin{cases} 1.0\ for\ T \leq 450\ °C \\ 1 - 5.8^{-3}(T - 450),\ for\ 450 < T \leq 550\ °C \end{cases}$$

Reliability modification factor

From the table below, $C_{rel} = 0.814$.

Reliability $1 - p_f$	C_r
0.5	1
0.9	0.897
0.95	0.868
0.99	0.814
0.999	0.753
0.9999	0.702
0.99999	0.659
0.999999	0.620

11.10.2.3. Mean stress correction

Approach	Equations
Modified Goodman	$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}}$
Gerber	$\sigma_{ar} = \frac{\sigma_a}{1 - \left(\frac{\sigma_m}{\sigma_u}\right)^2}, for\ \sigma_m \geq 0$
Morrow	$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}}$
SWT	$\sigma_{ar} = \sqrt{\sigma_{max}\sigma_a}$ $= \sqrt{(\sigma_m + \sigma_a)\sigma_a}$ $= \sigma_{max}\sqrt{\frac{1-R}{2}}$
Walker	$\sigma_{ar} = \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} \quad (\sigma_{max} > 0)$ $= \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} \quad (\sigma_{max} > 0)$ $\gamma = -0.000200\sigma_u + 0.8818 \quad (\sigma_u\ in\ MPa)$

11.10.2.4. Equation for the S-N curve

$$\sigma_{ar}^m N = C$$
$$\sigma_{ar,1}^m N_1 = \sigma_{ar,2}^m N_2$$
$$m = \frac{\log\left(\frac{N_2}{N_1}\right)}{\log\left(\frac{\sigma_{ar,1}}{\sigma_{ar,2}}\right)}$$
$$N_R = \begin{cases} \left(\frac{\sigma_{ar,1}}{\sigma_{ar}}\right)^m N_1 & 0.9\sigma_u \geq \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$

In this case, the slope of the S-N curve is calculated from the applicable fatigue strengths at 1 000 and 1 000 000 cycles as follows:

$$\sigma_{er} = \sigma'_{\mathrm{erb}} C_{size} C_{load} C_{surf} C_T C_{rel}$$

$$\sigma'_{erb} = \frac{\sigma_{erb}}{K_f}$$

$$\sigma_{er} = \frac{0.5\sigma_u}{K_f} C_{size} C_{load} C_{surf} C_T C_{rel}$$

$$= \frac{0.5 \times 1050}{1.98} \times 0.71 \times 0.7 \times 0.7 \times 1.0 \times 0.814$$

$$= 75\ MPa$$

$$\sigma_{ar,10^3} = \sigma'_{arb,10^3} C_{load} C_T C_{rel}$$

$$\sigma'_{arb,10^3} = \frac{0.9\sigma_u}{K'_f}$$

$$\sigma_{ar,10^3} = \frac{0.9\sigma_u}{K'_f} C_{load} C_T C_{rel}$$

$$= \frac{0.9 \times 1050}{1.37} \times 0.7 \times 1.0 \times 0.814$$

$$= 393.0\ MPa$$

The slope of the S-N curve:

$$m = \frac{\log\left(\frac{N_2}{N_1}\right)}{\log\left(\frac{\sigma_{ar,1}}{\sigma_{ar,2}}\right)}$$

$$= \frac{\log \frac{10^6}{10^3}}{\log \frac{393.0}{75}}$$

$$= 4.17$$

$$N_R = \begin{cases} \left(\frac{75}{\sigma_{ar}}\right)^{4.17} 10^6 & 0.9\sigma_u \geq \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$

11.10.2.5. Damage for one repetition

σ_a [MPa]	σ_m [MPa]	n_i [cyles/block]
100	200	10 000
50	0	5 000
100	100	20 000
200	-100	2 000

$$\sigma_a = 100\ MPa, \sigma_m = 200\ MPa, n_i = 10\ 000\ cycles$$

Goodman mean stress correction:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}}$$

$$= \frac{100}{1 - \frac{200}{1050}}$$

$$= 123.5\ MPa$$

Endurance/fatigue life:

$$N_R = \begin{cases} \left(\frac{75}{\sigma_{ar}}\right)^{4.17} 10^6 & 0.9\sigma_u \geq \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$

$$= \left(\frac{75}{123.5}\right)^{4.17} 10^6$$

$$= 124\ 955\ cycles$$

Damage:

$$d_i = \frac{n_i}{N_i}$$

$$d_1 = \frac{10\ 000}{124\ 955}$$

$$= 0.0800$$

$$\sigma_a = 50\ MPa, \sigma_m = 0\ MPa, n_i = 5\ 000\ cycles$$

Goodman mean stress correction:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} = \frac{50}{1 - \frac{0}{1050}} = 50\ MPa$$

Endurance/fatigue life:

$$N_R = \begin{cases} \left(\frac{75}{\sigma_{ar}}\right)^{4.17} 10^6 & 0.9\sigma_u \geq \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$
$$= \infty$$

Damage:

$$d_i = \frac{n_i}{N_i}$$
$$d_2 = \frac{5\ 000}{\infty} = 0$$

$$\sigma_a = 100\ MPa, \sigma_m = 100\ MPa, n_i = 20\ 000\ cycles$$

Goodman mean stress correction:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} = \frac{100}{1 - \frac{100}{1050}} = 110.5\ MPa$$

Endurance/fatigue life:

$$N_R = \begin{cases} \left(\frac{75}{\sigma_{ar}}\right)^{4.17} 10^6 & 0.9\sigma_u \geq \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$
$$= \left(\frac{75}{110.5}\right)^{4.17} 10^6$$
$$= 198\ 694\ cycles$$

Damage:

$$d_i = \frac{n_i}{N_i}$$
$$d_3 = \frac{20\ 000}{198\ 694} = 0.1007$$

$$\sigma_a = 200\ MPa, \sigma_m = -100\ MPa, n_i = 2\ 000\ cycles$$

Goodman mean stress correction:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_u}} = \frac{200}{1 - \frac{-100}{1050}} = 182.6\ MPa$$

Endurance/fatigue life:

$$N_R = \begin{cases} \left(\frac{75}{\sigma_{ar}}\right)^{4.17} 10^6 & 0.9\sigma_u \geq \sigma_{ar} \geq \sigma_{er} \\ \infty & \sigma_{ar} < \sigma_{er} \end{cases}$$
$$= \left(\frac{75}{182.6}\right)^{4.17} 10^6$$
$$= 24\ 465\ cycles$$

Damage:

$$d_i = \frac{n_i}{N_i}$$

$$d_4 = \frac{2\,000}{24\,465} = 0.0817$$

11.10.2.6. Total damage per repetition

The total damage according to the Palmgren-Miner rule is:

$$\begin{aligned} D &= \sum d_i \\ &= \sum \frac{n_i}{N_i} \\ &= d_1 + d_2 + d_3 + d_4 \\ &= 0.0800 + 0 + 0.1007 + 0.0817 \\ &= 0.262 \end{aligned}$$

The number of repetitions to failure is then 3.8:

$$\begin{aligned} B_f &= \frac{1}{D} \\ &= \frac{1}{0.262} \\ &= 3.8 \end{aligned}$$

11.10.3. Check with Excel

For the supplied stress spectrum, the life is 12 Blocks for a 1% probability of failure (99% probability of survival).

Table 9: Stress spectrum for one block

S_{ut}	1050	MPa	S_e^*	265	MPa	
	152.3948	ksi	$S_{10^3}^*$	689	MPa	
a	0.003384	in	C_{rel}	0.814		
	0.085962	mm	C_{size}	0.71		
r	4	mm	C_T	1		
K_t	2		C_{load}	0.7		
K_f	1.98		$C_{surface}$	0.7		
q	0.38		S_{10^3}	392	MPa	
K_f^*	1.37		S_e	75	MPa	
			m	4.18		
σ_a [MPa]	σ_m [MPa]	n_i [cyles/block]	S_a [MPa]	N_i [cycles]	$D_i = \frac{n_i}{N_i}$	
100	200	10 000	124	125 204	0.07987	
50	0	5 000	50	INF	0	
100	100	20 000	111	199 273	0.100365	
200	-100	2 000	183	24 454	0.081786	
			Total damage =		0.262021	
				Period =	1	Block
				Life =	3.816483	Block

11.10.4. Conclusion

The fatigue life of the component for 99% probability of survival is 3.8 repetitions.

11.11. Multiaxial fatigue

Consider the following loading on a thin-wall pipe:

- Pressure, that causes the biaxial stresses in the longitudinal and circumferential directions
- Torsion

If the torsion is constant and the pressure cycles, the direction of the principal axis change direction as the pressure goes through zero (in which case it will be at 45°).

11.11.1. Octahedral stress approach

This approach is suitable where:

- Cyclic loads are completely reversed and have the same frequency
- Cyclic loads are either in-phase (0° phase shift) or 180° out-of-phase with one another.

For zero mean

The octahedral stress is used to calculate the effective stress amplitude:

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{1a}-\sigma_{2a})^2 + (\sigma_{2a}-\sigma_{3a})^2 + (\sigma_{3a}-\sigma_{1a})^2} \tag{77}$$

For non-zero mean

The effective mean stress is:

$$\bar{\sigma}_m = \sigma_{1m} + \sigma_{2m} + \sigma_{3m} \tag{78}$$

The equivalent stress amplitude is then calculated as in Equation (77), or from the following equation if the principal stresses are not known:

$$\bar{\sigma}_a = \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_{xa}-\sigma_{ya}\right)^2 + \left(\sigma_{ya}-\sigma_{za}\right)^2 + (\sigma_{za}-\sigma_{xa})^2 + 6(\tau_{xya}^2 + \tau_{yza}^2 + \tau_{zxa}^2)} \tag{79}$$

Mean stress correction is then done as before using one of the mean stress correction equations. If the Morrow equation is applied, the equivalent completely reversed uniaxial stress amplitude is given by:

$$\begin{aligned} \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} &= 1 \\ \sigma_{ar} &= \frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_f'}} \\ &= \frac{\bar{\sigma}_a}{1-\frac{\bar{\sigma}_m}{\sigma_f'}} \end{aligned} \tag{80}$$

Use σ_{ar} on the relevant fatigue curve.

Pure shear

For pure shear, $\tau_{xya} = \tau_{xym} \neq 0$, the octahedral equation becomes:

$$\begin{aligned} \bar{\sigma}_a &= \frac{1}{\sqrt{2}}\sqrt{6\left(\tau_{xya}^2\right)} \\ &= \sqrt{3}\tau_{xya} \quad \bar{\sigma}_m = 0 \end{aligned} \tag{81}$$

The mean stress $\bar{\sigma}_m = 0$ even if a mean shear stress is present. This agrees with experimental observation.

11.11.2. Sine's method

Sine's observed the following:

- Torsional mean stress did not affect fatigue life of a specimen subject to alternating torsion or bending stress.
- Tensile mean stress reduces fatigue life of a component subject to cyclic torsional loading.
- Tensile mean stress decrease and compressive mean stress increase fatigue life of a specimen under uniaxial loading.

The Sine's relationship:

$$\frac{1}{3}((P_1-P_2)^2 + (P_2-P_3)^2 + (P_1-P_3)^2)^{0.5} + \alpha\left(S_x + S_y + S_z\right) \le A \tag{82}$$

Where:

P_1, P_2, P_3 Amplitudes of the alternating principal stresses

S_x, S_y, S_z Orthogonal (any coordinate system) static (mean) stresses

α Material constant which gives variation of the permissible range of stress with static stress

A Material constant proportional to the reversed fatigue strength

The constants can be found easily:

- A can be found from the fatigue strength of a completely reversed uniaxial test.
- α is only needed when there are non-zero mean static stresses present.

Disadvantages:

- The model is only dependent on the strain at two instants in time which shows path independence that has been shown as incorrect by experimental data.
- Limited to applications in which the principal axes of the alternating stress components are fixed to the body.
    - This can be done by modification into the maximum range of shear stress criterion.

11.11.3. Maximum range of shear stress criterion

This criterion eliminates the restriction that the principal axes of the body must remain fixed to the body. The relationship is obtained by modification of the first term of the Sine's equation:

$$\frac{1}{6}((\Delta S_{11} - \Delta S_{22})^2 + (\Delta S_{22} - \Delta S_{33})^2 + (\Delta S_{11} - \Delta S_{33})^2 + 6[\Delta S_{12}^2 + \Delta S_{23}^2 + \Delta S_{31}^2])^{0.5} + \alpha(S_x + S_y + S_z) \leq A \quad (83)$$

The terms in brackets is maximized with respect to time where:

$$\begin{array}{ll} Stress\ differences & \Delta S_{ij} = \sigma_{ij}(t_1) - \sigma_{ij}(t_2) \\ Stress\ component\ at\ t_1 & \sigma_{ij}(t_1) \\ Stress\ component\ at\ t_2 & \sigma_{ij}(t_2) \end{array} \quad (84)$$

11.11.4. Maximum range of shear strain

This method is used in the low cycle fatigue regime. The effective strain range is calculated as:

$$\Delta\bar{\varepsilon} = \frac{\sqrt{2}}{3}((\Delta\varepsilon_{11} - \Delta\varepsilon_{22})^2 + (\Delta\varepsilon_{22} - \Delta\varepsilon_{33})^2 + (\Delta\varepsilon_{11} - \Delta\varepsilon_{33})^2 + 6[\Delta\varepsilon_{12}^2 + \Delta\varepsilon_{23}^2 + \Delta\varepsilon_{31}^2])^{0.5} \leq A \quad (85)$$

The terms in brackets is maximized with respect to time where:

$$\begin{array}{ll} Strain\ differences & \Delta\varepsilon_{ij} = \varepsilon_{ij}(t_1) - \varepsilon_{ij}(t_2) \\ Strain\ component\ at\ t_1 & \varepsilon_{ij}(t_1) \\ Strain\ component\ at\ t_2 & \varepsilon_{ij}(t_2) \end{array} \quad (86)$$

11.11.5. Critical plane approach

This approach is based on the observation that fatigue cracks initiate on planes of maximum shear. Brown & Millar used an approach where the critical plane is considered at the maximum shear strain plane, and the crack initiation is driven by the shear and tensile or normal strain acting on it:

$$\frac{\Delta\gamma}{2} + \Delta\varepsilon_n = C \quad (87)$$

Where:

$\frac{\Delta\gamma}{2}$ Is the shear strain amplitude on the maximum shear strain plane

$\Delta\varepsilon_n$ Is the tensile normal strain range to this plane

C Is the material constant

11.12. Conclusion

- Endurance limit only exist in plain carbon and low-alloy steels
- Following factors will reduce the endurance limit

  - Tensile mean stress, large section size, rough surface finish, chrome and nickel plating (except in corrosive environment), decarburization due to forging and hot rolling, severe grinding
- Following factors tend to increase the endurance limit:
  - Nitriding, flame and induction hardening, carburization, shot peening, cold rolling
  - Chrome and nickel plating for materials in a corrosive environment

The stress-strain relationship for isotropic materials are:

$$\varepsilon_1 = \frac{\sigma_1}{E}$$
$$\varepsilon_2 = -\frac{\nu}{E}(\sigma_1)$$
$$\varepsilon_3 = -\frac{\nu}{E}(\sigma_1)$$

12. STRAIN-LIFE ANALYSIS

This section introduces strain life analysis. MSV 780 students must have an in-depth understanding of the strain-life method. Students in welding courses only need to take note of the strain-life method and understand the governing principles in this method.

Presentation used in class:	Investmech - Fatigue (Strain life analysis) R0.0
Download link for the Excel sheet for strain life sequence effects:	Document name: Strain life example - Sequence effect.xls Link: http://www.investmech.com/fatigue.html
Supporting chapters in textbook:	Chapter 12 in the text book titled: 'Plastic Deformation Behavior and Models for Materials' form part of this section, and, must be thoroughly studied as the principles of the chapter will be confirmed in the semester test. This chapter is for self-study. Chapter 14: Strain-based approach to fatigue

The objectives of this section are to:

- Understand the strain life method
- Apply the relevant equations for
  - Zero and non-zero means
  - Multiaxial stress
- Apply strain-based method for fatigue life estimates

12.1. Introduction

Text book Section 14.1.

Strain-based fatigue:

- Considers plastic deformation that may occur in localized regions:
  - Edges of beams
  - Stress raisers
  - Notches
- Stress-strain relationships in these regions are applied in fatigue
- Models local yielding:
  - Case for ductile materials at relatively short lives
- Also applies where there are little plasticity & long lives
  - Therefore, strain-based approach is a comprehensive approach that can be used in the place of stress-based approach
- Initially developed in 1950's & early 1960's in response to need to analyse fatigue problems involving short fatigue lives:
  - Nuclear reactors
  - Jet engines
- Best method to model short fatigue lives
- Best method to model sequence effect of occasional severe events

Table 10: Comparison of the stress- and strain-based approach to fatigue life estimation

Stress-based approach	Strain-based approach
Use nominal stress	Use local stress and strains (local yielding)
Elastic concentration factors	Cyclic stress-strain relationship at notch
Nominal stress vs life (S-N)	Strain vs life ($\varepsilon - N$)
Good for long lives	Good for short and long lives
Mean nominal stress	More rational & accurate handling of mean stress effect by employing the local mean stress at the notch
No specific analysis of crack propagation (growth)	No specific analysis of crack propagation (growth)
Can not model the sequence effect of severe events	Best method to model sequence effects of severe events

12.1.1. Cyclic stress-strain curve (constitutive model)

The following cyclic stress-strain curve is applied:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \tag{88}$$

12.2. Engineering and true stress and strain

Selfstudy: Dowling Chapter 4.

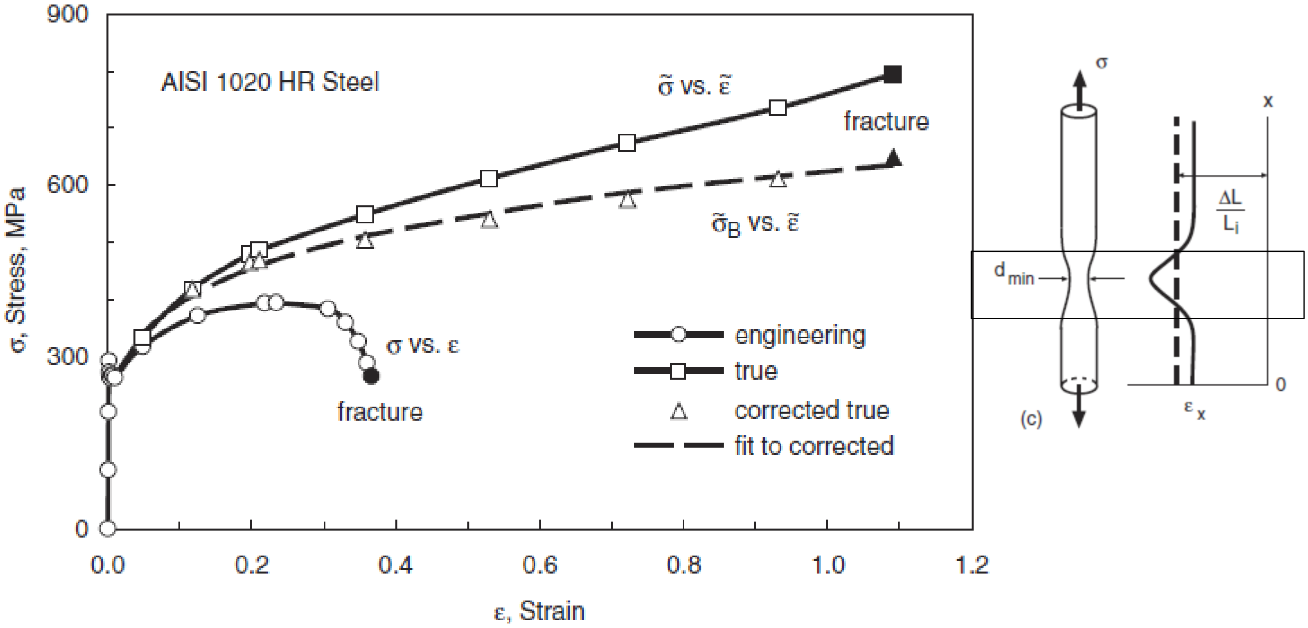
See Table 11 and Figure 48 for an explanation of the calculation of engineering and true stress-strain relationships of a material.

