Investmech: Force, moment, torque and stress distributions

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Topics

- Shear force distribution
- Bending moment distribution
  - Bernoulli-Euler beam modelling
- Normal stress distribution
- Bending stress distribution
- Torque shear stress distribution
- Shear force stress distribution

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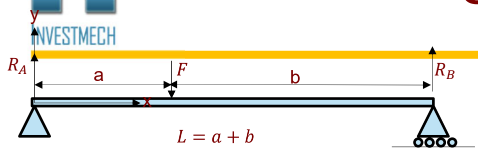
Shear force diagram



From Newton's law for forces:

$$R_A - F + R_B = 0$$

From Newton's law for moments around A:

$$R_B L - Fa = 0$$

$$R_B = \frac{Fa}{L}$$

Substitute into the first equation above:

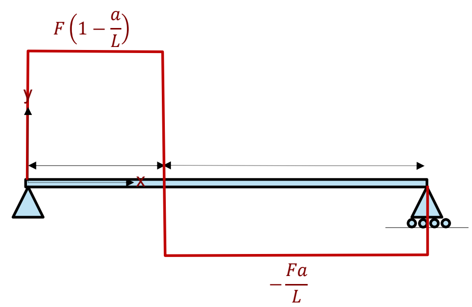
$$R_A = F - R_B$$

$$= F - \frac{Fa}{L}$$

$$= F\left(1 - \frac{a}{L}\right)$$

Shear force diagram:

- R_A for $x \in [0; a]$
- $-R_B$ for $x \in [a; L]$



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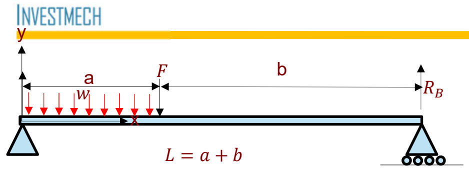
Add distributed load



Shear force distribution over $x \in [0; a]$:

$$V(x) = R_A - wx$$

$$= R_A - wx$$

Shear force distribution over $x \in [a, L]$:

$$V(x) = R_A - wa - F$$

From Newton's law for forces:

$$R_A - F + R_B - wa = 0$$

From Newton's law for moments around A:

$$R_B L - Fa - wa \cdot \frac{a}{2} = 0$$

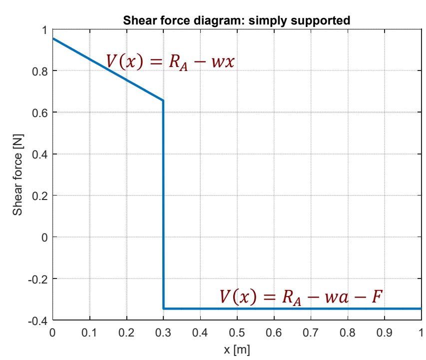
$$R_B = \frac{Fa + \frac{wa^2}{2}}{L}$$

Substitute into the first equation above:

$$R_A = F - R_B + wa$$

$$= F - \frac{Fa + \frac{wa^2}{2}}{L} + wa$$

$$= F\left(1 - \frac{a}{L}\right) + w\left(\frac{a^2}{L} + a\right)$$



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Bending moment diagram

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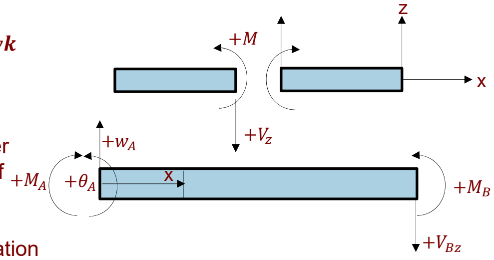
Deflections in 3D are: $u\boldsymbol{i} + v\boldsymbol{j} + w\boldsymbol{k}$
Euler-Bernoulli beam theory:

$$E(x)I(x)\frac{d^2w}{dx^2} = M(x)$$

For y-axis pointing towards reader
Sign of M depends on direction of z-axis

If z-axis points upwards, the equation becomes (y-axis pointing away from reader):

$$E(x)I(x)\frac{d^2w}{dx^2} = M_y(x)$$



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Apply to cantilever beam

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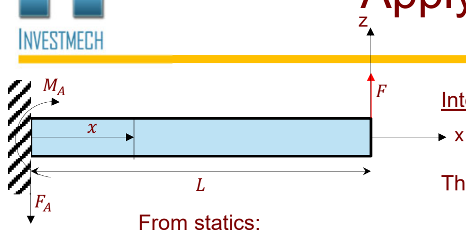