At the end of the test there is large amount of necking; which result in a tensile hoop stress; therefore, the state of stress is not uniaxial anymore. Bridgman correction can be done on steels (Dowling, 2013, p. 146):

$$\begin{aligned} \tilde{\sigma}_B &= B\tilde{\sigma} \\ B &= \begin{cases} 1 & \tilde{\varepsilon} < 0.12 \\ 0.0648x^3 + 0.0461x^2 - 0.205x + 0.825 & 0.12 \leq \tilde{\varepsilon} \leq 3 \\ not\ defined\ yet & \tilde{\varepsilon} > 3 \end{cases} \\ x &= \log_{10}\tilde{\varepsilon} \end{aligned} \tag{89}$$

Table 11: Engineering and true stress and strain

Engineering A_i is the original cross-sectional area L_i is the original length	**True** A is the current cross-sectional area L is the current length
Engineering stress: $\sigma = \frac{P}{A_i}$	True stress: $\tilde{\sigma} = \frac{P}{A}$
Engineering strain: $\varepsilon = \frac{\Delta L}{L_i}$	True strain: $\tilde{\varepsilon} = \frac{\Delta L_1}{L_1} + \frac{\Delta L_2}{L_2} + \frac{\Delta L_3}{L_3} + \cdots = \sum \frac{\Delta L_j}{L_j}$ $= \int_{L_i}^{L} \frac{1}{L} dL$ $= \ln\left(\frac{L}{L_i}\right)$ $= \ln(1+\varepsilon)$
Constant volume assumption relationships	
Constant volume assumption $A_i L_i = AL$ $\frac{A_i}{A} = \frac{L}{L_i} = \frac{L_i + \Delta L}{L_i} = 1 + \varepsilon$ $\tilde{\sigma} = \sigma(1+\varepsilon)$ $\tilde{\varepsilon} = \ln\left(\frac{A_i}{A}\right)$	Constant volume assumption and round sections $\tilde{\varepsilon} = \ln\left(\frac{A_i}{A}\right)$ $= \ln\left(\frac{\frac{\pi}{4} d_i^2}{\frac{\pi}{4} d^2}\right)$ $= 2\ln\left(\frac{d_i}{d}\right)$



Source: (Dowling, 2013, p. 143)

Figure 48: Engineering and true stress-strain curves from a tension test on hot-rolled AISI 1020 steel

12.3. Stress-strain curves

12.3.1. Elastic, perfectly plastic

Reference: Prescribed textbook Section 12.2.1

In this case:

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_o \\ \sigma_o & \sigma \geq \sigma_o \end{cases} \tag{90}$$

12.3.2. Elastic, linear-hardening relationship

Section 12.2.2 of prescribed textbook give the following:

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_o \\ (1-\delta)\sigma_o + \delta E\varepsilon & \sigma \geq \sigma_o \end{cases} \tag{91}$$

12.3.3. Elastic, Power-hardening relationship

Reference: Prescribed textbook Section 12.2.3

In this case it is assumed that the stress is the strain raised to a power (the strain hardening exponent) once yielding starts to occur.

The stress-strain relationship is:

$$\sigma = \begin{cases} E\varepsilon & \sigma \leq \sigma_o \\ H_1\varepsilon^{n_1} & \sigma \geq \sigma_o \end{cases} \tag{92}$$

Or, in terms of strain:

$$\varepsilon = \begin{cases} \frac{\sigma}{E} & \sigma \leq \sigma_o \\ \left(\frac{\sigma}{H_1}\right)^{\frac{1}{n_1}} & \sigma \geq \sigma_o \end{cases} \tag{93}$$

The yield strength can be estimated as follows:

$$\sigma_o = E\left(\frac{H_1}{E}\right)^{\frac{1}{1-n_1}} \tag{94}$$

12.3.4. Ramberg-Osgood relationship

Reference: Prescribed textbook Section 12.2.4

The Ramberg-Osgood relationship is:

$$\begin{aligned} \varepsilon &= \varepsilon_e + \varepsilon_p \\ &= \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{\frac{1}{n}} \end{aligned} \tag{95}$$

It provides a single smooth curve and does not exhibit a distinct yield point. The yield strength is then estimated as:

$$\sigma_o = H(0.002)^n \tag{96}$$

Ramberg-Osgood & the power-hardening rules are equivalent when the plastic portion dominates. In this case: $H_1 = H, n_1 = n$

12.4. Notch stress and strain for local yielding – Neuber's rule

Reference: Prescribed textbook Sections 13.5.3 to 13.5.6.

Normally numerical analysis by FEA is done to calculate stress and strain in the notch. This is time consuming and expensive for nonlinear elasto-plastic stress-strain relationships, because repetitive cycles must be simulated.

When plastic deformation begins at the notch, $\frac{\sigma}{S} < k_t$ because of the flatter stress-strain curve due to yielding. For linear-elastic analysis k_t remains constant.

Neuber's rule is one approximate method for estimating notch stresses and strains. It is also widely used. Neuber's rule states that the geometric mean of the stress and strain concentration factors remains equal to the theoretical stress concentration factor, k_t, during plastic deformation:

$$\begin{aligned} k_t &= \sqrt{k_\sigma k_\varepsilon} \\ k_\sigma &= \frac{\sigma}{S}, for\ \sigma \leq \sigma_o \\ k_\varepsilon &= \frac{\varepsilon}{e}, for\ \sigma \geq \sigma_o \end{aligned} \tag{97}$$

Where:

e nominal strain (away from the notch)

S average or nominal stress (away from the notch)

ε point strain (strain in the notch)

σ point stress (stress in the notch)

σ_o yield strength

12.4.1. Fully plastic yielding

For fully plastic yielding $k_\sigma \to 1$, therefore, it is clear that the strain concentration factor is limited by k_t^2:

$$\begin{aligned} k_\sigma &= \frac{\sigma}{S} = 1 \\ k_t &= \sqrt{k_\sigma k_\varepsilon} \\ &= \sqrt{1 \cdot k_\varepsilon} \\ k_\varepsilon &= k_t^2 \end{aligned} \tag{98}$$

12.4.2. No fully plastic yielding

This is the case for notch analysis. The nominal stress does not exceed the yield strength of the material; however, local plasticity occurs at the notch.

In this case:

$$
\begin{aligned}
e &= \frac{S}{E} \\
k_\sigma &= \frac{\sigma}{S} \\
k_\varepsilon &= \frac{\varepsilon}{e} \\
&= \frac{\varepsilon E}{S} \\
k_t &= \sqrt{k_\sigma k_\varepsilon} \\
&= \sqrt{\frac{\sigma}{S} \cdot \frac{\varepsilon E}{S}} \\
&= \sqrt{\frac{\sigma \varepsilon E}{S^2}} \\
\boldsymbol{\sigma\varepsilon} &= \frac{(\boldsymbol{k_t S})^2}{\boldsymbol{E}}
\end{aligned}
\tag{99}
$$

Note:

- Equation (109) also gives the correct solution consistent with linear-elastic behaviour if the local stress does not exceed the yield strength.
- Equation (109) will underestimate strains if there is fully plastic yielding. In this case the equation does not apply.

12.4.3. Elastic, perfect plastic material beyond yielding

In this case the strains are calculated by using: $\sigma = \sigma_o$, the yield strength of the material.

$$
\begin{aligned}
\sigma\varepsilon &= \frac{(k_t S)^2}{E} \\
\varepsilon &= \frac{(k_t S)^2}{\sigma_o E}
\end{aligned}
\tag{100}
$$

12.4.4. Notch stress and strain and the Ramberg-Osgood stress-strain curve

The first equation is then the Ramberg-Osgood stress-strain relationship:

$$
\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}} \tag{101}
$$

The second equation is then:

$$
\sigma\varepsilon = \frac{(k_t S)^2}{E} \tag{102}
$$

Substitution gives the following function from which the notch stress can be solved using the fzero function in Matlab, or, plot and read off the graphs, etc.:

$$
\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}} = \frac{(k_t S)^2}{\sigma E} \tag{103}
$$

The notch strain is then:

$$
\begin{aligned}
\sigma\varepsilon &= \frac{(k_t S)^2}{E} \\
\varepsilon &= \frac{(k_t S)^2}{\sigma E}
\end{aligned}
\tag{104}
$$

12.4.5. Neuber's rule during cyclic loading

In this case Neuber's rule becomes:

$$
\Delta\sigma\Delta\varepsilon = \frac{(k_t \Delta S)^2}{E} \tag{105}
$$

The Ramberg-Osgood relationship becomes:

$$
\begin{aligned}
\Delta\varepsilon &= \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}} \\
\varepsilon_a &= \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}
\end{aligned}
\tag{106}
$$

12.4.6. Discussion

- Neuber's rule, Equation (97), can be used for all loads except for fully plastic yielding.
- Neuber's rule may be applied for complex situations (e.g. combined bending & torsion, multiaxial loading). Application is difficult for nonproportional loading with principal axes varying in direction during stressing.
- Neuber's rule is an approximation. More accurate results may be obtained by FEA using accurate constitutive models, or, from strain measurements.

12.4.7. Apply the rules

<u>Problem statement</u>

A notched member shown below has an elastic stress concentration factor $k_t = 2.2$. The material has an elastic, power-hardening stress-strain curve with:

- $E = 200\ GPa$
- $H_1 = 834\ MPa$
- $n_1 = 0.2$

Estimate the residual stress and strain at the notch using the power-hardening and Ramberg-Osgood stress-strain relations if the member is loaded to a nominal stress of:

a) $S' = 200\ MPa$

b) $S' = 300\ MPa$

<u>Solution</u>

Step 1: Determine if the stress at the notch exceeds the yield strength:

Power-hardening stress-strain curve:

$$
\begin{aligned}
\sigma_o &= E\left(\frac{H_1}{E}\right)^{\frac{1}{1-n_1}} \\
&= 200 \times 10^3 \left(\frac{843}{200 \times 10^3}\right)^{\frac{1}{1-0.2}} \\
&= 214.8\ MPa
\end{aligned}
$$

Ramberg-Osgood stress-strain:

The yield strength is estimated from the following for the Ramberg-Osgood approach:

$$\begin{aligned}\sigma_o &= H(0.002)^n \\ &= 843(0.002)^{0.2} \\ &= 243\ MPa\end{aligned}$$

The stress at the notch for linear-elastic stress-strain is:

$$\begin{aligned}\sigma &= k_t S \\ &= 2.2 \times 200 \\ &= 440\ MPa\end{aligned}$$

This exceeds the yield strength. Note, this check is only required for the power-hardening stress-strain relationship.

Step 2: Calculate notch stress and strain after the first reversal of 200 MPa

Power-hardening stress-strain relationship for stress above σ_o:

In this case we have:

$$\begin{aligned}\sigma &= H_1 \varepsilon^{n_1} \\ &= 843\varepsilon^{0.2}\end{aligned}$$

And

$$\begin{aligned}\sigma\varepsilon &= \frac{(k_t S)^2}{E} \\ &= \frac{(2.2 \times 200)^2}{200 \times 10^3} \\ &= 0.968\end{aligned}$$

Therefore:

$$\begin{aligned}H_1 \varepsilon^{n_1+1} &= \frac{(k_t S)^2}{E} \\ \varepsilon &= \left[\frac{(k_t S)^2}{H_1 E}\right]^{\frac{1}{1+n_1}} \\ 843\varepsilon^{1+0.2} &= 0.968 \\ \varepsilon &= 0.0035 \\ \sigma &= 272.8\ MPa\end{aligned}$$

Ramberg-Osgood stress-strain relationship:

For the loading, the stress in the notch is calculated as follows for the Ramberg-Osgood stress-strain relationship:

$$\frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}} = \frac{(k_t S)^2}{\sigma E}$$

$$f(\sigma) = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}} - \frac{(k_t S)^2}{\sigma E} = 0$$

This function is solved with the Matlab fzero function for the loading stress 200 MPa:

```
kt=2.2;E=200e3;H=834;n=0.2;
f=@(S,sigma) sigma./E+(sigma./H).^(1/n)-(kt*S)^2./(sigma*E);
f200=@(sigma) f(200,sigma);
fzero(f200,300)
```

The answer was found 252.9 MPa.

Therefore:

$$\begin{aligned}\varepsilon &= \frac{(2.2 \times 200)^2}{252.9 \times 200E3} \\ &= 0.0038\end{aligned}$$

Step 3: Calculate the notch stress and strain after the second reversal of 200 MPa back to zero – for unloading

The nominal stress range in this case is $\Delta S = 200\ MPa$. The direction is $\psi = -1$.

Power-hardening stress-strain relationship:

Yielding on unloading will occur only if the stress changes by more than $2\sigma_o$.

$$\begin{aligned}k_t \Delta S &= 2.2 \times 200 \\ &= 440\ MPa\end{aligned}$$

$$\begin{aligned}2\sigma_o &= 2 \times 272.8 \\ &= 546\ MPa\end{aligned}$$

Therefore, yielding will not occur on the unloading stress range. In this case:

$$\begin{aligned}\Delta\sigma &= k_t \Delta S \\ &= 440\ MPa\end{aligned}$$

The notch stress after the unloading is then:

$$\begin{aligned}\sigma_2 &= \sigma_1 + \psi\Delta\sigma \\ &= 272.8 + \psi 440 \\ &= -167.2\ MPa\end{aligned}$$

For strain:

$$\begin{aligned}\Delta\varepsilon &= \frac{\Delta\sigma}{E} \\ &= \frac{440}{200 \times 10^3} \\ &= 0.0022\end{aligned}$$

And the residual strain is:

$$\begin{aligned}\varepsilon_2 &= \varepsilon_1 + (\psi)\Delta\varepsilon \\ &= 0.0035 - 0.0022 \\ &= 0.0013\end{aligned}$$

Therefore, the material at the notched elongated, and, upon unloading, the remaining material tried to return to its original position resulting in a compressive stress in the notch.

Ramberg-Osgood stress-strain curve:

For stress and strain after the first monotonic load, the cyclic form of the Ramberg-Osgood equation applies:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}}$$

Further, Neuber's equation becomes:

$$\begin{aligned}\Delta\sigma\Delta\varepsilon &= \frac{(k_t \Delta S)^2}{E} \\ \Delta\varepsilon &= \frac{(k_t \Delta S)^2}{\Delta\sigma E}\end{aligned}$$

The function of which the root must be found is:

$$f(\Delta\sigma) = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}} - \frac{(k_t \Delta S)^2}{\Delta\sigma E} = 0$$

```
kt=2.2;E=200e3;H=834;n=0.2;
df=@(dsigma,dS,kt,H,E,n)
dsigma/E+2*(dsigma/(2*H)).^(1/n)-
(kt*dS)^2./(dsigma*E);
fzero(@(x) df(x,200,kt,H,E,n), 300)
```

The notch stress range is then: $\Delta\sigma = 358\ MPa$, and the residual stress on unloading:

$$\begin{aligned}\sigma_2 &= \sigma_1 + \psi\Delta\sigma \\ &= 252.9 - 358.1 \\ &= -105\ MPa\end{aligned}$$

The strain range is:

$$\begin{aligned}\Delta\varepsilon &= \frac{(k_t \Delta S)^2}{\Delta\sigma E} \\ &= \frac{(2.2 \times 200)^2}{358 \times 200 \times 10^3} \\ &= 0.0027\end{aligned}$$

The notch strain on unloading is then:

$$\begin{aligned}\varepsilon_2 &= \varepsilon_1 + \psi\Delta\varepsilon \\ &= 0.0038 - 0.0027 \\ &= 0.0011\end{aligned}$$

As shown, assuming that the strain hardening exponent and H is equal for the power-hardening and Ramberg-Osgood yields approximately the same residual states after unloading.

Step 4: Calculate the stress after the 300 MPa first reversal

The first calculation that should occur is to calculate if the linear-elastic notch stress exceeds the yield strength, which will be the case for the 300 MPa nominal stress.

As before, the following equations are used:

$$\begin{aligned}\sigma &= H_1 \varepsilon^{n_1} \\ &= 843\varepsilon^{0.2}\end{aligned}$$

And

$$\sigma\varepsilon = \frac{(k_t S)^2}{E}$$

Therefore:

$$H_1 \varepsilon^{n_1+1} = \frac{(k_t S)^2}{E}$$

$$\varepsilon = \left[\frac{(k_t S)^2}{H_1 E}\right]^{\frac{1}{1+n_1}}$$

From which the strain after the first reversal becomes:

$$\varepsilon = 0.0070$$

The stress after the first reversal is then:

$$\begin{aligned}\sigma &= 843 \times 0.0070^{0.2} \\ &= 308.9\ MPa\end{aligned}$$

Step 5: Calculate the strain and stress range after the load release and calculate the residual stress and strain

As before, the following equations are used:

$$\frac{\Delta\sigma}{2} = H_1 \left(\frac{\Delta\varepsilon}{2}\right)^{n_1}$$

$$\Delta\sigma = 2\mathrm{H}_1 \left(\frac{\Delta\varepsilon}{2}\right)^{n_1}$$

$$\Delta\sigma = 2 \times 843 \left(\frac{\Delta\varepsilon}{2}\right)^{n_1}$$

And

$$\Delta\sigma\Delta\varepsilon = \frac{(k_t \Delta S)^2}{E}$$

Therefore:

$$H_1 2 \left(\frac{\Delta\varepsilon}{2}\right)^{1+n_1} = \frac{(k_t \Delta S)^2}{E}$$

$$\Delta\varepsilon = 2\left[\frac{(k_t \Delta S)^2}{2 H_1 E}\right]^{\frac{1}{1+n_1}}$$

For the parameters given, the strain range is solved as $\Delta\varepsilon = 0.0079$ using:

```
2*((kt*300)^2/(2*H*E))^(1/(1+n))
```

The stress range is: $\Delta\sigma = 551.4\ MPa$.

The residual stress and strain are then:

$$\begin{aligned}\varepsilon_2 &= \varepsilon_1 + \psi\Delta\varepsilon \\ &= 0.0070 - 0.0079 \\ &= -0.0009\end{aligned}$$

$$\begin{aligned}\sigma_2 &= \sigma_1 + \psi\Delta\sigma \\ &= 308.9 - 551.4 \\ &= -242\ MPa\end{aligned}$$

Note the compressive residual strain and stress in this case. This is because a large part of the component was subject to plastic deformation by the 300 MPa that exceeds the material yield strength.

12.5. Strain vs Life curves

Strain-based fatigue:

- Considers plastic deformation that may occur in localized regions where fatigue cracks begin (at stress concentrations).
- Permits detailed consideration of fatigue situation where local yielding occurs.
- Also applies where there is little plasticity at long lives.
- Is a comprehensive approach that can be used in place of the stress-based approach.
- Employs the local mean stress at the notch.

12.5.1. Strain-life tests & equations

Use strain amplitude vs cycles to failure, analogous to the stress-based $S - N$ curves.

Derive from tests under completely reversed strain ($R = -1$) between constant strain limits, stress is measured, as described in ASTM E606.

Fatigue life, N_f, defined as occurring when there is substantial cracking of the specimen.

Amplitudes of stress (σ_a), strain (ε_a), and plastic strain (ε_{pa}) are measured from hysteresis loop.

The strain amplitude consists of the elastic and plastic strain parts:

$$\begin{aligned} \varepsilon_a &= \varepsilon_{ea} + \varepsilon_{pa} \\ &= Elastic + Plastic \\ &= \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} \\ &= \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \end{aligned} \tag{107}$$

Neuber's rule is used to model the stress at the notch.

The intercept constants $\frac{\sigma_f'}{E}$ and ε_f' are by convention calculated at one reversal or half cycle $N_f = 0.5$ as shown in Figure 49.

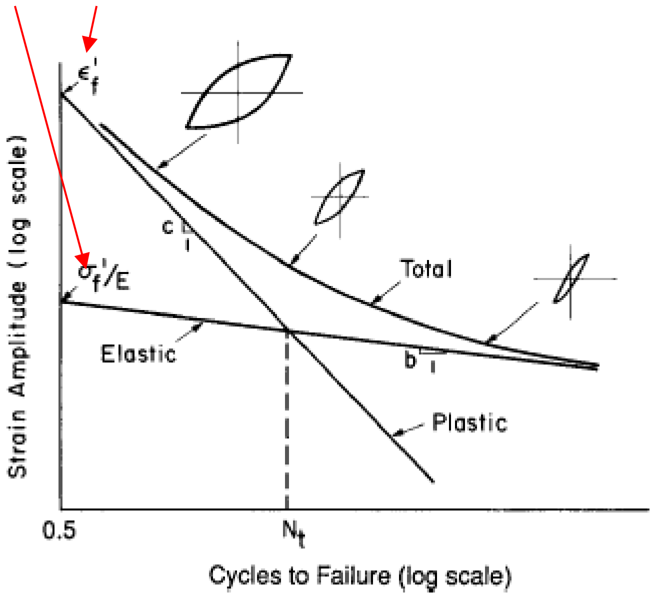


Figure 49: Elastic, plastic, and total strain vs life curves

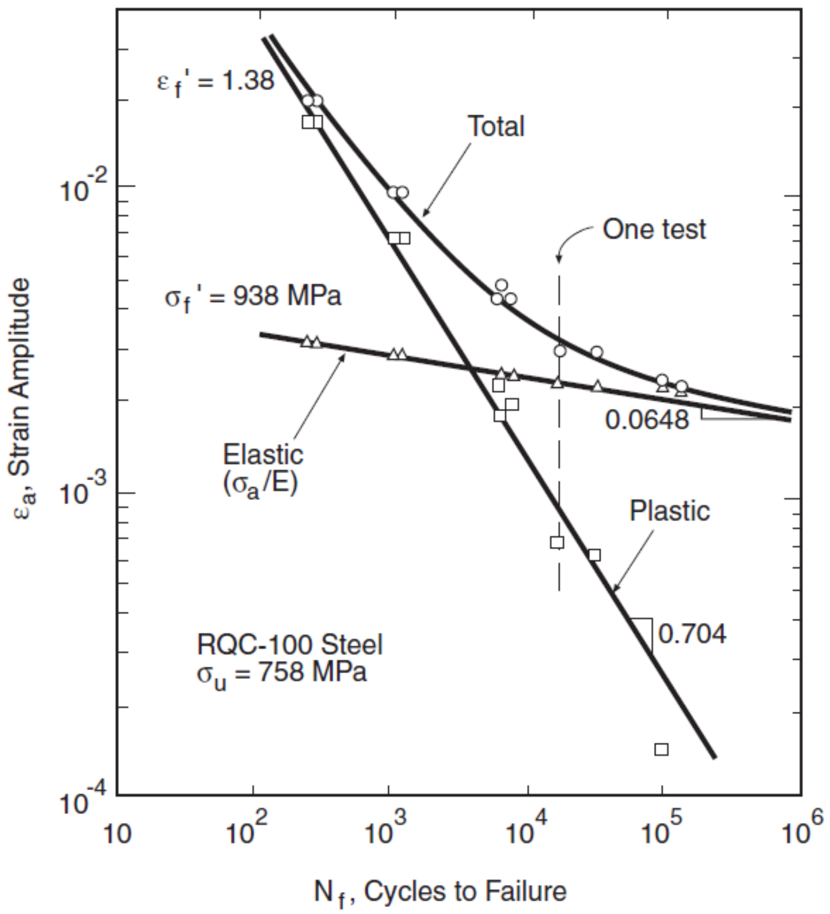
Hysteresis loop used for strain-based fatigue life modelling taken at a cycle number 50% x N_f, is considered to represent approximately stable behaviour after most cycle-dependent hardening or softening is compete.

The fatigue life can now be estimated from:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{108}$$

Generally called the Coffin-Manson relationship.

Note how this equation includes the stress-life relationship as first term ($\sigma_a = \sigma_f'(2N_f)^b$). Therefore, this relationship can be used to predict short and long lives. Figure 50 shows the elastic, plastic, and total strain amplitude vs cycles to failure (fatigue life) for RQC-100 steel.



Source: (Dowling, 2013, p. 749)

Figure 50: Strain vs life curves for RQC-100 steel

Cyclic stress-strain and strain-life constants are summarised in Table 12.

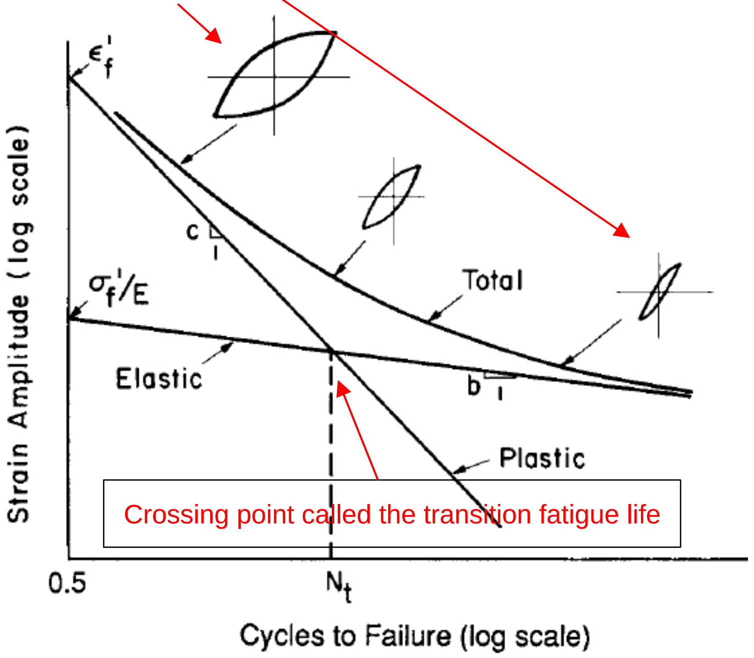
Table 12: Cyclic stress-strain and strain-life constants for selected engineering metals

Material	Source	Tensile properties				Cyclic $\sigma - \varepsilon$ cuve			Strain-life curve			
		σ_o	σ_u	$\tilde{\sigma}_{fB}$	%RA	E	H'	n'	σ_f'	b	ε_f'	c
Steels												
SAE 1015 (normalized)	8	228	415	726	68	207 000	1 349	0.282	1 020	-0.138	0.439	-0.513
Man-Ten[2] (hot-rolled)	7	322	557	990	67	203 000	1 096	0.187	1 089	-0.115	0.912	-0.606
RQC-100 (roller Q&T)	2	683	758	1 186	64	200 000	903	0.0905	938	-0.0648	1.38	-0.704
SAE 1045 (HR & norm)	6	382	621	985	51	202 000	1 258	0.208	948	-0.092	0.260	-0.445
Source: (Dowling, 2013, pp. 751, Table 14.1)												

12.5.2. Comments

At long lives, the elastic term is dominant, and the curve approximates the elastic strain line. The hysteresis loop in this case is thin.

At short lives, plastic strains are large compared with elastic strains, curve approaches the plastic straight line & hysteresis loops are fat.



Source: (Dowling, 2013, p. 748)

Figure 51: Elastic, plastic & total strain versus life

At the crossing point, the **transition fatigue life** can be calculated as follows, as the elastic and plastic fatigue lives are equal at this point:

$$\begin{aligned}
\frac{\sigma_f'}{E}(2N_t)^b &= \varepsilon_f'(2N_t)^c \\
\frac{(2N_t)^b}{(2N_t)^c} &= \frac{\varepsilon_f' E}{\sigma_f'} \\
(2N_t)^{b-c} &= \frac{\varepsilon_f' E}{\sigma_f'} \\
2N_t &= \left(\frac{\varepsilon_f' E}{\sigma_f'}\right)^{\frac{1}{b-c}} \\
N_t &= \frac{1}{2}\left(\frac{\varepsilon_f' E}{\sigma_f'}\right)^{\frac{1}{b-c}} \\
&= \frac{1}{2}\left(\frac{\sigma_f'}{\varepsilon_f' E}\right)^{\frac{1}{c-b}}
\end{aligned} \tag{109}$$

Transition fatigue life, N_t, locates boundary between fatigue behaviour involving substantial plasticity and behaviour involving little plasticity.

Therefore, the transition fatigue life is the most logical point for separating low-cycle and high-cycle fatigue.

At lives less than N_t the strain-based approach is needed.

At lives longer than N_t the stress-based approach based on elastic analysis is sufficient.

Figure 52 shows that the transition fatigue life decreases with increasing hardness because hardness varies inversely with ductility.

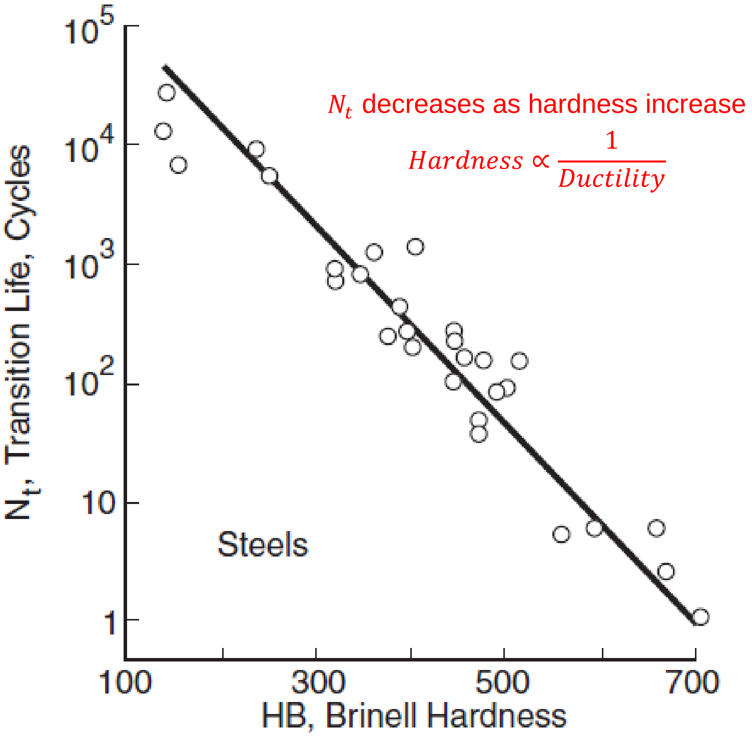


Figure 52: Transition fatigue life vs hardness for a wide range of steels

The plastic strain term of the cyclic stress-strain curve gives:

$$\varepsilon_{pa} = \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$
$$\sigma_a = H'\varepsilon_{pa}^{n'} \tag{110}$$

To find a relationship between the stable cyclic strain hardening exponent, n' and the stable cyclic plastic strain coefficient, H', and material properties, the following applies

The elastic and plastic strain life equations give:

$$\sigma_a = \sigma_f'(2N_f)^b$$
$$2N_f = \left(\frac{\sigma_a}{\sigma_f'}\right)^{\frac{1}{b}}$$
$$and$$
$$\varepsilon_{pa} = \varepsilon_f'(2N_f)^c$$
$$2N_f = \left(\frac{\varepsilon_{pa}}{\varepsilon_f'}\right)^{\frac{1}{c}} \tag{111}$$
$$from\ which\ it\ is\ clear\ that$$
$$\left(\frac{\sigma_a}{\sigma_f'}\right)^{\frac{1}{b}} = \left(\frac{\varepsilon_{pa}}{\varepsilon_f'}\right)^{\frac{1}{c}}$$
$$\sigma_a = \sigma_f'\left(\frac{\varepsilon_{pa}}{\varepsilon_f'}\right)^{\frac{b}{c}}$$

Substitution into Equation (110) gives:

(112)

$$Therefore$$

$$H' = \frac{\sigma_f'}{\left(\varepsilon_f'\right)^{\frac{b}{c}}}$$

$$n' = \frac{b}{c}$$

Thus, from the equation above:

- Only four of the six constants $H', n', \sigma_f', b, \varepsilon_f'\ and\ c$ are independent.
- Common practise to make three separate fits of data so that abovementioned relationships are satisfied approximately.
- Reported values of the six constants may not be exactly mutually consistent as implied by Equation 77.

Ductile materials at very short lives may have strain sufficiently large that true stresses and strains differ significantly from the engineering stresses and strains. In such cases replace $\sigma_a, \varepsilon_{pi}$ & ε_a with the true stress and strain $\tilde{\sigma}_a, \tilde{\varepsilon}_{pa}$ & $\tilde{\varepsilon}_a$.

If a tensile test is interpreted as a fatigue test where failure occurs and $N_f = 0.5$:

- Intercept constants σ_f' & ε_f' should be the same as the true fracture stress and strain from the tension test: $\tilde{\sigma}_f$ & $\tilde{\varepsilon}_f$.
- Constants σ_f' & ε_f' are normally found from curve fitting actual fatigue data and then calculate at $N_f = 0.5$, there is often reasonable agreement with $\tilde{\sigma}_f$ & $\tilde{\varepsilon}_f$.

12.5.3. Trends for engineering materials

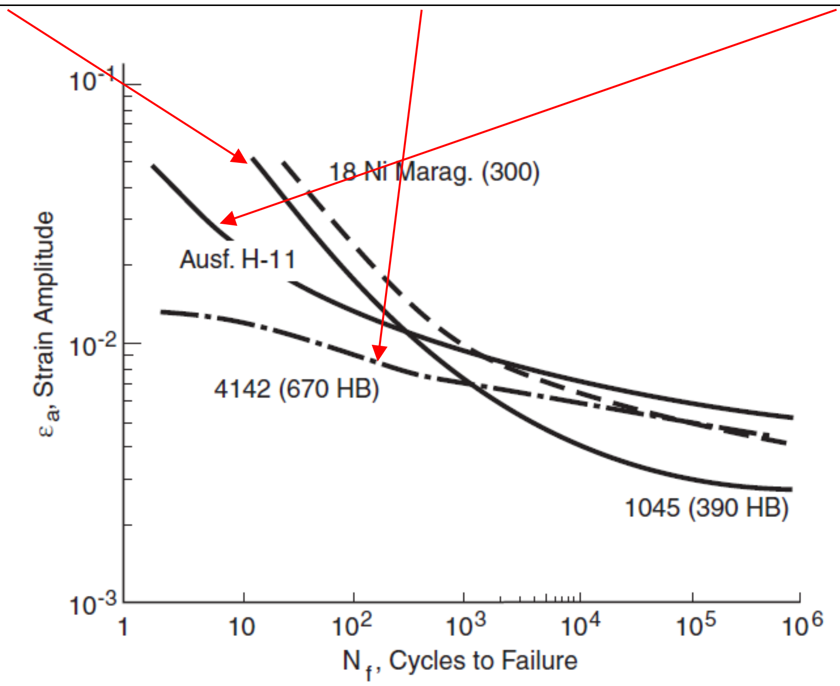
Intercept constants for strain-life curve expected to be similar to the true fracture stress and strain from a tension test:

$$\sigma_f' = \tilde{\sigma}_f \qquad (113)$$
$$\varepsilon_f' = \tilde{\varepsilon}_f$$

Table 13 presents a comparison between ductile, brittle and tough materials.

Table 13: Ductile, brittle and tough materials

Ductile metal	Brittle material	Tough material
Has high $\tilde{\varepsilon}_f$ and low value of $\tilde{\sigma}_f$	Has high $\tilde{\sigma}_f$ and low value of $\tilde{\varepsilon}_f$	Intermediate for both
Plastic strain-life curve tends to be high & elastic strain-life curve low. Hence, steep strain-life curve	Flatter strain-life curve	Strain-life curve between two extremes
Transition fatigue life, N_t, relatively long	Transition fatigue life, N_t, relatively short	Transition fatigue life, N_t, between the two extremes
$b \sim -0.12$ for soft metals	$b \sim -0.05$ for hardened metals	In between
Elastic strain slope: $b \sim -0.085$		
Narrow range for engineering metals plastic strain slop: $-0.5 \leq c \leq -0.8$ Typical value $c = -0.6$		
Strain-life curves for wide variety of engineering metals tend to all pass $(\varepsilon_a; N_f) = (0.01; 1\,000)$		



Source: (Dowling, 2013, p. 756)

Figure 53: Strain-life curves for four hardened steels

For steels with ultimate tensile strengths $\sigma_u \leq 1\,400\ MPa$ a fatigue limit occurs near 10^6 cycles at a stress amplitude around $\sigma_a = \frac{\sigma_u}{2}$. One point that must satisfy the stress-life relationship if $\sigma_f' = \tilde{\sigma}_f$ gives:

$$\sigma_a = \sigma_f'(2N_f)^b \quad (114)$$

$$
\begin{aligned}
\log_{10}\frac{\sigma_a}{\sigma_f'} &= b\log_{10}2N_f \\
b &= \frac{\log_{10}\sigma_a - \log_{10}\sigma_f'}{\log_{10}2N_f} \\
&= \frac{\log_{10}\frac{\sigma_a}{\sigma_f'}}{\log_{10}2N_f} \\
&= \frac{1}{6.301}\log_{10}\frac{\sigma_a}{\sigma_f'} \\
&= \frac{1}{6.301}\log_{10}\frac{0.5\sigma_u}{\sigma_f'}
\end{aligned}
$$

For any other point on the stress-life fatigue curve where the fatigue limit (long-life fatigue strength) is given $\sigma_e = m_e\sigma_u$ at N_e is given, the factor b is:

$$
b = \frac{\log_{10}\frac{m_e\sigma_u}{\sigma_f'}}{\log_{10}(2N_e)} \tag{115}
$$

12.5.4. Effects on strain-life fatigue curve, surface and size effects

12.5.4.1. Surface effects

Surface effects only the fatigue limit (endurance limit). The fracture strength is not affected. At high strains the effect of the surface roughness is eliminated as the high strain propagates sub-surface cracks.

The approach followed is to reduce the fatigue strength of the elastic part in the equation at the endurance limit to $m_s\sigma_e$, where m_s is the surface effect factor, that is then used to calculate a new slope for the S-N curve:

$$
\begin{aligned}
\sigma_e &= \sigma_f'(2N_e)^b \\
m_s\sigma_e &= \sigma_f'(2N_e)^{b_s} \\
&\mathit{Dividing\ the\ equations\ gives}: \\
m_s &= \frac{\sigma_f'(2N_e)^b}{\sigma_f'(2N_e)^{b_s}} \\
m_s &= (2N_e)^{b_s-b} \\
\log m_s &= (b_s - b)\log 2N_e \\
\frac{\log m_s}{\log 2N_e} &= b_s - b \\
b_s &= b + \frac{\log m_s}{\log 2N_e}
\end{aligned} \tag{116}
$$

12.5.4.2. Size effects

Limited results available on the effect of size on the strain-life curve.

Test results on shafts indicate that the whole strain-life curve must be lowered by the size effect factor.

The slope does not change.

This is can be done by lowering the intercept constants σ_f' and ε_f' as follows:

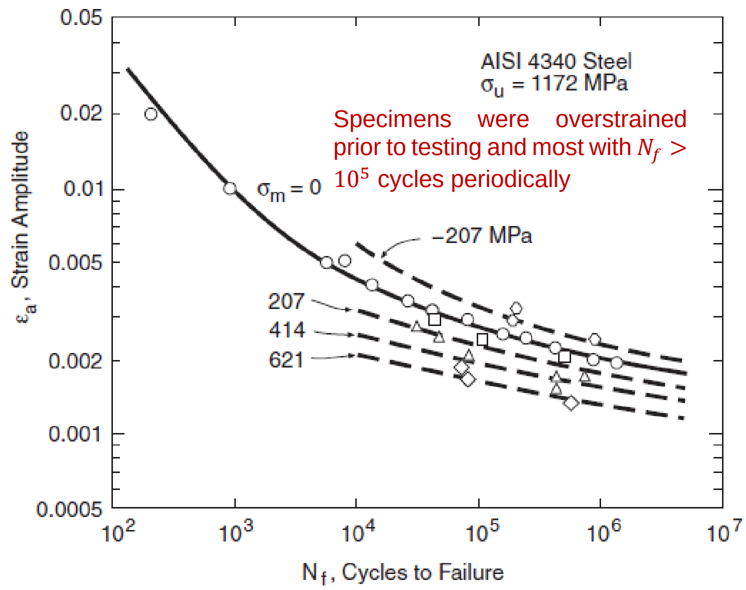
$$
\begin{aligned}
\sigma_{fd}' &= m_d\sigma_f' \\
\varepsilon_d' &= m_d\varepsilon_f'
\end{aligned} \tag{117}
$$

One study for shafts ≤ 250 mm that included data on low-carbon and low-alloy steels resulted in the following reduction factor in terms of the minimum diameter d for shafts containing fillet radii or circumferential grooves:

$$
m_d = \left(\frac{d}{25.4\,mm}\right)^{-0.093} \tag{118}
$$

12.6. Mean stress effects

Experimental results of mean stress effect on the strain-based fatigue life of an alloy is shown in Figure 54 which shows, as was the case with the stress-life equations, that compressive mean stress is beneficial and tensile mean stress detrimental.



Source: (Dowling, 2013, p. 759)

Figure 54: Mean stress effect on the strain-life curve of an alloy steel, with dashed curves from Morrow mean stress correction

Cyclic strain test with non-zero mean strain:

- Cycle dependent relaxation of mean stress likely
- Plastic strain amplitude is small:
  - Some mean stress will remain.
  - Life affected by the mean stress.

Cyclic stress-controlled test:

- No relaxation of mean stress possible
- Cycle-dependent creep can occur
- Plasticity in localized regions
  - Where large cyclic creep deformations are generally prevented by the stiffness of surrounding elastic material
- Results from controlled stress test are of present interest only if the failure is not dominated by cycle-dependent creep

12.6.1. Mean stress effects in the strain-life equations

The elastic stress life part of the strain-life equation is given below. To quantify the effect of mean stress, an additional equation is added for the equivalent completely reversed stress amplitude σ_{ar}:

$$\begin{aligned}\sigma_{ar} &= \sigma_f'\left(2N_f\right)^b \\ \sigma_{ar} &= f(\sigma_a, \sigma_m)\end{aligned} \tag{119}$$

Where $f(\sigma_a, \sigma_m)$ is the stress—based fatigue mean stress correction equations (Goodman, Gerber, Morrow, SWT, Walker, etc.).

To include the mean stress effect in the strain-life relationship in terms of σ_a and σ_m:

$$\begin{aligned}\sigma_{ar} &= f(\sigma_a, \sigma_m) \\ &= \sigma_a \cdot \frac{f(\sigma_a, \sigma_m)}{\sigma_a} \\ &= \sigma_f'\left(2N_f\right)^b \\ \sigma_a &= \sigma_{ar} \times \frac{\sigma_a}{f(\sigma_a, \sigma_m)} \\ &= \sigma_f'\left(2N_f\right)^b \times \frac{\sigma_a}{f(\sigma_a, \sigma_m)} \\ \sigma_a &= \sigma_f'\left[2N_f\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{\frac{1}{b}}\right]^b = \sigma_f'(2N^*)^b \\ N^* &= N_f\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{\frac{1}{b}} \\ N_f &= N^*\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{-\frac{1}{b}}\end{aligned} \tag{120}$$

Where:

N^* Zero-mean-stress-equivalent life

N_f Fatigue life as affected a non-zero mean stress in this case.