From statics:

$$M_A = FL$$
$$F_A = F$$

The bending moment at position x:

$$M_y(x) = -F_A x + M_A$$
$$= -Fx + FL$$

Euler-Bernoulli for this coordinate system:

$$E(x)I(x)\frac{d^2w}{dx^2} = M_y(x)$$
$$\frac{d^2w}{dx^2} = \frac{M}{E(x)I(x)}$$
$$= \frac{F}{EI}(-x + L)$$

Clockwise moment at x is + for chosen coordinate system

Integrate once to calculate beam slope:

$$\frac{dw}{dx} = \frac{F}{EI}\left(-\frac{1}{2}x^2 + Lx\right) + A$$

The rotation is zero at $x = 0$ (built in):

$$\theta_A = \frac{dw}{dx}[x = 0]$$
$$= A = 0$$

$\theta(x = L) = \frac{1}{2}\frac{FL^2}{EI}$: tip will lift up, + slope

Integrate to calculate deflection:

$$w = \frac{F}{EI}\left(-\frac{1}{6}x^3 + \frac{1}{2}Lx^2\right) + B$$

The deflection is zero at $x = 0$. Therefore,

$$B = 0$$

Deflection at the end is:

$$w = \frac{1}{3}\cdot\frac{FL^3}{EI}$$

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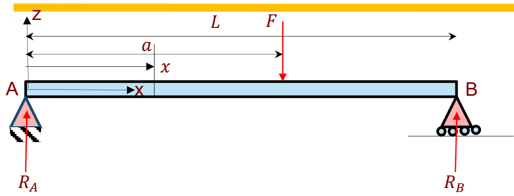
Simply supported beam



Apply the static equations

- Sum of forces:

$$\sum F_z = 0 = R_A + R_B - F$$

$$R_A + R_B = F$$

- Sum of moments around A:

$$\sum M_y = 0$$

$$Fa - LR_B = 0$$

$$R_B = \frac{Fa}{L}$$

Therefore:

$$R_A = F - \frac{Fa}{L}$$

Bending moment equations

From A to F x∈[0; a]:

$$M_y = R_A x$$

$$= \left(F - \frac{Fa}{L}\right) x$$

From F to B x ∈[a; L]:

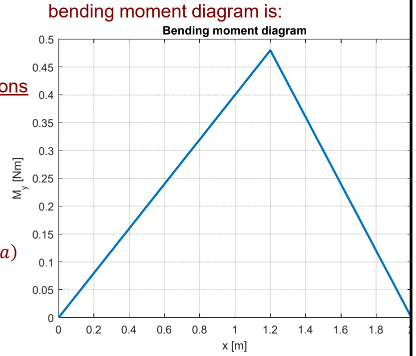
$$M_y = R_A x - F(x - L)$$

$$= F\left(1 - \frac{a}{L}\right) x - F(x - a)$$

$$= -\frac{Fa}{L} x + Fa$$

$$= Fa\left(1 - \frac{x}{L}\right)$$

For F = 1 N; a = 1.2 m & L = 2 m, the bending moment diagram is:



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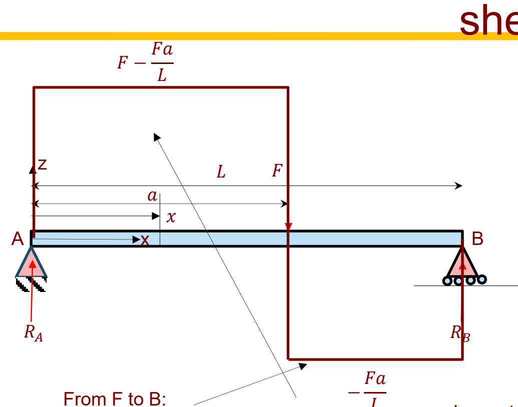
Shear force diagram and shear force to bending moment by integration of shear force