The effect on the strain-life equation is then:

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c \tag{121}$$

The approach taken is as follows:

Step 1: Calculate N^* as if the mean stress is zero:

$$\begin{aligned}\varepsilon_a &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c \\ 0 &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c - \varepsilon_a\end{aligned} \tag{122}$$

Step 2: Calculate the N_f to include mean stress effect

$$N_f = N^*\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{-\frac{1}{b}} \tag{123}$$

12.6.2. Morrow mean stress compensation

The completely reversed stress amplitude according to Morrow is:

$$\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\sigma_f'}} \tag{124}$$

Substitution into Equation (123) gives:

$$\begin{aligned} N_{mi}^* &= N_f \left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{\frac{1}{b}} \\ &= N_f \left(\frac{\sigma_a}{\left(\frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_f'}}\right)}\right)^{\frac{1}{b}} \\ N_{mi}^* &= N_f \left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{1}{b}} \\ N_f &= N_{mi}^* \left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{-\frac{1}{b}} \\ N_f &= N_{mi}^* \left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}} \end{aligned} \tag{125}$$

The strain-life equation then becomes:

$$\begin{aligned} \varepsilon_a &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c \\ &= \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \varepsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}(2N_f)^c \end{aligned} \tag{126}$$

For some materials, especially aluminium alloys where σ_f' differ significantly from the true fracture strength, $\tilde{\sigma}_{fB}$, the true fracture strength is used in the relationship:

$$\begin{aligned} &\sigma_{ar} = \frac{\sigma_a}{1 - \frac{\sigma_m}{\tilde{\sigma}_f}} \\ &and \\ &N_{mf}^* = N_f \left(1 - \frac{\sigma_m}{\tilde{\sigma}_{fB}}\right)^{\frac{1}{b}} \end{aligned} \tag{127}$$

Works well for steels.
Inaccurate for aluminium alloys – can be improved by using $\sigma_f' = \tilde{\sigma}_{fB}$

12.6.3. Modified Morrow mean stress compensation

The modified Morrow relationship removes the mean stress effect at the plastic strain part:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \varepsilon_f'(2N_f)^c \tag{128}$$

The equation has the effect of reducing the estimated effect of mean stress at short lives.
Test results showed a good correlation with this formula.

12.6.4. Smith, Watson & Topper (SWT) mean stress compensation

In this approach it is assumed that the life for any situation of mean stress depends on:

$$\sigma_{max}\varepsilon_a = h''(N_f) \tag{129}$$

Where $\sigma_{max} = \sigma_m + \sigma_a$ and $h''(N_f)$ indicates a function of fatigue life N_f.

For zero mean, the equation has the same form as before for completely reversed stress.
If σ_{ar}, the completely reversed stress amplitude and ε_{ar}, the completely reversed strain amplitude, is applied to have the same life as the $\sigma_{max}\varepsilon_a$ combination, the relationship becomes:

$$\begin{aligned}\sigma_{max}\varepsilon_a &= \sigma_{ar}\varepsilon_{ar} \\ &= \sigma_f'(2N_f)^b\varepsilon_{ar} \\ &= \sigma_f'(2N_f)^b\left(\frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c\right) \\ &= \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}\end{aligned} \tag{130}$$

Agreement with test data is good, but not as good as Morrow.

Acceptable results for wide range of materials

Accurate for steels

Good for aluminium alloys

12.6.5. Walker mean stress compensation

As before,

$$\begin{aligned}\sigma_{ar} &= \sigma_{max}^{1-\gamma}\sigma_a^{\gamma} & (\sigma_{max} > 0) \\ &= \sigma_{max}\left(\frac{1-R}{2}\right)^{\gamma} & (\sigma_{max} > 0)\end{aligned} \tag{131}$$

Where $\gamma = -0.000200\sigma_u + 0.8818 \quad (\sigma_u \; in \; MPa)$

Substitution gives:

$$\begin{aligned}N_w^* &= N_f\left(\frac{\sigma_a}{f(\sigma_a, \sigma_m)}\right)^{\frac{1}{b}} \\ &= N_f\left(\frac{\sigma_a}{\sigma_{max}^{1-\gamma}\sigma_a^{\gamma}}\right)^{\frac{1}{b}} \\ &= N_f\left(\frac{\sigma_a^{1-\gamma}}{\sigma_{max}^{1-\gamma}}\right)^{\frac{1}{b}} \\ &= N_f\left(\frac{\sigma_a}{\sigma_{max}}\right)^{\frac{1-\gamma}{b}} \\ &= N_f\left(\frac{1-R}{2}\right)^{\frac{1-\gamma}{2}} \\ N_f &= N_w^*\left(\frac{1-R}{2}\right)^{\frac{\gamma-1}{b}}\end{aligned} \tag{132}$$

Substitution into the stress-strain relationship gives:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(\frac{1-R}{2}\right)^{1-\gamma}(2N_f)^b + \varepsilon_f'\left(\frac{1-R}{2}\right)^{\frac{c(1-\gamma)}{b}}(2N_f)^c \tag{133}$$

The following method can also be applied:

Step 1: Calculate N^* as if the mean stress is zero:

$$\begin{aligned}\varepsilon_a &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c \\ 0 &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c - \varepsilon_a\end{aligned} \tag{134}$$

Step 2: Calculate the N_f to include mean stress effect from $N_w^* = N^*$ above:

$$\begin{aligned}N_f &= N_w^*\left(\frac{\sigma_a}{\sigma_{max}}\right)^{\frac{\gamma-1}{b}} \\ &= N_w^*\left(\frac{1-R}{2}\right)^{\frac{\gamma-1}{b}}\end{aligned} \tag{135}$$

Test results on steels, aluminum alloys, and one titanium alloy gave excellent results (Dowling, 2013, p. 765). Note, if $\gamma = 0.5$, the equation corresponds to the stress-based SWT relationship.

Most accurate of all methods mention if γ can be accurately estimated.

12.6.6. Strain-life example

Problem statement

RQC-100 steel is subjected to cycling with a strain amplitude of $\varepsilon_a = 0.0045$ and a tensile mean stress of $\sigma_m = 120\ MPa$. How many cycles can be applied before fatigue cracking is expected?

Use Morrow, modified Morrow, SWT, and Walker mean stress correction methods

Solution

Material properties

The material properties for RQC-100 steel is found from Dowling Table 14.1 as:

Parameter	Value	Unit
σ_o	683	MPa
σ_u	758	MPa
σ_{fB}	1 186	MPa
$\%RA$	64	
E	200 x 10^3	MPa
H'	903	MPa
n'	0.0905	
σ_f'	938	MPa
b	-0.0648	
ε_f'	1.38	
c	-0.704	

Morrow mean stress compensation

The Morrow equation is given as follows from which the zero-mean-stress-equivalent endurance N^* can be calculated:

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

The number of cycles to crack initiation is then:

$$N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

Apply Morrow

$$0.0045 = \frac{938}{200 \times 10^3}(2N^*)^{-0.0648} + 1.38(2N^*)^{-0.704}$$

Using Matlab code:

```
f=@(N,epsa,sfa,E,b,epsfa,c)
abs(sfa/E*(2*N).^b+epsfa*(2*N).^c - epsa);
Nstar=fminbnd(@(N)
f(N,0.0045,sfa,E,b,epsfa,c), 5000,10000)
```

From which the result was found:

$$N^* = 5\ 635\ cycles$$

The Morrow equation is now applied to calculate the fatigue life:

$$N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

$$= 5\ 635 \times \left(1 - \frac{120}{938}\right)^{-\frac{1}{-0.0648}}$$

$$= 5\ 635 \times \left(1 - \frac{120}{938}\right)^{\frac{1}{0.0648}}$$

$$= 681\ cycles$$

```
Nf=Nstar*(1-120/sfa)^(-1/b)
```

Modified Morrow mean stress compensation

The modified Morrow equation is:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(2N_f)^b + \varepsilon_f'(2N_f)^c$$

$$0.0045 = \frac{938}{200 \times 10^3}\left(1 - \frac{120}{938}\right)(2N_f)^b + 1.38 \times (2N_f)^c$$

This was solved in Matlab as shown below and the answer was found 4 598 cycles:

```
fmm=@(Nf,epsa,sm,sfa,E,b,epsfa,c)   sfa/E*(1-sm/sfa)*(2*Nf).^b+epsfa*(2*Nf).^c - epsa;
Nff=fzero(@(Nf)
fmm(Nf,0.0045,120,sfa,E,b,epsfa,c),[10,8e6])
```

SWT mean stress compensation

The equation for the SWT mean stress correction is:

$$\sigma_{max}\varepsilon_a = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

As shown, the maximum stress in the notch is required, which can be determined from the Ramberg-Osgood equation as follows:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

The equation can then be rewritten in the following format and solved with Matlab:

$$f(\varepsilon_a) = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} - \varepsilon_a = 0$$

The following solver found $\sigma_a = 513\ MPa$

```
ramberg=@(siga,epsa,E,Ha,na)
siga/E+(siga/Ha).^(1/na)-epsa;
sigmaa=fzero(@(siga)
ramberg(siga,0.0045,E,Ha,na), [1,1000])
```

The maximum stress is then:

$$\sigma_{max} = \sigma_a + \sigma_m = 513 + 120 = 633\ MPa$$

Therefore:

$$\sigma_{max}\varepsilon_a = \frac{(\sigma_f')^2}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

$$633 \times 0.0045 = \frac{938^2}{200 \times 10^3}(2N_f)^{2\times -0.0648} + 938 \times 1.38 \times (2N_f)^{-0.0648-0.704}$$

From this the number of cycles to crack initiation in the notch is:

$$N_f = 3\,446\ cycles$$

The following code was used:

```
fswt=@(N,smax,epsa,sfa,E,b,epsfa,c) sfa^2/E*(2*N).^(2*b)+sfa*epsfa*(2*N).^(b+c) -smax*epsa;
Nf=fzero(@(N) fswt(N,633,0.0045,sfa,E,b,epsfa,c),[1000,10000])
```

Remember to modify the interval in the solver above by looking at the result on a graph.

Walker mean stress compensation

The Walker mean stress correction applied to the strain-based fatigue life:

$$\varepsilon_a = \frac{\sigma_f'}{E}\left(\frac{1-R}{2}\right)^{1-\gamma}\left(2N_f\right)^b + \varepsilon_f'\left(\frac{1-R}{2}\right)^{\frac{c(1-\gamma)}{b}}\left(2N_f\right)^c$$

The constant γ:

$$\begin{aligned}\gamma &= -0.000200\sigma_u + 0.8818 \quad (\sigma_u\ in\ MPa)\\ &= -0.0002 \times 758 + 0.8818\\ &= 0.7302\end{aligned}$$

As shown, the stress ratio in the notch is required, which can be determined from the Ramberg-Osgood equation as follows:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$

The equation can then be rewritten in the following format and solved with Matlab:

$$f(\varepsilon_a) = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} - \varepsilon_a = 0$$

The following solver found $\sigma_a = 513\ MPa$

```
ramberg=@(siga,epsa,E,Ha,na) siga/E+(siga/Ha).^(1/na)-epsa;
sigmaa=fzero(@(siga) ramberg(siga,0.0045,E,Ha,na), [1,1000])
```

The maximum stress is then:

$$\begin{aligned}\sigma_{max} &= \sigma_a + \sigma_m\\ &= 513 + 120\\ &= 633\ MPa\end{aligned}$$

The minimum stress is then

$$\begin{aligned}\sigma_{min} &= \sigma_m - \sigma_a\\ &= 120 - 513\\ &= -393\ MPa\end{aligned}$$

The stress ratio is then:

$$\begin{aligned}R &= \frac{\sigma_{min}}{\sigma_{max}}\\ &= -\frac{393}{633}\\ &= -0.6209\end{aligned}$$

The following instructions was used to solve for the number of cycles to crack initiation $N_f = 2\,349\ cycles$.

```
fwalker=@(N,epsa,sfa,E,R,gamma,b,epsfa,c) sfa/E*((1-R)/2)^(1-gamma)*(2*N).^b+epsfa*((1-R)/2)^(c*(1-gamma)/b)*(2*N).^c-epsa;
=fzero(@(N) fwalker(N,0.0045,sfa,E,-0.6209,0.7302,b,epsfa,c),[1,100000])
```

Note: The following method can also be applied:

Step 1: Calculate N^* as if the mean stress is zero:

$$\begin{aligned}\varepsilon_a &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c\\ 0 &= \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c - \varepsilon_a\end{aligned} \tag{136}$$

This was solved as $N_w^* = 5\,635\ cycles$

Step 2: Calculate the N_f to include mean stress effect

$$\begin{aligned}N_f &= N_w^*\left(\frac{\sigma_a}{\sigma_{max}}\right)^{\frac{\gamma-1}{b}}\\ &= N_w^*\left(\frac{1-R}{2}\right)^{\frac{\gamma-1}{b}}\\ &= 5\,635 \times \left(\frac{1+0.6209}{2}\right)^{\frac{0.7302-1}{-0.0648}}\\ &= 2\,349\ cycles\end{aligned} \tag{137}$$

Discussion

The fatigue lives were calculated as follows:

Morrow:	681 cycles
Modified Morrow:	4 598 cycles
SWT:	3 446 cycles
Walker:	2 349 cycles

The Morrow and modified Morrow predicted fatigue life differ significantly due to the relatively short life involved. The SWT and Walker mean stress compensated fatigue lives lie between the two Morrow estimates, with the **Walker estimate likely being the most accurate of the four.**

12.7. Multi-axial stress effects

Research is continuing in this field – the reason for the papers to be studied as part of the assignment.

Uncertainty exists as to the best procedure for complex non-proportional loadings, where the ratios of the principal stresses change, and where the principal axis may also rotate.

12.7.1. Effective strain approach

For situations where all cyclic loadings have the same frequency and are either in-phase or 180° out-of-phase, the effective strain amplitude that is proportional to the cyclic amplitude of the octahedral shear strain as:

$$\bar{\varepsilon}_a = \frac{\bar{\sigma}_a}{E} + \bar{\varepsilon}_{pa} \tag{138}$$

Where, from Dowling Eqs 12.21 & 12.22 we have the effective stress and plastic strain:

$$\begin{aligned} \bar{\sigma}_a &= \frac{1}{\sqrt{2}}\sqrt{(\sigma_{1a}-\sigma_{2a})^2+(\sigma_{2a}-\sigma_{3a})^2+(\sigma_{3a}-\sigma_{1a})^2} \\ \bar{\varepsilon}_{pa} &= \frac{\sqrt{2}}{3}\sqrt{\left(\varepsilon_{p1a}-\varepsilon_{p2a}\right)^2+\left(\varepsilon_{p2a}-\varepsilon_{p3a}\right)^2+\left(\varepsilon_{p3a}-\varepsilon_{p1a}\right)^2} \end{aligned} \tag{139}$$

The strain-life fatigue curve is as before:

$$\bar{\varepsilon}_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \varepsilon_f'\left(2N_f\right)^c \tag{140}$$

Where the first and second terms correspond to the elastic and plastic components of the effective strain:

$$\begin{aligned} \bar{\sigma}_a &= \sigma_f'\left(2N_f\right)^b \\ \bar{\varepsilon}_a &= \varepsilon_f'\left(2N_f\right)^c \end{aligned} \tag{141}$$

12.7.1.1. Plane stress state

In this case the following applies:

$$\begin{aligned} \sigma_{2a} &= \lambda\sigma_{1a} \\ \sigma_{3a} &= 0 \\ \varepsilon_{1a} &= \varepsilon_{1ea} + \varepsilon_{1pa} \end{aligned} \tag{142}$$

Where subscript a indicates amplitude. For 180° out-of-phase stress, use $\lambda = -1$. For pure shear, $\lambda = -1$.

For linear elastic isotropic materials (see Dowling Chapter 12 Eq 12.19):

$$\begin{aligned} \varepsilon_{ex} &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \\ \varepsilon_{ey} &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \\ \varepsilon_{ez} &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_y + \sigma_x\right)\right] \\ \gamma_{exy} &= \frac{\tau_{xy}}{G}, \gamma_{eyz} = \frac{\tau_{yz}}{G}, \gamma_{exz} = \frac{\tau_{xz}}{G} \end{aligned} \tag{143}$$

In this case, the linear elastic equation becomes:

$$\begin{aligned}\varepsilon_{ex} &= \frac{1}{E}[\sigma_x - \nu(\lambda\sigma_x)] \\ &= \frac{\sigma_x}{E}(1-\lambda\nu) \\ \varepsilon_{ey} &= \frac{1}{E}[\lambda\sigma_x - \nu(\sigma_x)] \\ &= \frac{\sigma_x}{E}(\lambda-\nu) \\ \varepsilon_{ez} &= \frac{1}{E}[0 - \nu(\lambda\sigma_x + \sigma_x)] \\ &= \frac{\sigma_x}{E}(-\nu(\lambda+1))\end{aligned} \tag{144}$$

From the deformation plasticity theory (Dowling 12.3.1) we have:

$$\begin{aligned}\varepsilon_{px} &= \frac{1}{E_p}\left[\sigma_x - 0.5\left(\sigma_y + \sigma_z\right)\right] \\ \varepsilon_{py} &= \frac{1}{E_p}\left[\sigma_y - 0.5\left(\sigma_x + \sigma_z\right)\right] \\ \varepsilon_{pz} &= \frac{1}{E_p}\left[\sigma_z - 0.5\left(\sigma_y + \sigma_x\right)\right] \\ \gamma_{pxy} &= \frac{3\tau_{xy}}{E_p}, \gamma_{pyz} = \frac{3\tau_{yz}}{E_p}, \gamma_{pxz} = \frac{3\tau_{xz}}{E_p}\end{aligned} \tag{145}$$

Which becomes the following in this case:

$$\begin{aligned}\varepsilon_{px} &= \frac{1}{E_p}[\sigma_x - 0.5(\lambda\sigma_x)] \\ &= \frac{\sigma_x}{E_p}(1-0.5\lambda) \\ \varepsilon_{py} &= \frac{1}{E_p}[\lambda\sigma_x - 0.5(\sigma_x)] \\ &= \frac{\sigma_x}{E_p}(\lambda-0.5) \\ \varepsilon_{pz} &= \frac{1}{E_p}[0 - 0.5(\lambda\sigma_x + \sigma_x)] \\ &= \frac{\sigma_x}{E_p}(-0.5(\lambda+1))\end{aligned} \tag{146}$$

The effective stress for this situation is then:

$$\bar{\sigma} = \sigma_1\sqrt{1-\lambda+\lambda^2} \tag{147}$$

And strain-life curve:

$$\bar{\varepsilon}_{1a} = \frac{\frac{\sigma_f'}{E}(1-\nu\lambda)\left(2N_f\right)^b + \varepsilon_f'(1-0.5\lambda)\left(2N_f\right)^c}{\sqrt{1-\lambda+\lambda^2}} \tag{148}$$

Note, the equation above can be used with the Ramberg-Osgood stress-strain curve for biaxial loading where all stresses and strains are treated as amplitude.

For the special case of shear, $\lambda = -1$, and:

$$\gamma_{xya} = \frac{\sigma_f'}{\sqrt{3}G}\left(2N_f\right)^b + \sqrt{3}\varepsilon_f'\left(2N_f\right)^c \tag{149}$$

Where

γ_{xya} is the shear strain amplitude

12.7.1.2. Average of the amplitudes compensation

The average of the amplitudes of the principal normal stresses is:

$$\sigma_{ha} = \frac{\sigma_{1a} + \sigma_2 + \sigma_{3a}}{3} \tag{150}$$

The relative value of σ_{ha} may be expressed as a triaxiality factor:

$$T = \frac{3\sigma_{ha}}{\bar{\sigma}_a} \tag{151}$$

For the plane stress state above, the triaxiality factor is:

$$T = \frac{1 + \lambda}{\sqrt{1 - \lambda + \lambda^2}} \tag{152}$$

Which implies that:

- Pure planar shear, $\lambda = -1\,, T = 0$
- Uniaxial stress, $\lambda = 0, T = 1$
- Equal biaxial stress, $\lambda = 1, T = 2$
- The life is shorter for larger values of T

To include average of the amplitudes, Marloff suggests:

$$\begin{aligned} \bar{\varepsilon}_a &= \frac{\sigma_f'}{E}\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'\left(2N_f\right)^c \\ \bar{\varepsilon}_{1a} &= \frac{\frac{\sigma_f'}{E}(1 - \nu\lambda)\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'(1 - 0.5\lambda)\left(2N_f\right)^c}{\sqrt{1 - \lambda + \lambda^2}} \\ \gamma_{xya} &= \frac{\sigma_f'}{\sqrt{3}G}\left(2N_f\right)^b + 2^{1-T}\sqrt{3}\varepsilon_f'\left(2N_f\right)^c \end{aligned} \tag{153}$$

12.7.1.3. Mean stress compensation

Assume that the controlling mean stress is the noncyclic component of the hydrostatic stress and modify the strain-life curves according to the mean stress correction method selected.

Morrow

$$\begin{aligned} \bar{\varepsilon}_a &= \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}\left(2N_f\right)^c \\ \bar{\varepsilon}_{1a} &= \frac{\frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(1 - \nu\lambda)\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}(1 - 0.5\lambda)\left(2N_f\right)^c}{\sqrt{1 - \lambda + \lambda^2}} \end{aligned} \tag{154}$$

Modified Morrow

$$\begin{aligned} \bar{\varepsilon}_a &= \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'\left(2N_f\right)^c \\ \bar{\varepsilon}_{1a} &= \frac{\frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\sigma_f'}\right)(1 - \nu\lambda)\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'(1 - 0.5\lambda)\left(2N_f\right)^c}{\sqrt{1 - \lambda + \lambda^2}} \end{aligned} \tag{155}$$

SWT

$$\bar{\sigma}_{max}\bar{\varepsilon}_a = \frac{\left(\sigma_f'\right)^2}{E}\left(2N_f\right)^{2b} + 2^{1-T}\sigma_f'\varepsilon_f'\left(2N_f\right)^{b+c}$$

Walker

$$\bar{\varepsilon}_a = \frac{\sigma_f'}{E}\left(\frac{1 - R}{2}\right)^{1-\gamma}\left(2N_f\right)^b + 2^{1-T}\varepsilon_f'\left(\frac{1 - R}{2}\right)^{\frac{c(1-\gamma)}{b}}\left(2N_f\right)^c \tag{156}$$

12.7.2. Critical plane approaches

Where loading is nonproportional.