From A to F:

$$M_y = \int_{x=0}^{x} \left(F - \frac{Fa}{L}\right) dx$$

$$= \left(F - \frac{Fa}{L}\right) x$$

From F to B:

$$M_y = \int_{x=a}^{x} \left(-\frac{Fa}{L}\right) dx + \left(F - \frac{Fa}{L}\right) a$$

$$= -\frac{Fa}{L} x + \frac{Fa^2}{L} + \left(Fa - \frac{Fa^2}{L}\right)$$

$$= Fa\left(1 - \frac{x}{L}\right)$$

In most cases, this step is not followed and the shear force and bending moment diagram is constructed by addition and subtraction as demonstrated up to this slide

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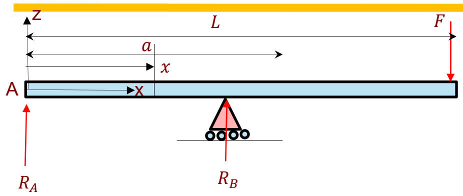
Sliding beam



$$R_B = \frac{L}{L_1}F$$
$$R_A + R_B = F$$
$$R_A = F\left(1 - \frac{L}{L_1}\right)$$

For interval A-B:

$$M_y = R_A x$$
$$\frac{d^2w}{dx^2} = \frac{1}{EI}R_A x$$
$$\frac{dw}{dx} = \frac{1}{EI}\left(\frac{1}{2}R_A x^2 + C_1\right)$$
$$w = \frac{1}{EI}\left(\frac{1}{6}R_A x^3 + C_1 x + C_2\right)$$

But, $w(0) = 0$ and $w(L_1) = 0$

$$C_2 = 0$$
$$C_1 = -\frac{1}{6}R_A L_1^2$$

For interval B-C:

$$M_y = R_A x + R_B(x - L_1)$$
$$\frac{d^2w}{dx^2} = \frac{1}{EI}(R_A x + R_B(x - L_1))$$
$$\frac{dw}{dx} = \frac{1}{EI}\left(\frac{1}{2}R_A x^2 + \frac{1}{2}R_B x^2 - R_B L_1 x + C_3\right)$$
$$w = \frac{1}{EI}\left(\frac{1}{6}R_A x^3 + \frac{1}{6}R_B x^3 - \frac{1}{2}R_B L_1 x^2 + C_3 x + C_4\right)$$

But, $\frac{dw}{dx}_{AB} = \frac{dw}{dx}_{BC}$ at $x = L_1$ and $w(L_1) = 0$

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THIS MUST BE COMPLETED

AT Point B: $\frac{dw}{dx}_{AB} = \frac{dw}{dx}_{BC}$ at $x = L_1$ and $w(L_1) = 0$

$$\frac{dw(L_1)}{dx} = \frac{1}{2}R_A x^2 - \frac{1}{6}R_A L_1^2 = \frac{1}{2}R_A x^2 + \frac{1}{2}R_B x^2 - R_B L_1 x + C_3$$
$$C_3 = -\frac{1}{6}R_A L_1^2 - \frac{1}{2}R_B {L_1}^2 + R_B L_1^2$$
$$= L_1^2\left(-\frac{1}{6}R_A + \frac{1}{2}R_B\right)$$

and

$$w(L_1) = \frac{1}{EI}\left(\frac{1}{6}R_A x^3 - \frac{1}{6}R_A L_1^2 x\right) = \frac{1}{EI}\left(\frac{1}{6}R_A x^3 + \frac{1}{6}R_B x^3 - \frac{1}{2}R_B L_1 x^2 + C_3 x + C_4\right)$$
$$-\frac{1}{6}R_A L_1^3 = \frac{1}{6}R_B L_1^3 - \frac{1}{2}R_B L_1^3 + C_3 L_1 + C_4$$
$$C_4 = -\frac{1}{6}R_A L_1^3 - \frac{1}{6}R_B L_1^3 + \frac{1}{2}R_B L_1^3 + C_3 L_1$$

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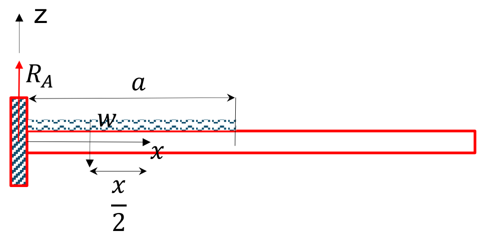
Bending moment distribution on a beam



At position x, formulate the bending moments by just looking towards the origin of the coordinate system

For $x \in [0; a]$:

$$M_y = M_{R_A} + M_w$$
$$= R_A x - wx \cdot \frac{x}{2}$$

For $x \in [a; L]$:

$$M_y = M_{R_A} + M_w$$
$$= R_A x - wa \cdot \left(x - \frac{a}{2}\right)$$

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Where to put splices?