Stresses & strains during cyclic loading determined for various planes – use stresses & strains on the most severely loaded plane for fatigue life estimate.

Fatemi & Soci suggest (see Figure 55 for stress directions):

$$\gamma_{ac}\left(1+\frac{\alpha\sigma_{max,c}}{\sigma_o'}\right)=\frac{\tau_f'}{G}\left(2N_f\right)^b+\gamma_f'\left(2N_f\right)^c \tag{157}$$

Where γ_{ac} is the largest shear strain amplitude for any plane, and $\sigma_{max,c}$ is the peak tensile stress normal to the plane of γ_{ac} occurring at any time during the γ_{ac} cycle.

$\alpha = 0.6\ to\ 1.0$, depending on the material. σ_o' is the yield strength for the cyclic stress-strain curve. As shown, the crack opening stress will reduce the fatigue life.

In the absence of τ_f' and γ_f', use the values as derived from Equation (140):

$$\begin{aligned}\gamma_{ac}\left(1+\frac{\alpha\sigma_{max,c}}{\sigma_o'}\right)&=\frac{\tau_f'}{G}\left(2N_f\right)^b+\gamma_f'\left(2N_f\right)^c\\&=\frac{\sigma_f'}{\sqrt{3}G}\left(2N_f\right)^b+\sqrt{3}\varepsilon_f'\left(2N_f\right)^c\\\tau_f'&=\frac{\sigma_f'}{\sqrt{3}}\\\gamma_f'&=\sqrt{3}\varepsilon_f'\end{aligned} \tag{158}$$

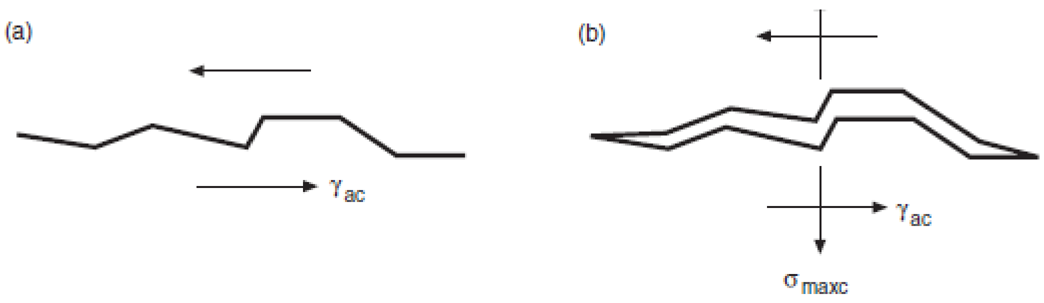


Figure 55: Crack under pure shear and normal & shear

Figure 55 shows two modes of crack initiation and early growth:

- Mode II: growth on planes of high shear stress
- Mode I: growth on planes of high tensile stress

The SWT parameter can be employed for tensile stress dominated cracking. In this case:

- ε_a is the largest amplitude of normal strain for any plane
- σ_{max} is the maximum normal stress on the same plane as ε_a, specifically the peak during the ε_a-cycle

Chu recommends the following multiaxial fatigue criterion that considers both the shear and normal stress cracking modes:

$$2\tau_{max}\gamma_a+\sigma_{max}\varepsilon_a=f(N_f) \tag{159}$$

The critical plane is were $2\tau_{max}\gamma_a+\sigma_{max}\varepsilon_a$ is the largest.

$f(N_f)$ can be obtained from uniaxial test date and can be thought of as a generalization of the SWT parameter.

12.7.2.1. Summary

Effective strain approach limited in its applicability to combined loading.

Reasonable for combined loadings that are in-phase and 180° out-of-phase provided there are no steady (mean) loading which cause substantial rotation of the principal axes during cyclic loading, because, rotation of the principal axes causes nonproportional loading.

12.8. Variable amplitude loading

Reference: (Dowling, 2013, p. 775 Section 14.5.2)

The process followed is:

1. Reorder the stress history to start and return to the peak or valley with the largest absolute value
2. Rainflow counting
    a. Because the stress history was reordered to start and end with the largest peak (or valley), closed stress-strain hysteresis loops will result.
3. Calculate notch stress and strain, while observing the memory effect at the points where hysteresis loops close.
    a. The first reversal is calculated from the monotone stress-strain curve.
    b. Cyclic responses are calculated from the cyclic stress-strain curves.
    c. The strain amplitude and mean stress of each closed stress-strain hysteresis loop is calculated.
    d. Determine by Rainflow counting. This is to ensure that the effect of the large hysteresis response is included in the assessment.
4. Use Palmgren-Miner rule for damage.
5. Calculate repetitions, B_f, to failure.

12.9. Strain-life estimates for structural components

12.9.1. Nominal loading and theoretical stress concentration factor

Calculate the nominal stresses, S_{max}, S_a from the loading. Or, if needed, other parameters.

Obtain the theoretical (or linear elastic) stress concentration factor for the material, k_t.

12.9.2. Obtain material properties

Material parameters are needed for the:

- Constitutive model: Stress-strain relationship. Use the Ramberg-Osgood equation. It needs: E, H' & n'. Get from Dowling Table 14.1 or from literature.

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}}$$

- Strain-life curve: It needs, $E, \sigma_f', b, \varepsilon_f'$ & c. Get from Dowling Table 14.1 or from literature. The pressure vessel code has long list of strain life parameters.

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

12.9.3. Equation for the notch stress and strain response

Apply the Neuber equation for both S_{max} & S_a:

$$\sigma_{max}\varepsilon_{max} = \frac{(k_t S_{max})^2}{E}$$

$$\sigma_a \varepsilon_a = \frac{(k_t S_a)^2}{E}$$

12.9.4. Reorder the stress history

Reorder the stress history so that the:

1. The history starts at 0.
2. Highest absolute value of the stress history is at the beginning of the signal (see Figures 56 and 57).
3. Highest absolute value is added at the end of the signal.

The Matlab commands to reorder the signal.

```
S=rand(20,1);plot(S); grid
ii=find(max(abs(S))==abs(S));SS=[S(ii(1):end)' S(1:ii(1))'];
figure,plot(SS);grid
[ext,exttime]=sig2ext(SS, 1, 40);
hold on, plot(exttime+1,ext,'ro');grid
```

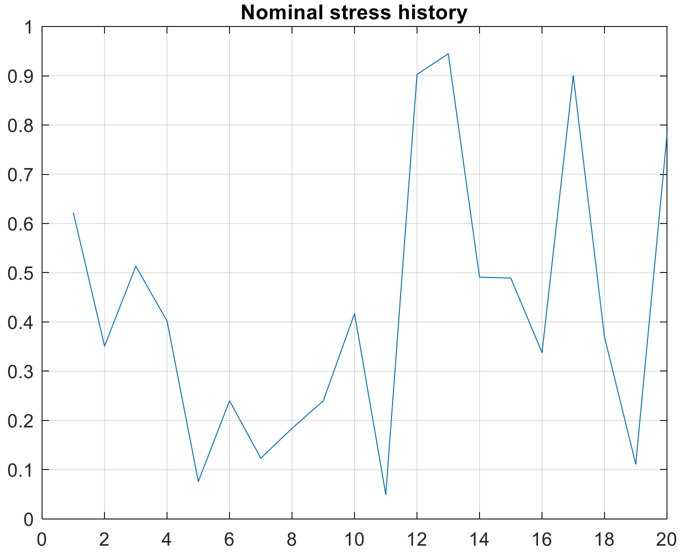


Figure 56: Original stress history

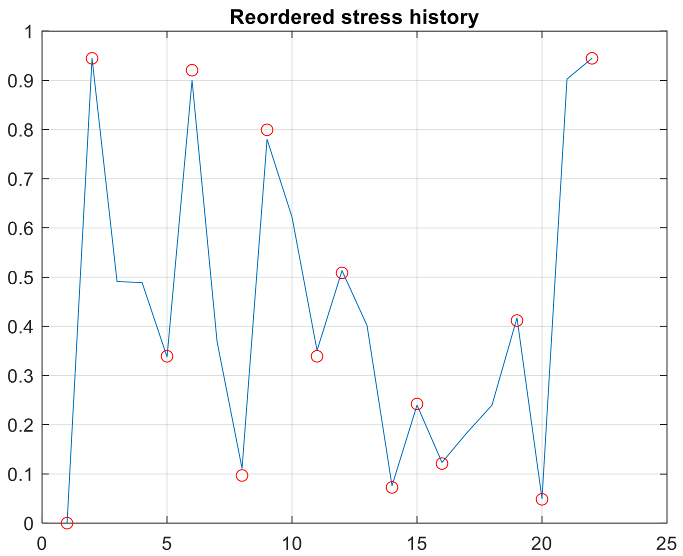


Figure 57: Reordered nominal stress history

12.9.5. Cycle counting

Use Rainflow counting to determine the presence of cyclic stress-strain responses. Not required where the sequence effect is modelled.

To model the memory effect, the stress signal needs to be followed to determine the reference stress from which the cycles need to be calculated. See the demonstration in class.

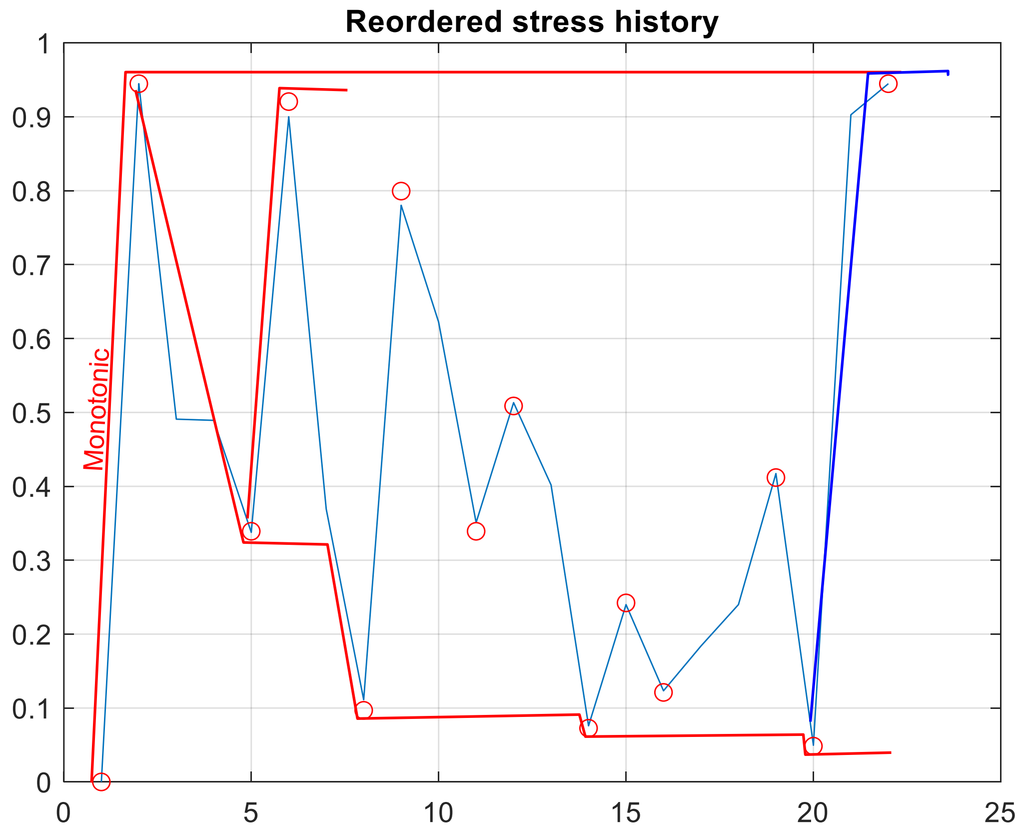


Figure 58: Orders for modelling the memory effect

12.9.6. Calculate the stress and strain response in the notch for the first monotonic part

<u>Ramberg-Osgood and Neuber's rule</u>

The Ramberg-Osgood stress-strain relationship is given as:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H'}\right)^{\frac{1}{n'}} = f(\sigma)$$

Neuber's rule:

$$\varepsilon\sigma = \frac{(k_t S)^2}{E}$$
$$\varepsilon = \frac{(k_t S)^2}{\sigma E} = g(S, \sigma)$$

The notch stress-strain response is calculated by solving the following equation with a numerical optimization function by finding σ_a that results in $f_e = 0$:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$
$$= f(\sigma_a)$$
$$\sigma_a \varepsilon_a = \frac{(k_t S_a)^2}{E}$$
$$\varepsilon_a = \frac{(k_t S_a)^2}{\sigma_a E}$$
$$= \mathrm{g}(\mathrm{S_a})$$
$$\left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right] = \frac{(k_t S_a)^2}{\sigma_a E}$$
$$0 = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$
$$= g(S_a, \sigma_a) - f(\sigma_a)$$

The calculate the strain after the first cycle as:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = f(\sigma_a)$$

See the previous examples in the notes for other stress-strain relationships.

12.9.7. *Calculate the notch stress-strain response for cyclic loading*

Ramberg-Osgood and Neuber's rule

The notch stress-strain response is calculated by solving the following equation with a numerical optimization function by finding $\Delta\sigma$ that results in $f_e = 0$:

$$\frac{\Delta\varepsilon}{2} = f\left(\frac{\Delta\sigma}{2}\right)$$
$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}}$$
$$= f\left(\frac{\Delta\sigma}{2}\right)$$
$$\frac{\Delta\sigma}{2}\left(\frac{\Delta\varepsilon}{2}\right) = \frac{\left(k_t \frac{\Delta S}{2}\right)^2}{E}$$
$$\left(\frac{\Delta\varepsilon}{2}\right) = \frac{\left(k_t \frac{\Delta S}{2}\right)^2}{\frac{\Delta\sigma}{2} E}$$
$$= \mathrm{g}\left(\frac{\Delta \mathrm{S}}{2}, \frac{\Delta\sigma}{2}\right)$$
$$\Delta\varepsilon = 2 \times \frac{2\left(k_t \frac{\Delta S}{2}\right)^2}{\Delta\sigma E}$$
$$= \frac{(k_t \Delta S)^2}{\Delta\sigma E}$$
$$= 2\mathrm{g}\left(\frac{\Delta \mathrm{S}}{2}, \frac{\Delta\sigma}{2}\right)$$
$$\frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}} = \frac{(k_t \Delta S)^2}{2\Delta\sigma E}$$

$$0 = \frac{(k_t \Delta S)^2}{2\Delta\sigma E} - \left[\frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}}\right]$$
$$= \mathrm{g}\left(\frac{\Delta S}{2}, \frac{\Delta\sigma}{2}\right) - f\left(\frac{\Delta\sigma}{2}\right)$$
$$\sigma_a = \frac{\Delta\sigma}{2}$$

Then calculate the cyclic strain range as:

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{\frac{1}{n'}}$$
$$\varepsilon_a = \frac{\Delta\varepsilon}{2}$$

See the previous examples in the notes for other stress-strain relationships.

The endpoint of the stress and strain is then:

$$\sigma_{i+1} = \sigma_i + \psi\Delta\sigma$$
$$\varepsilon_{i+1} = \varepsilon_i + \psi\Delta\varepsilon$$

Calculate the means:

$$\sigma_{mi} = \frac{(\sigma_i + \sigma_{i+1})}{2}$$
$$\varepsilon_{mi} = \frac{(\varepsilon_i + \varepsilon_{i+1})}{2}$$

12.9.8. Zero-mean-stress-equivalent and mean-stress compensated fatigue life

The zero-mean-stress-equivalent fatigue life is:

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$
$$\frac{(k_t S_a)^2}{\sigma_a E} = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$
$$f_{errsl} = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c - \frac{(k_t S_a)^2}{\sigma_a E}$$

The mean-stress compensated fatigue life is:

$$N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

12.10. Strain-life: Example 1

<u>Problem statement</u>

One repetition of the nominal stress on a Ti-6Al-4V alloy is shown in the table below. The elastic stress concentration factor (also called the theoretical stress concentration factor) is $k_t = 2.6$. Estimate the number of repetitions required to cause fatigue cracking at the notch. Use the Ramberg-Osgood stress-strain relationship and the Neuber equation for notch response.

- The stress history is shown in Figure 59.
- Which was then reordered to start and end with the first peak or valley with the maximum absolute value as shown in Figure 60.
- In this case, no extrema were calculated, and exact values are used. The stress history is short enough for this.

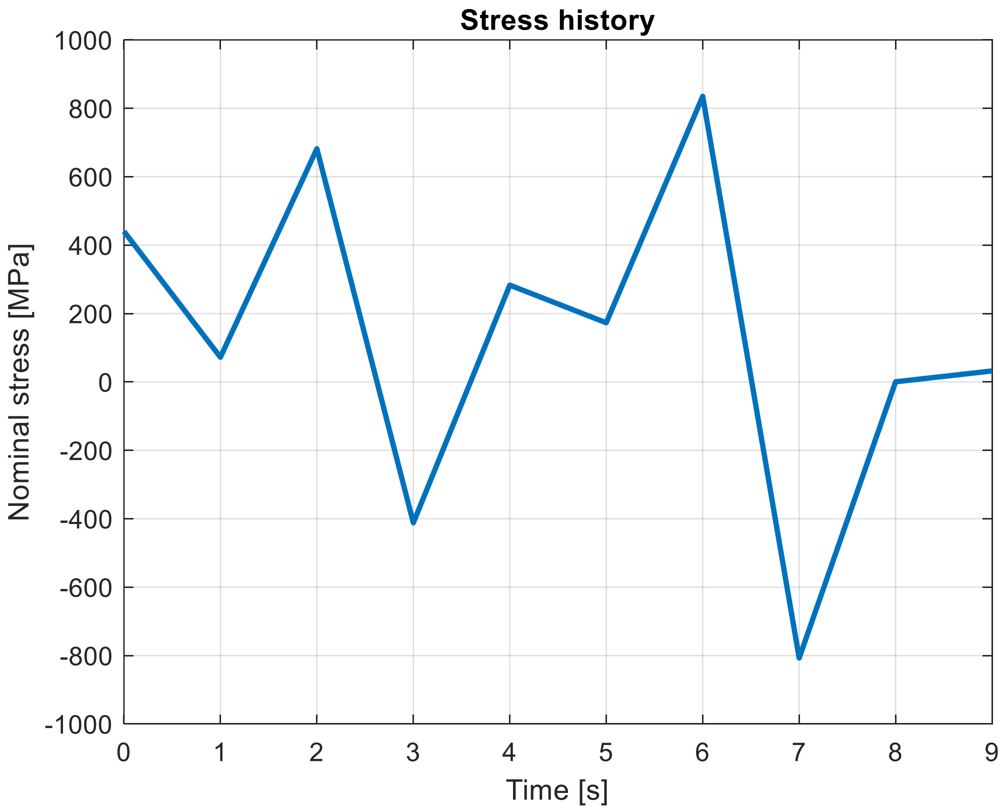


Figure 59: Original stress history - 1 repetition

Figure 60: Reordered stress history - for one repetition

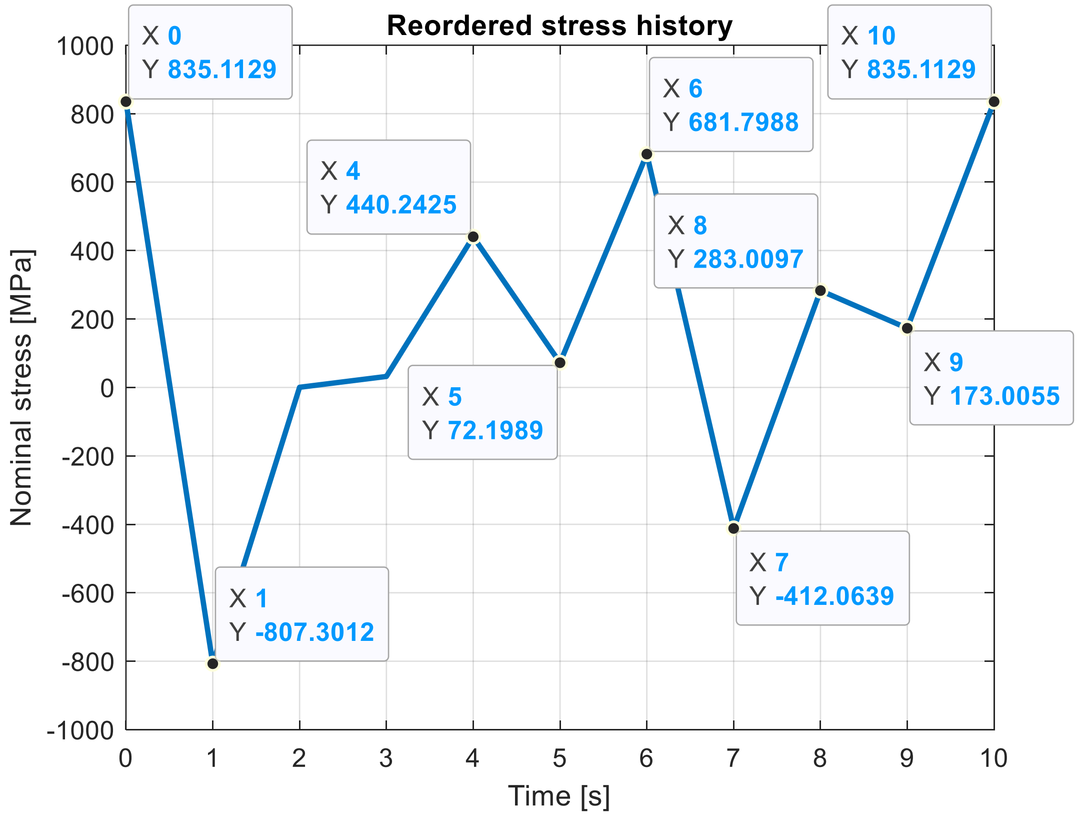


Figure 61: Stress history with stresses indicated

<u>Solution</u>

The material properties for Ti-6Al-4V is as follows from Dowling Table 14.1:

$$E = 117 \times 10^3\ MPa$$
$$H' = 1\,772\ MPa$$
$$n' = 0.106$$
$$\sigma_f' = 2030\ MPa$$
$$b = -0.104$$
$$\varepsilon_f' = 0.841$$
$$c = -0.688$$