- Demonstrate in Prokon
- There is an example in the class notes

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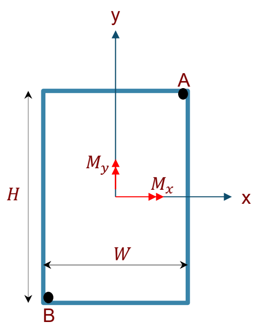
Bending stress distribution



General equation:

$$\sigma_{b,i} = \frac{M_x y_i}{I_{xx}} - \frac{M_y x_i}{I_{yy}}$$

Where:

x_i: x-coordinate of the point where stress is required
y_i: y-coordinate of the point where stress is required
I_{xx}: Second moment of area around the x-axis [m^4]
I_{yy}: Second moment of area around the y-axis [m^4]

Applied to Point A:

$$\sigma_{b,A} = \frac{M_x y_A}{I_{xx}} - \frac{M_y x_A}{I_{yy}}$$

$$x_A = \frac{W}{2}, y_A = \frac{H}{2}$$

Applied to Point B:

$$\sigma_{b,B} = \frac{M_x y_B}{I_{xx}} - \frac{M_y x_B}{I_{yy}}$$

$$x_B = -\frac{W}{2}, y_B = -\frac{H}{2}$$

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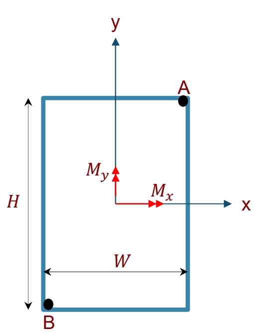
Bending stress distribution



Say $H = 100; W = 50\ mm$, then:

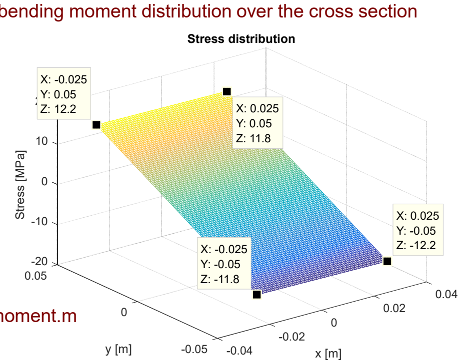
$$I_{xx} = \frac{1}{12}WH^3 = 4.1667 \times 10^{-6}\ m^4$$

$$I_{yy} = \frac{1}{12}HW^3 = 1.25 \times 10^{-5}\ m^4$$

Bending stresses at A and B for $M_x = 1000\ Nm$ & $M_y = 100\ Nm$

$$\sigma_{b,A} = 11.8\ MPa; \sigma_{b,B} = -11.8\ MPa$$

The bending moment distribution over the cross section



Investmech algorithm: stressbendingmoment.m

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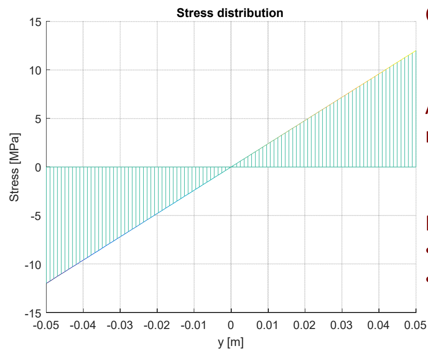
Bending stress distribution at x=0



General equation:

$$\sigma_{b,i} = \frac{M_x y_i}{I_{xx}} - \frac{M_y x_i}{I_{yy}}$$

At $x = 0$, the bending stress distribution reduces to:

$$\sigma_{b,i} = \frac{M_x y_i}{I_{xx}}$$

For a positive bending moment M_{xx}:

- Tensile stress for positive y
- Compressive stress for negative y

Investmech algorithm: stressbendingunsymmetric.m

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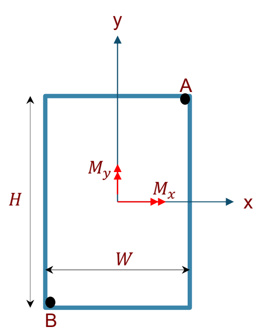
Class problem – do in class



You have square cross section with $W = 50\ mm$ and $H = 100\ mm$

What are the stresses at Points A and B on the figure for moments $M_x = 1\,000\ Nm$ and $M_y = 100\ Nm$ applied simultaneously?