Section 14.2 Strain Versus Life Curves **751**

Table 14.1 Cyclic Stress–Strain and Strain–Life Constants for Selected Engineering Metals.[1]

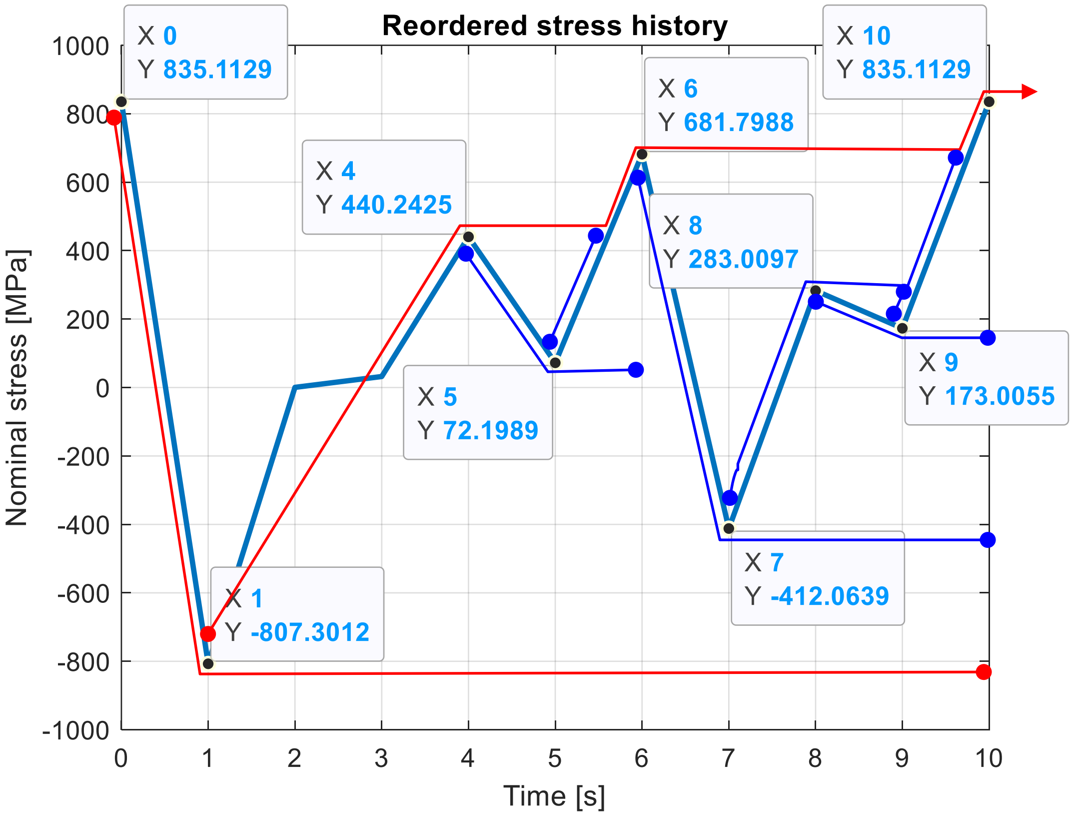
		Tensile Properties				Cyclic σ-ε Curve			Strain–Life Curve			
Material	Source	σ_o	σ_u	$\tilde{\sigma}_{fB}$	% *RA*	*E*	*H'*	*n'*	σ_f'	*b*	ε_f'	*c*
(a) Steels												
SAE 1015	(8)	228	415	726	68	207,000	1349	0.282	1020	−0.138	0.439	−0.513
(normalized)		(33.0)	(60.2)	(105)		(30,000)	(196)		(148)			
Man-Ten[2]	(7)	322	557	990	67	203,000	1096	0.187	1089	−0.115	0.912	−0.606
(hot rolled)		(46.7)	(80.8)	(144)		(29,500)	(159)		(158)			
RQC-100	(2)	683	758	1186	64	200,000	903	0.0905	938	−0.0648	1.38	−0.704
(roller Q & T)		(99.0)	(110)	(172)		(29,000)	(131)		(136)			
SAE 1045	(6)	382	621	985	51	202,000	1258	0.208	948	−0.092	0.260	−0.445
(HR & norm.)		(55.4)	(90.1)	(143)		(29,400)	(182)		(137)			
SAE 4142	(1)	1619	2450	2580	6	200,000	2810	0.040	2550	−0.0778	0.0032	−0.436
(As Q, 670 HB)		(235)	(355)	(375)		(29,000)	(407)		(370)			
SAE 4142	(1)	1688	2240	2650	27	207,000	4140	0.126	3410	−0.121	0.0732	−0.805
(Q & T, 560 HB)		(245)	(325)	(385)		(30,000)	(600)		(494)			
SAE 4142	(1)	1584	1757	1998	42	207,000	2080	0.093	1937	−0.0762	0.706	−0.869
(Q & T, 450 HB)		(230)	(255)	(290)		(30,000)	(302)		(281)			
SAE 4142	(1)	1378	1413	1826	48	207,000	2210	0.133	2140	−0.0944	0.637	−0.761
(Q & T, 380 HB)		(200)	(205)	(265)		(30,000)	(321)		(311)			
AISI 4340[2]	(3)	1103	1172	1634	56	207,000	1655	0.131	1758	−0.0977	2.12	−0.774
(Aircraft Qual.)		(160)	(170)	(237)		(30,000)	(240)		(255)			
AISI 4340	(1)	1371	1468	1557	38	200,000	1910	0.123	1879	−0.0859	0.640	−0.636
(409 HB)		(199)	(213)	(226)		(29,000)	(277)		(273)			
Ausformed H-11	(1)	2030	2580	3170	33	207,000	3475	0.059	3810	−0.0928	0.0743	−0.7144
(660 HB)		(295)	(375)	(460)		(30,000)	(504)		(553)			
(b) Other Metals												
2024-T351 Al	(1)	379	469	558	25	73,100	662	0.070	927	−0.113	0.409	−0.713
		(55.0)	(68.0)	(81.0)		(10,600)	(96.0)		(134)			
2024-T4 Al[3]	(4)	303	476	631	35	73,100	738	0.080	1294	−0.142	0.327	−0.645
(Prestrained)		(44.0)	(69.0)	(91.5)		(10,600)	(107)		(188)			
7075-T6 Al	(5)	469	578	744	33	71,000	977	0.106	1466	−0.143	0.262	−0.619
		(68.0)	(84)	(108)		(10,300)	(142)		(213)			
Ti-6Al-4V	(1)	1185	1233	1717	41	117,000	1772	0.106	2030	−0.104	0.841	−0.688
(soln. tr. & age)		(172)	(179)	(249)		(17,000)	(257)		(295)			
Inconel X	(1)	703	1213	1309	20	214,000	1855	0.120	2255	−0.117	1.16	−0.749
(Ni base, annl.)		(102)	(176)	(190)		(31,000)	(269)		(327)			

Notes: [1]The tabulated values either have units of MPa (ksi), or they are dimensionless. [2]Test specimens prestrained, except at short lives, also periodically overstrained at long lives. [3]For nonprestrained tests, use same constants, except $\sigma_f' = 900(131)$ and $b = -0.102$.
Sources: Data in (1) [Conle 84]; (2) author's data on the ASTM Committee E9 material; (3) [Dowling 73]; (4) [Dowling 89] and [Topper 70]; (5) [Endo 69] and [Raske 72]; (6) [Leese 85]; (7) [Wetzel 77] pp. 41 and 66; (8) [Keshavan 67] and [Smith 70].

<u>Evaluate the stress history</u>

Rainflow counting was performed on the stress signal as shown below:

1. The first step was to draw the start and stops for each reversal.
   a. Note, that a few points, and one turning point could have been left out if peak-valley reduction (signal to extrema) was done before the Rainflow counting.
2. Then number the peaks and valleys of the relevant cycles. In this case the time axis will be used as was calculated in Matlab. Otherwise use A, B, C, …. as per the examples in Dowling.

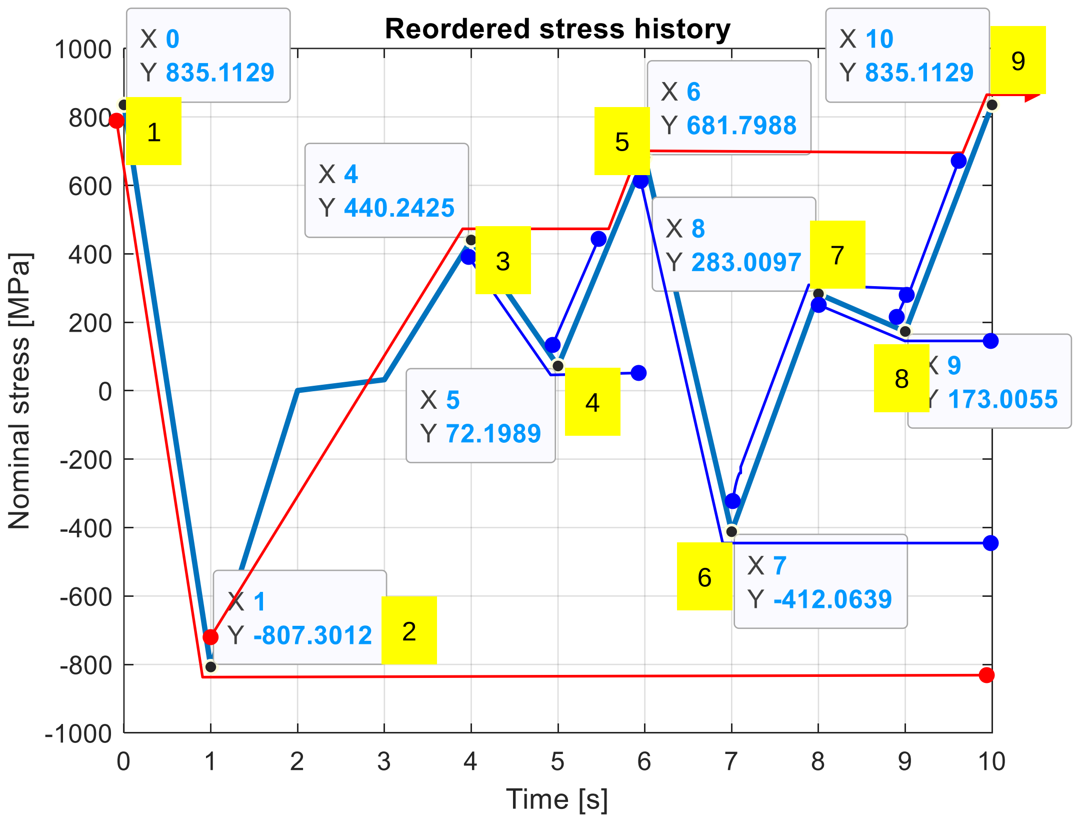


The stress-strain response is then as follows:

E	1.17E+05	MPa		kt	2.6
H'	1772	MPa			
n'	0.106				
σ_f'	2030	MPa			
b	-0.104				
ε_f'	0.841				
c	-0.688				
Stress	**From**		**To**		
	#	**Stress [MPa]**	**#**	**Stress [MPa]**	**Direction**
Monotone	0	0	1	835	1
Cyclic	0	835	1	-807	-1
	1	-807	4	440	1
	4	440	5	72	-1
	1	-807	6	682	1
	6	682	7	-412	-1
	7	-412	8	283	1
	8	283	9	173	-1
	1	-807	10	835	1

12.10.1. Stress-strain response at the notch

The purpose of this section is to calculate the stress and strain response at the notch, and to calculate the stress and strain at every point of interest. The points of interest are indicated in yellow in the figure below.



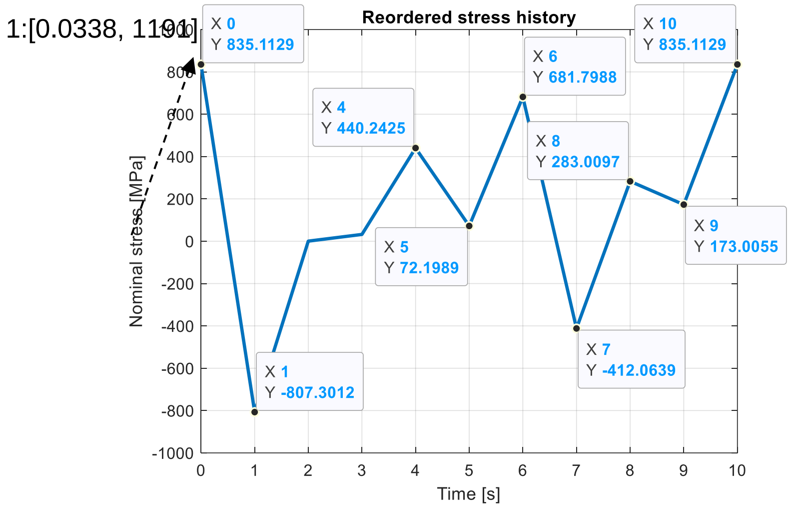
Reversal 1: From 0 to 1, monotonic stress-strain curve

For the first stress cycle, the nominal monotonic stress amplitude is $S_a = 835\ MPa$. The notch stress response is solved in Matlab as:

$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$
$$\sigma_{a1} = 1\ 192\ MPa$$

The notch strain amplitude for the stress reversal is then:

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$
$$= 0.0338$$



Reversal 2: 1 to 2, cyclic stress strain curve

The nominal monotonic stress amplitude is $S_a = \frac{835--807}{2} = 821\ MPa$.

Direction: $\psi = -1$.

The notch stress response is solved in Matlab as:

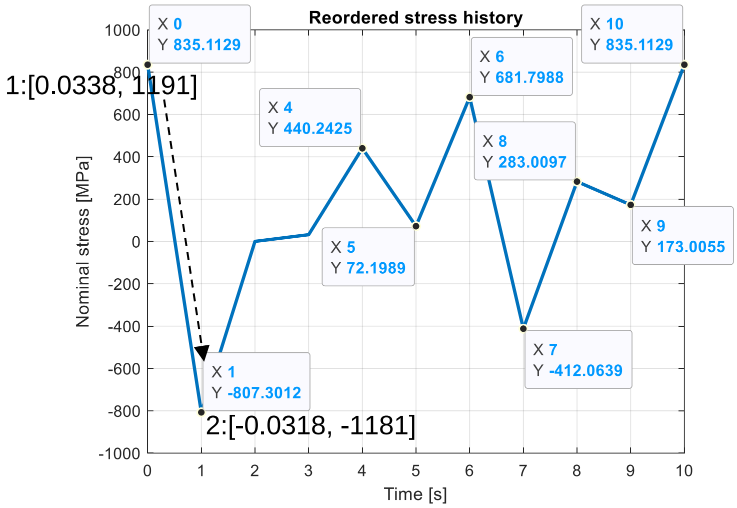
$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$
$$\sigma_{a1} = 1\ 186\ MPa$$

The notch strain amplitude for the stress reversal is then:

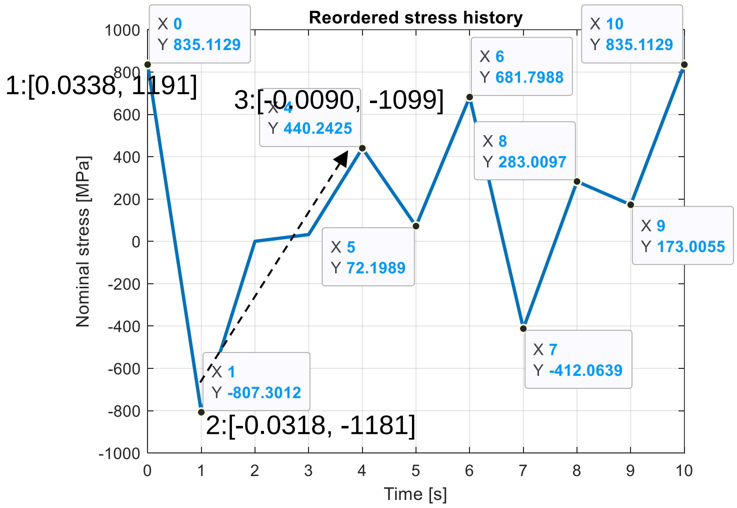
$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}$$
$$= 0.0328$$

Therefore, the stress and strain at the end of the reversal will be:

$$\sigma(2) = \sigma(1) + 2\psi\sigma_a(1)$$
$$= 1191 + 2(-1)(1186)$$
$$= -1\ 181\ MPa$$
$$\varepsilon(2) = \varepsilon(1) + 2\psi\varepsilon_a(1)$$
$$= -0.0318$$



Reversal 3: Reversal 2 to 3, cyclic stress strain curve



$$S_a = \frac{440 - -807}{2} = 624\ MPa$$

$$\psi = 1$$

$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$

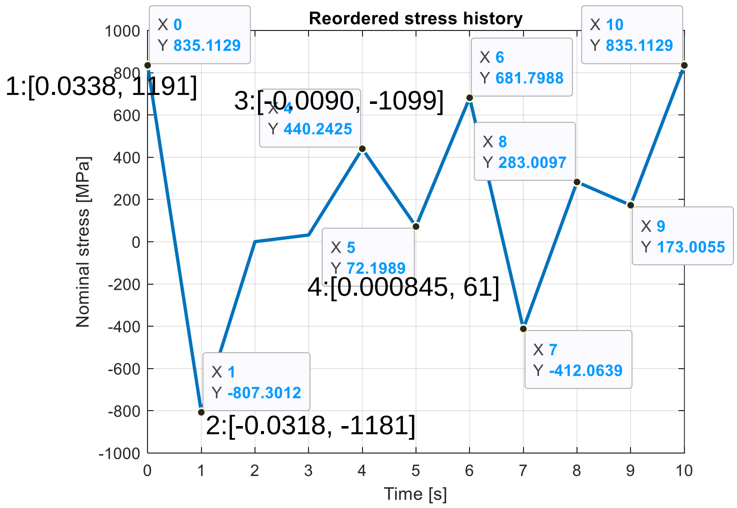
$$\sigma_a = 1\,099\ MPa$$

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.0204$$

$$\sigma(3) = \sigma(2) + 2\psi\sigma_a(3) = 1\,017\ MPa$$

$$\varepsilon(3) = \varepsilon(2) + 2\psi\varepsilon_a(3) = -0.00902$$

Reversal 4: 3 to 4, cyclic stress strain curve



$$S_a = \frac{440 - 72}{2} = 184\ MPa$$

$$\psi = -1$$

$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$

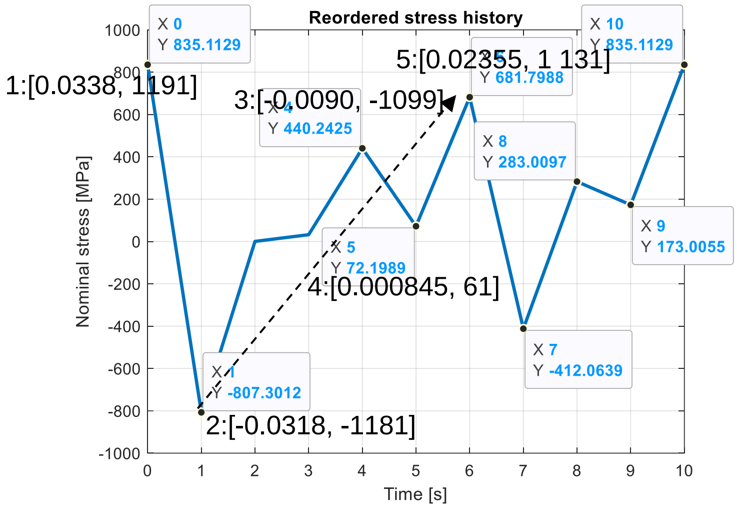
$$\sigma_a = 478\ MPa$$

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.0041$$

$$\sigma(3) = \sigma(2) + 2\psi\sigma_a(3) = 61\ MPa$$

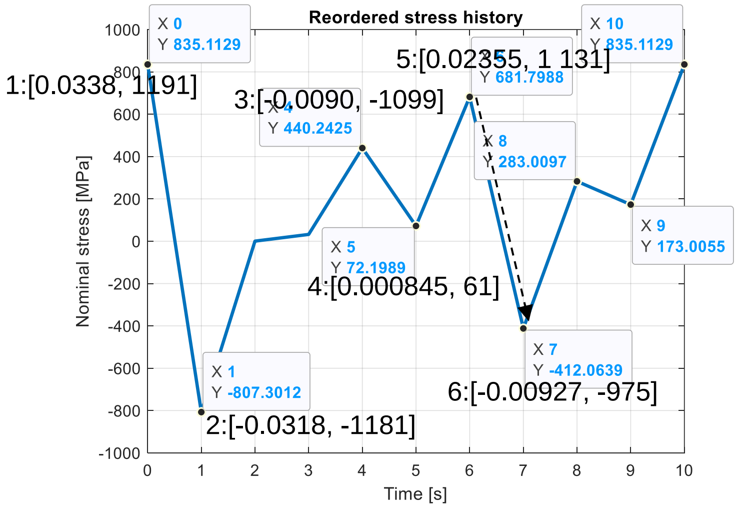
$$\varepsilon(3) = \varepsilon(2) + 2\psi\varepsilon_a(3) = 0.000845$$

Reversal 5: 2 to 5, cyclic stress strain curve



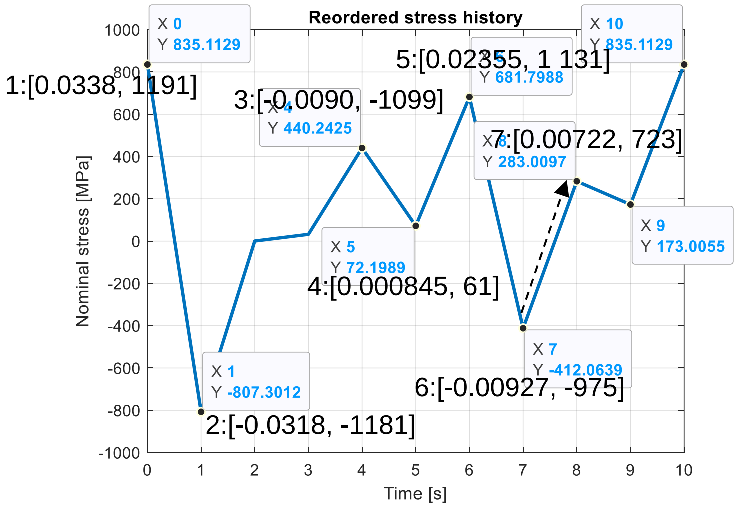
$$S_a = \frac{681 - -807}{2} = 745\ MPa$$
$$\psi = 1$$
$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$
$$\sigma_a = 1\,156\ MPa$$
$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.0277$$
$$\sigma(5) = \sigma(2) + 2\psi\sigma_a(5) = 1\,131\ MPa$$
$$\varepsilon(5) = \varepsilon(2) + 2\psi\varepsilon_a(5) = 0.02355$$

Reversal 6: 5 to 6, cyclic stress strain curve



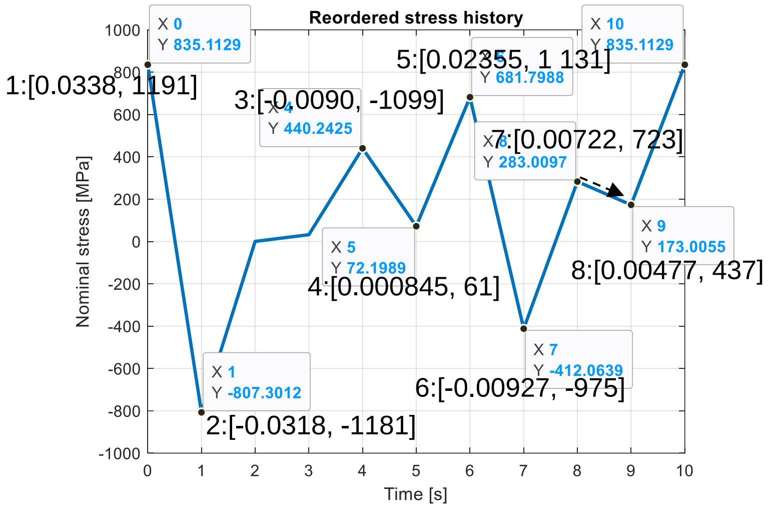
$$S_a = \frac{681 - -412}{2} = 547\ MPa$$
$$\psi = -1$$
$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$
$$\sigma_a = 1\,053\ MPa$$
$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.0164$$
$$\sigma(6) = \sigma(5) + 2\psi\sigma_a(6) = -975\ MPa$$
$$\varepsilon(6) = \varepsilon(5) + 2\psi\varepsilon_a(6) = -0.00927$$

Reversal 7: 6 to 7, cyclic stress strain curve



$$S_a = \frac{283 - -412}{2} = 348\ MPa$$
$$\psi = 1$$
$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$
$$\sigma_a = 849\ MPa$$
$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.00822$$
$$\sigma(5) = \sigma(2) + 2\psi\sigma_a(5) = 723\ MPa$$
$$\varepsilon(5) = \varepsilon(2) + 2\psi\varepsilon_a(5) = 0.00722$$

Reversal 8: 7 to 8, cyclic stress strain curve



$$S_a = \frac{283 - 173}{2} = 55\ MPa$$

$$\psi = -1$$

$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$

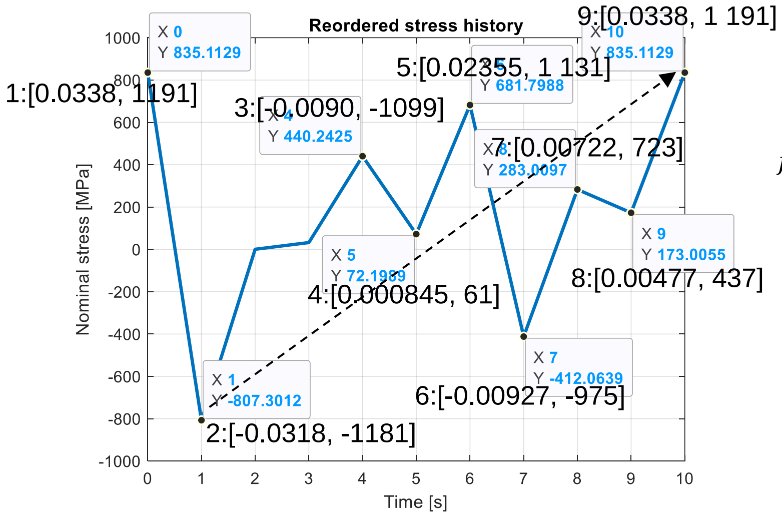
$$\sigma_a = 143\ MPa$$

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.00122$$

$$\sigma(8) = \sigma(7) + 2\psi\sigma_a(8) = 437\ MPa$$

$$\varepsilon(8) = \varepsilon(7) + 2\psi\varepsilon_a(8) = 0.00477$$

Reversal 9: 2 to 9, cyclic stress strain curve



$$S_a = \frac{835 - -807}{2} = 821\ MPa$$

$$\psi = 1$$

$$f_{error} = \frac{(k_t S_a)^2}{\sigma_a E} - \left[\frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}}\right]$$

$$\sigma_a = 1\,186\ MPa$$

$$\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{\frac{1}{n'}} = 0.0328$$

$$\sigma(9) = \sigma(2) + 2\psi\sigma_a(9) = 1\,191\ MPa$$

$$\varepsilon(9) = \varepsilon(2) + 2\psi\varepsilon_a(9) = 0.0338$$

12.10.2. Matlab commands: notch stress and strain at every point

Filename: strain_life_classproblem_2020_04_23.mlx

Class problem in strain-life
Date: 2020-04-23 done during class time.

```
E=117e3;
Ha=1772;na=0.106;sigfa=2030;b=-0.104;
epsfa=0.841;c=-0.688;
kt=2.6;
ferror=@(Sa,kt,E,sa,Ha,na) (kt*Sa)^2./sa/E-(sa/E+(sa/Ha).^(1/na));
epsa=@(sa,E,Ha,na) sa/E+(sa./Ha).^(1/na);
% Cycle 1: 0 to 835 MPa, Monotonic
Sa=835;sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=1
sa1=sa
epsa1=epsa(sa,E,Ha,na)
s1=0+psi*sa1
eps1=0+psi*epsa1
% Cycle 2: 835 to -807
Sa=(835--807)/2;sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=-1
sa2=sa
epsa2=epsa(sa,E,Ha,na)
s2=s1+psi*2*sa2
eps2=eps1+psi*2*epsa2
% Cycle
Sa=(440--807)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=1
sa3=sa
epsa3=epsa(sa,E,Ha,na)
s3=s2+psi*2*sa3
eps3=eps2+psi*2*epsa3

Sa=(440-72)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=-1
sa4=sa
epsa4=epsa(sa,E,Ha,na)
s4=s3+psi*2*sa4
eps4=eps3+psi*2*epsa4

Sa=(682--807)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=1
sa5=sa
epsa5=epsa(sa,E,Ha,na)
s5=s2+psi*2*sa5
eps5=eps2+psi*2*epsa5

Sa=(681--412)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=-1
sa6=sa
epsa6=epsa(sa,E,Ha,na)
s6=s5+psi*2*sa6
eps6=eps5+psi*2*epsa6

Sa=(283--412)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=1
sa7=sa
epsa7=epsa(sa,E,Ha,na)
s7=s6+psi*2*sa7
eps7=eps6+psi*2*epsa7

Sa=(283-173)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
```

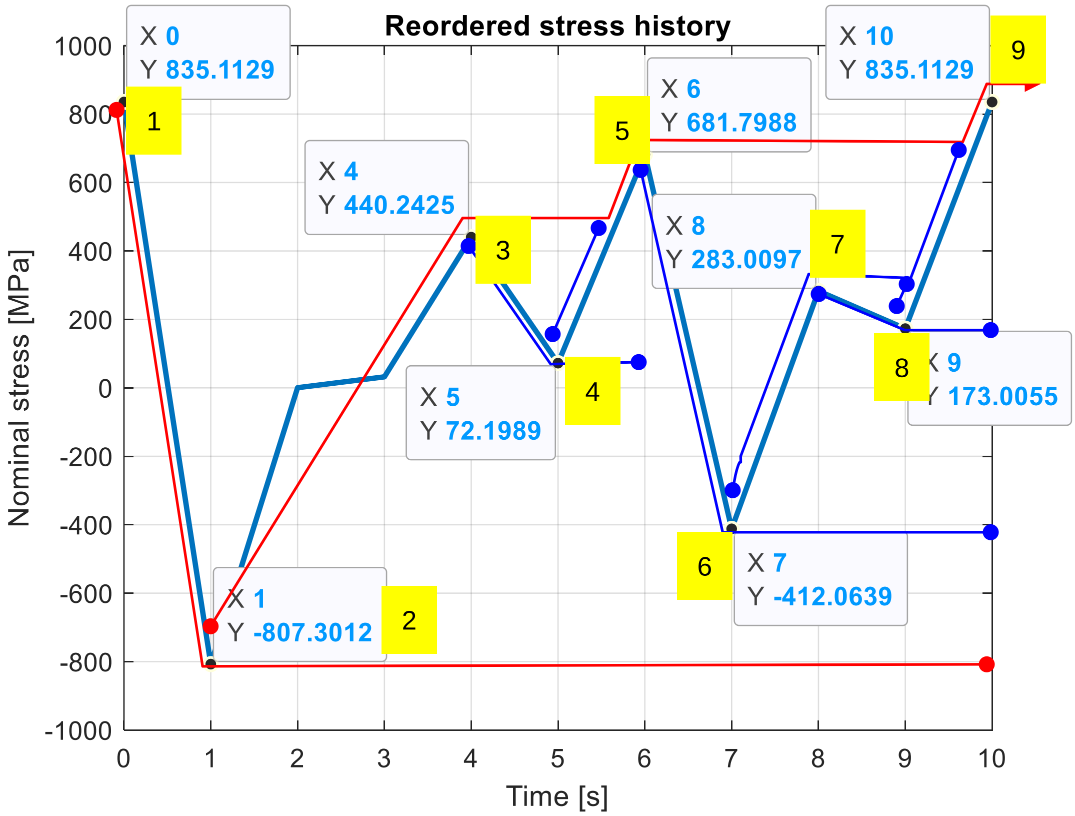
```
psi=-1
sa8=sa
epsa8=epsa(sa,E,Ha,na)
s8=s7+psi*2*sa8
eps8=eps7+psi*2*epsa8

Sa=(835--807)/2,sa=fzero(@(sa) ferror(Sa,kt,E,sa,Ha,na),[1 2000]);
psi=1
sa9=sa
epsa9=epsa(sa,E,Ha,na)
s9=s2+psi*2*sa9
eps9=eps2+psi*2*epsa9
```

12.10.3. Fatigue

From the figure below, the nominal stress cycles with number of cycles are as follows:

From	To	Number of cycles n_i
0	1	1
1	2	1
3	4	1
5	6	1
7	8	1



The steps to calculate the mean stress compensated fatigue life at the notch is as follows:

STEP 1: The zero-mean-stress-equivalent fatigue life is given by:

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

$$\frac{(k_t S_a)^2}{\sigma_a E} = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c$$

$$f_{errsl} = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c - \frac{(k_t S_a)^2}{\sigma_a E}$$

STEP 2: The mean-stress compensated fatigue life is:

$$N_f = N_{mi}^* \left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

This was programmed in the excel sheet on the next page.
The filename of the Excel Workbook is: Strainlife.xls

The use of the solver was demonstrated in class, as well as the use of Matlab to find the damage.

E	1.17E+05	MPa	Kt	2.6							
H'	1772	MPa									
n'	0.106										
σ_f'	2030	MPa									
b	-0.104										
ε_f'	0.841										
c	-0.688			The process here is followed to model the memory effect in the stress history.							

Stress	**From** #	**From** Stress [MPa]	**To** #	**To** Stress [MPa]	**Direction** ψ	**Nom Stress** S_a	**Notch Stress** σ_a	**Error function**	**Strain** ε_a	**Strain at notch** ε	**Stress at notch** σ
Monotone	0	0	1	835		835	1 191	5.77E-08	3.38E-02	3.38E-02	1 191
Cyclic	1	835	2	-807	-1	821	1 186	1.81E-07	3.28E-02	-3.18E-02	-1 181
Cyclic	2	-807	3	440	1	624	1 099	3.62E-08	2.04E-02	9.03E-03	1 017
Cyclic	3	440	4	72	-1	184	478	2.33E-08	4.09E-03	8.45E-04	61
Cyclic	2	-807	5	682	1	745	1 156	3.59E-07	2.77E-02	2.35E-02	1 131
Cyclic	5	682	6	-412	-1	547	1 054	3.63E-08	1.64E-02	-9.27E-03	-976
Cyclic	6	-412	7	283	1	348	849	4.86E-08	8.22E-03	7.17E-03	722
Cyclic	7	283	8	173	-1	55	143	-1.35E-07	1.22E-03	4.72E-03	436
Cyclic	8	-807	9	835	1	821	1 186	1.81E-07	3.28E-02	-1.05E-03	1 191
								1.06E-06			

Stress ranges for fatigue assessment

$$\varepsilon_a = \frac{\sigma_f'}{E}(2N^*)^b + \varepsilon_f'(2N^*)^c \qquad N_f = N_{mi}^*\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{-\frac{1}{b}}$$

	From	To	σ_a	ε_a	σ_m	n_i	N^*	Error	Nf	d	
0 to 1 M	0	1191	596	1.69E-02	596	1	4.13E+02	9.61E-07	1.46E+01	6.83E-02	
1 to 2 C	1191	-1181	1 186	3.28E-02	5	1	9.49E+01	8.84E-07	9.26E+01	1.08E-02	
3 to 4 C	1017	61	478	4.09E-03	539	1	6.15E+05	9.32E-07	3.17E+04	3.16E-05	
5 to 6 C	1131	-976	1 054	1.64E-02	78	1	4.46E+02	6.20E-07	3.06E+02	3.26E-03	
7 to 8 C	722	436	143	1.22E-03	579	1	6.00E+10	1.97E-07	2.38E+09	4.21E-10	
								3.59E-06	D =	8.24E-02	
									Life=	1.21E+01	repetitions

13. CREEP FATIGUE

Slides	Investmech - Fatigue (Creep fatigue) P R0.0.pptx
Chapter in prescribed handbook	15

Creep strain is further deformation of a material that occurs gradually with time. Applications:

- In applications involving high temperatures
- Steam turbines
- Jet and rocket engines
- Nuclear reactors
- Lightbulb filaments
- Gradual loosening of plastic eyeglass frames
- Plastic pipe
- Movement of ice in glaciers
- Etc

Metals and crystalline ceramics:

- Creep deformation of importance for $T > (0.3\ to\ .6) \times T_{melt}$

13.1. Strain behaviour under high temperature

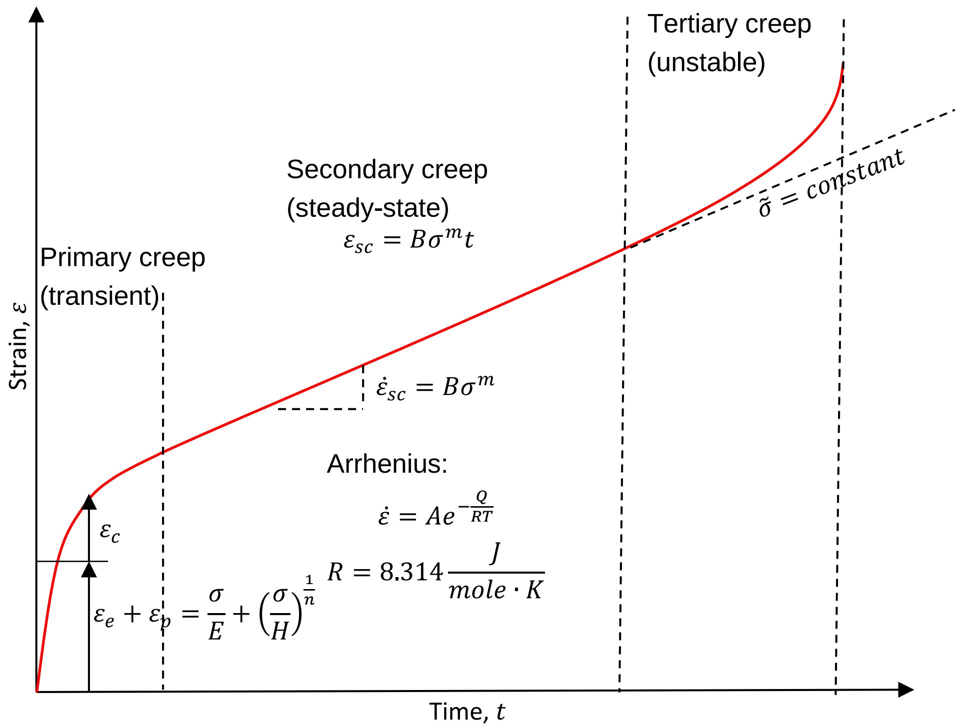


Figure 62: Stress-time response of a material above the creep temperature

13.2. Stress-strain-time models

13.2.1. Linear viscoelasticity

The relationship is:

$$\varepsilon = \frac{\sigma}{E_1} + \frac{\sigma t}{\eta_1} + \frac{\sigma}{E_2}\left(1 - e^{-\frac{E_2 t}{\eta_2}}\right) \tag{160}$$

The components are the elastic, secondary creep, and, tertiary creep strains:

$$\begin{aligned} \varepsilon_e &= \frac{\sigma}{E_1} & \dot{\varepsilon}_e &= 0 \\ \varepsilon_{sc} &= \frac{\sigma t}{\eta_1} & \dot{\varepsilon}_{sc} &= \frac{\sigma}{\eta_1} \\ \varepsilon_{tc} &= \frac{\sigma}{E_2}\left(1 - e^{-\frac{E_2 t}{\eta_2}}\right) & \dot{\varepsilon}_{tc} &= \frac{\sigma}{\eta_2} e^{\frac{-E_2 t}{\eta_2}} \end{aligned} \tag{161}$$

13.2.2. Non-linear creep relationships

Relationship 1

General stress-strain-time relationship:

$$\begin{aligned} \varepsilon &= \varepsilon_i + B\sigma^m t + D\sigma^\alpha\left(1 - e^{-\beta t}\right) \\ \varepsilon_i &= \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{\frac{1}{n}} \end{aligned} \tag{162}$$

Relationship 2

The Ramberg-Osgood relationship for creep is:

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H_c}\right)^{\frac{1}{n_c}} \tag{163}$$

The following

$$\begin{aligned} \varepsilon &= \varepsilon_i + D_3\sigma^\delta t^\phi \\ &= \frac{\sigma}{E} + D_3\sigma^\delta t^\phi \end{aligned} \tag{164}$$

Parameters for this equation can be found from:

- Dowling Table 15.4

This equation is equivalent to the Ramberg-Osgood equation with:

$$\begin{aligned} n_c &= \frac{1}{\delta} \\ H_c &= \frac{1}{(D_3 t^\phi)^{\frac{1}{\delta}}} \end{aligned} \tag{165}$$

13.3. Variable step loading

Rough estimates using the Palmgren-Miner rule for the time-fraction rule:

$$\begin{aligned} \sum \frac{\Delta t_i}{t_{ri}} &= 1 \\ B_f\left(\sum \frac{\Delta t_i}{t_{ri}}\right) &= 1 \end{aligned} \tag{166}$$

Where:

B_f is the number of repetitions to failure

Δt is the time spent at stress level σ_i and temperature T_i

t_{ri} is the rupture life at stress level σ_i at temperature T_i

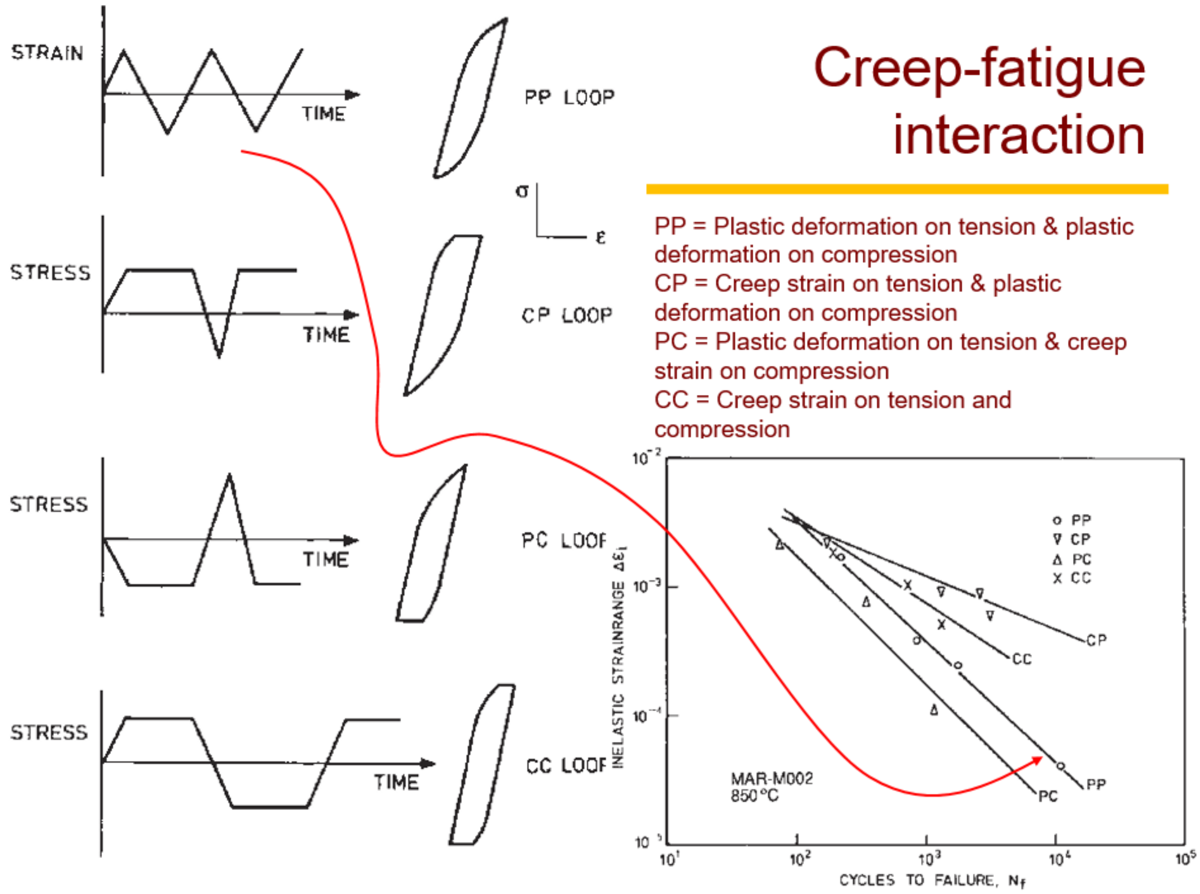
13.4. Creep-fatigue interaction

The Palmgren-Miner rule is applied for a rough estimation of fatigue life:

$$\sum \frac{\Delta t_i}{t_{ri}} + \sum \frac{N_i}{N_{fi}} = 1 \quad (167)$$

Why this approach is not accurate:

1. The physical processes of creep and fatigue are distinct.
2. Creep damage may involve grain boundary cracking in engineering metals, whereas damage due to the fatigue portion may be concentrated in slip bands within the crystal grains.
3. The frequency of cycling is important:
    a. Slow frequencies: creep have more time to contribute to damage.
    b. High frequencies, fatigue might cause crack initiation before significant onset of creep.
    c. In these cases the frequency-modified fatigue approach is applied.
4. Time variation of stress and strain is important – see the figure below.
    a. Loading with creep in compression only (PC) is most severe.
    b. Loading with creep in tension only (CP), gives the longest fatigue life.
5. Oxide layer cracking. Oxidising environment causes an oxide surface layer to form during compressive creep loading, which cracks under subsequent rapid loading into tension. Early cracking results.



Source: (Dowling, 2013, p. 835)

Figure 63: Effect of intermittent creep in tension, compression, or bothy, during cyclic loading of cast Ni-based alloy MAR-M002 at 850 °C

13.5. Creep fatigue curves

Creep fatigue curves of the type shown in Figure 64 are expensive to determine due to the large quantity of tests required. Note that in this case, the life is governed by the amount of creep that occurs at the alternating and mean stress states.

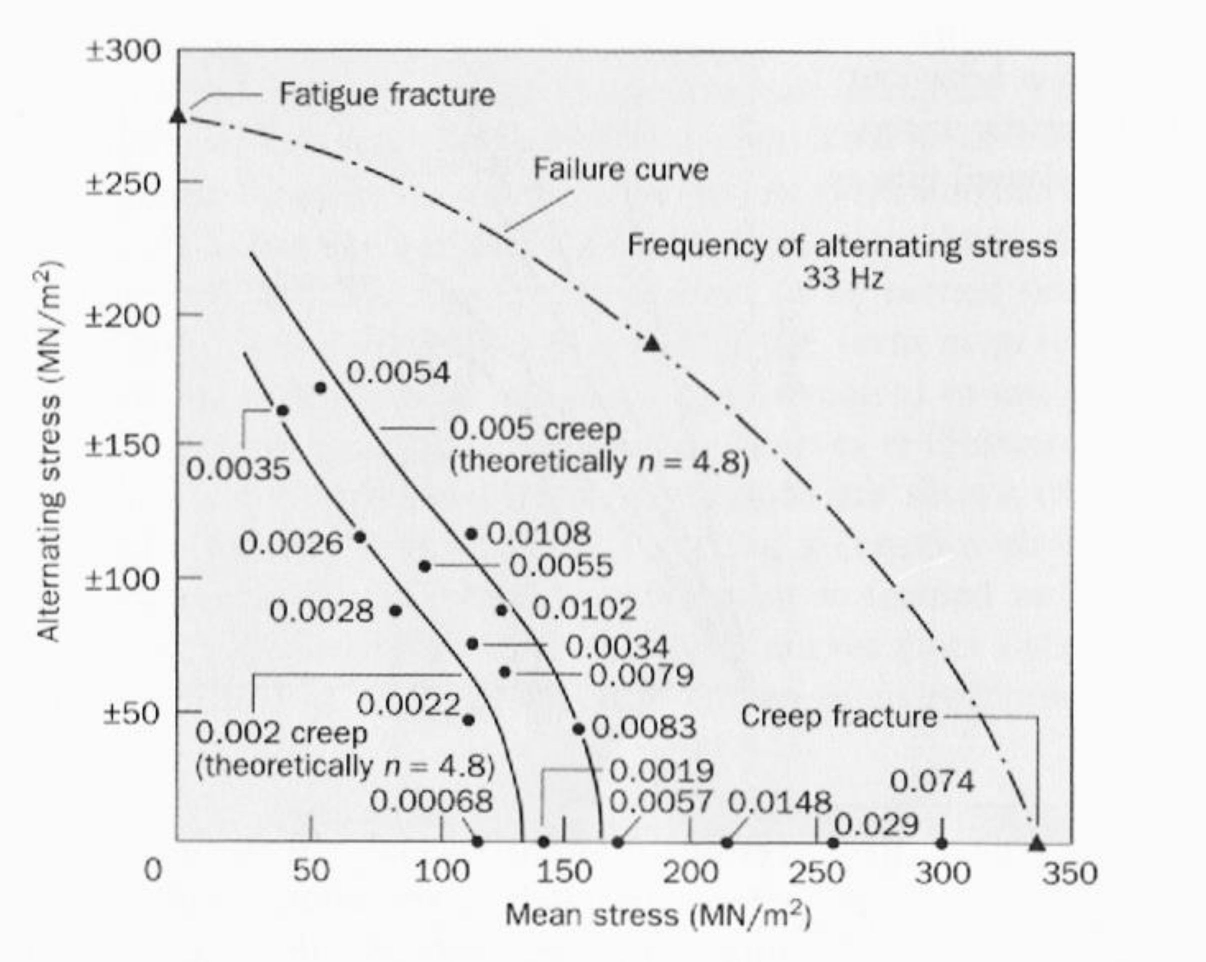
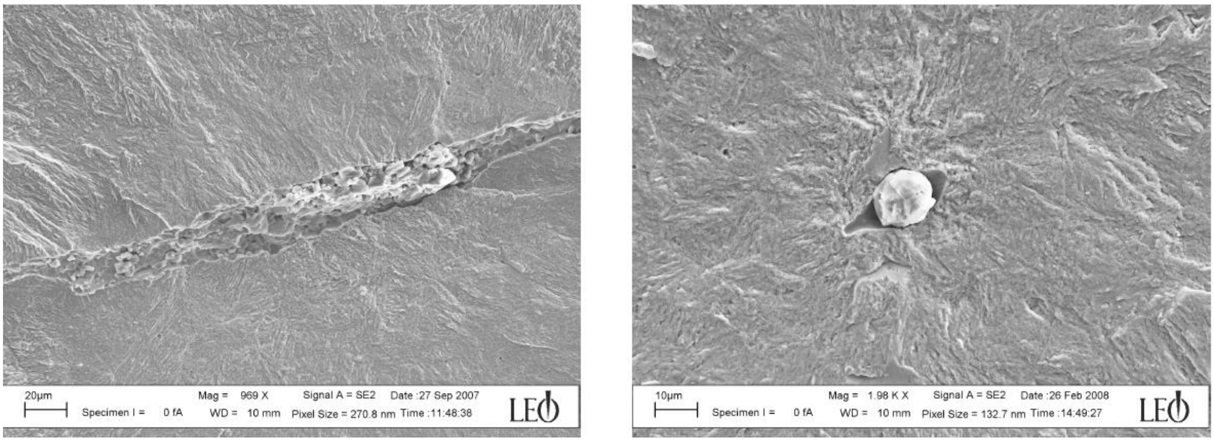


Figure 64: Total creep of 0.26% carbon steel occurring in 100 hours at 400 °C

14. VERY HIGH CYCLE FATIGUE (VHCF)

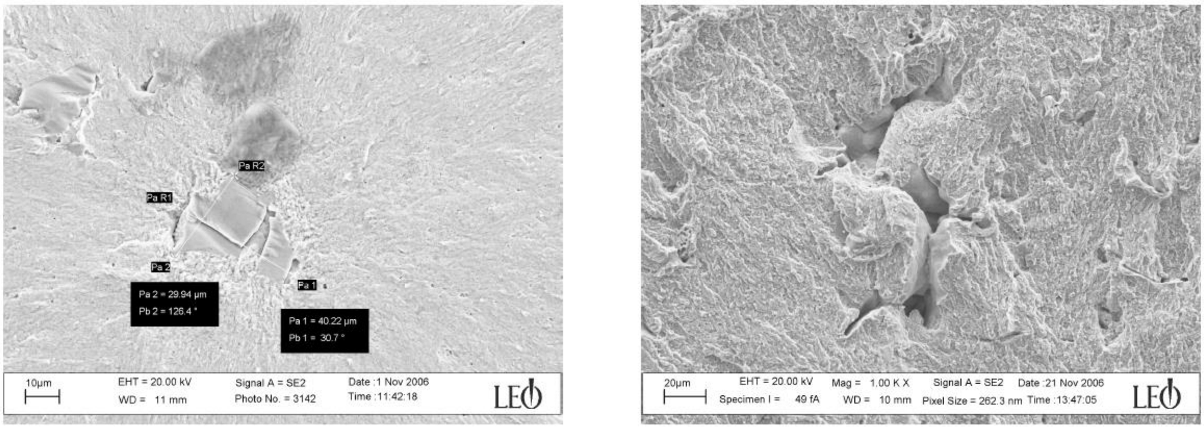
According to Kazymyrovych (2008):

- To study fatigue life above 10^9 cycles
- Finding contradicts presence of a fatigue limit at typically 10×10^6 cycles
- Tested using ultrasonic fatigue testing equipment
- Causes & mechanisms of VHCF failures investigated by means of high-resolution electron microscopy
- Mostly originates at slag inclusions (stringer type or single particle) – See Figure 65



Source: (Kazymyrovych, 2008, p. 6)

Figure 65: Fatigue failure originating from slag inclusions of stringer type and single particle



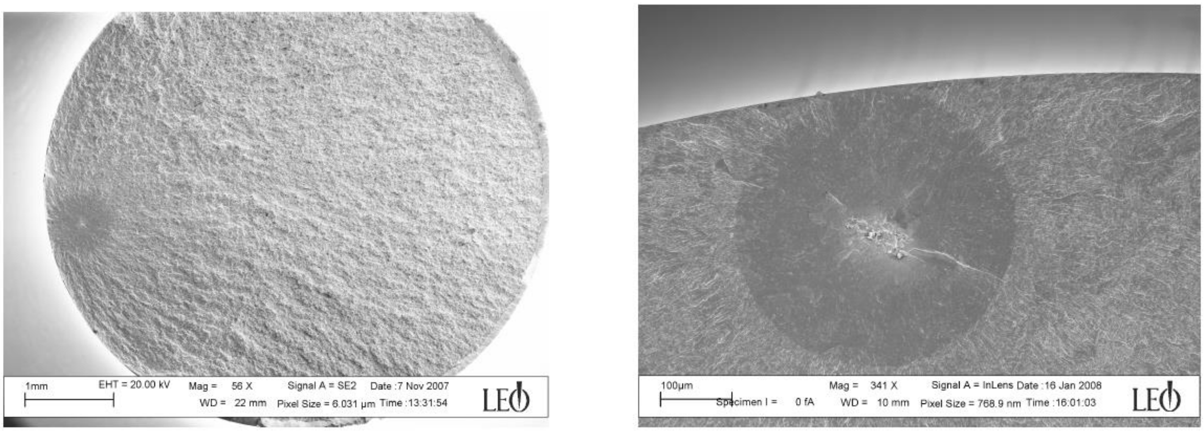
Source: (Kazymyrovych, 2008, p. 7)

Figure 66: Fatigue failure initiated from carbide and void

14.1. Fracture morphology

According to Kazymyvorych (2008, pp. 7-8)

- "Fish-eye" around the fatigue crack initiation point
  - Has relatively flat morphology
- Changes shape when the fatigue crack reaches the surface
  - Crack growth rates differ within and outside the "fish-eye"
- If "fish-eye" is to be considered as an evaluating parameter of the fatigue process, then only "fish-eyes" located away from the surface should be used
- "Fish-eyes" reaching the surface
  - Size is largely defined by the distance from the fatigue initiating defect to the surface, rather than by critical crack size that marks the shift in growth mechanism for the internal crack
  - Crack grows beyond the "fish-eye" boundary until it reaches a critical size and rapid failure occurs



Source: (Kazymyrovych, 2008, p. 8)

Figure 67: Typical fatigue failure with the "fish-eye" dye to very high-cycle loading

14.2. Crack initiation and growth

Two main stages according to Kazymyvorych (2008, p. 8):

- Crack growth within the "fish-eye"
  - 10 – 20 nm/cycle
- Crack growth outside the "fish-eye"
  - 30 – 140 nm/cycles
- Portions of fatigue life for the crack to grow within & outside the "fish-eye" are insignificant in comparison to total fatigue life
  - Less than 1% of total life
- Concept of VHCF initiation zone
  - Area around fatigue origin where striations are not distinguishable in SEM
  - Estimated at 50 μm in diameter
  - Conclusion that in the VHCF regime, ~100% of fatigue life is consumed during the crack initiation

For a specimen almost 10^9 cycles were applied during the initiating stage, i.e. prior to the fatigue crack reaching ~ 50 μm. Kazymyvorych: ' If assumed that the fatigue crack grows with each load cycle by the smallest possible step of 0.1 nm – the interatomic distance, then it would take less than 10^6 load cycles for the crack to grow through the initiating zone" (2008, p. 10). There must be mostly non-propagating cycles during initiation stage.

According to Kazymyvorych, <u>if hydrogen-assisted growth is assumed</u> (2008, p. 11):

- It is possible that during non-propagating cycles the H-atoms diffuse to the most stressed regions – the crack tip
- When H-atom concentration is high enough, the crack propagates for a short distance
- New period of crack stagnation and H-atom diffusion begins

Dislocation rearrangement (Kazymyrovych, 2008, p. 11):

- During non-propagating cycles, continuous dislocation rearrangement takes place
- When favourable positioning is achieved – crack advances by small step

Non-propagating cycles constitute about 99% of VHCF life – then the total fatigue life should be related to the stress state at the crack tip during the initiation stage

14.2.1. Murakami

Suggests that in the VHCF range the crack growth during the initiation stage is enabled by synergistic effect of cycle stresses and hydrogen that is trapped by the inclusion

Without hydrogen the crack equal to the size of VHCF initiating inclusion would be non-propagating

Destructive effect of hydrogen results in the crack growing to a certain size when it is big enough to propagate entirely due to the applied stress

Optically Dark Area (ODA) – represents hydrogen-assisted crack growth: appears dark on optical microscope, but is light when viewed with SEM (Scanning Electron Microscope)

15. DESIGN TO AVOID FATIGUE FAILURE

Consider:

- Stress concentrations: eliminate or reduce their effect
- Stress distributions in members (beams, etc.): put joints (including splices) at points of lower bending moments, shear force, etc.
- Residual stress.

Study Section 10.9 of Dowling (Dowling, 2013).

16. REFERENCES

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List of symbols and abbreviations

Conversion factors used by Investmech is available at is available at this link.

k_1	Magnification factor for nominal stress ranges to account for secondary bending moments in trusses
k_f	Long life fatigue notch factor
k_f'	Short life fatigue notch factor
k_{fm}	Mean stress fatigue notch factor
k_s	Reduction factor for fatigue stress to account for size effects
m	Slope of the fatigue curve
m_e	Fatigue limit ratio ($= \sigma_{erb}/\sigma_u$)
m_d	Fatigue limit reduction factor for size
m_s	Fatigue limit reduction factor for surface
m_t	Fatigue limit reduction factor for load type
m_1	Slope of the fatigue curve for stress range above the constant amplitude fatigue limit
m_2	Slope of the fatigue curve for stress ranges between the cut-off limit and constant amplitude fatigue limit
n_1, n, n'	Strain-hardening exponent
$\hat{N}$	Service life [cycles]

N_e	Fatigue limit life [cycles]
N_R	Design life time expressed as number of cycles related to a constant stress range.
N_f	Endurance or failure life [cycles]
q	Notch sensitivity
Q_k	Characteristic value of a single action [N]

Ramberg-Osgood $\sigma - \varepsilon$ curve:

E, H, n	Monotonic behaviour
E', H', n'	Cyclic behaviour
$\overline{\Delta S}, S_{ar}$	Equivalent stress [MPa]
S_{er}	Fatigue limit for notched specimen [MPa]
X_N	Safety factor in life
X_S	Safety factor in stress

Greek symbols:

α	Peterson constant
β	Geometric stress concentration factor Also used for the Neuber constant
γ_{Ff}	Partial factor for equivalent constant amplitude stress ranges $\Delta\sigma_E, \Delta\tau_E$
γ_{Mf}	Partial factor for fatigue strength $\Delta\sigma_C, \Delta\tau_C$
δ	Slope reduction factor
ε	Strain. If there is a subscript it indicates the direction of the strain. For example, ε_x is strain in the x-direction.
η	Eta
θ	Angle [°]
λ_i	Damage equivalent factors
ν	Poisson ratio
σ	Point stress [MPa]
σ_a	Stress amplitude [MPa], subscripts indicate directions
$\hat{\sigma}_a$	Service stress amplitude [MPa]
σ_{ar}	Equivalent completely reversed stress amplitude [MPa]
σ_{aq}	Equivalent constant amplitude stress [MPa]
σ_e	Endurance limit or fatigue limit [MPa], subscript b as in σ_{erb} indicate the endurance limit from a polished rotating bending test
σ_{er}	Adjusted fatigue limit [MPa]
σ_{erb}	Fatigue limit of a polished specimen on a rotating bending test [MPa]

σ_o'	Cyclic yield strength [MPa]
$\Delta\sigma$	Direct stress range [Pa]
$\Delta\tau$	Shear stress range [Pa]
$\Delta\sigma_{eq}$	Equivalent stress range for connections in webs of orthotropic decks [Pa]
$\Delta\sigma_C, \Delta\tau_C$	Reference value of the fatigue strength at $N_C = 2 \times 10^6$ cycles [Pa]
$\Delta\sigma_D, \Delta\tau_D$	Fatigue limit for constant amplitude stress range at N_D cycles [Pa]
$\Delta\sigma_E, \Delta\tau_E$	Equivalent constant amplitude stress range related to n_{max} [Pa]
$\Delta\sigma_{E2}, \Delta\tau_{E2}$	Equivalent constant amplitude stress range related to 2 million cycles [Pa]
$\Delta\sigma_L, \Delta\tau_L$	Cut-off limit for stress ranges at N_L cycles [Pa]
$\Delta\sigma_{C,red}$	Reduced reference value of the fatigue strength [Pa]

Note, the lists above do not contain symbols that are clearly defined in the section or that are indicated on sketches.