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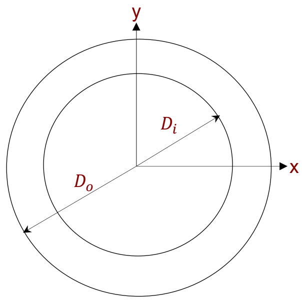
Bending stress: Circular tube

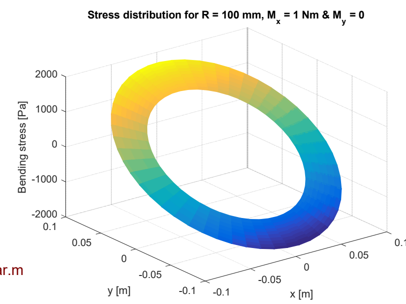
General equation:

$$\sigma_{b,i} = \frac{M_x y_i}{I_{xx}} - \frac{M_y x_i}{I_{yy}}$$

Where:

$$I_{xx} = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$
$$I_{yy} = I_{xx}$$
$$I_{zz} = I_{xx} + I_{yy}$$
$$= 2I_{xx}$$





Investmech algorithm: stresscircular.m

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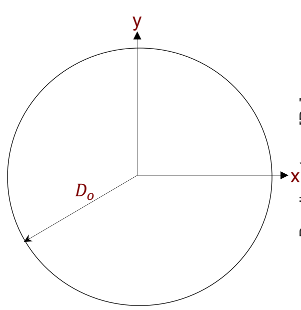
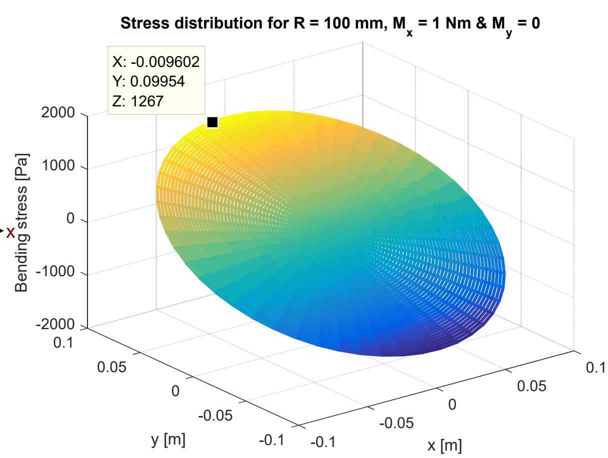
Bending stress: Solid circular bar





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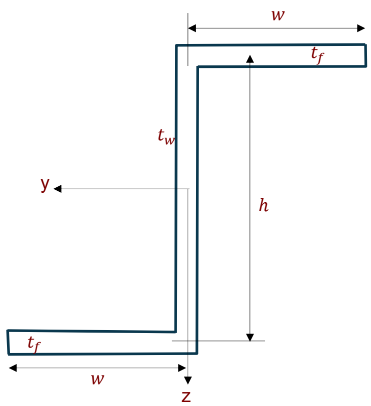
Bending stress in unsymmetrical sections



The equation for unsymmetrical bending under M_y:

$$\sigma = \frac{F}{A} + \frac{M_y I_{zz} + M_z I_{yz}}{I_{zz} I_{yy} - I_{yz}^2} z - \frac{M_z I_{yy} + M_y I_{yz}}{I_{zz} I_{yy} - I_{yz}^2} y$$

Where:

$$I_{yy} = \frac{1}{12} t_w h^3 + w t_f \left(\frac{h}{2}\right)^2 \times 2$$

$$I_{zz} = \frac{1}{12} h t_w^3 + 2 \times \frac{1}{3} t_f w^3$$

$$I_{yz} = t_f w \left(\frac{h}{2} \cdot \frac{w}{2}\right) + t_f w \left(\frac{-h}{2} \cdot \frac{-w}{2}\right)$$

$$= \frac{1}{2} t_f w^2 h$$

See stress distribution on next page

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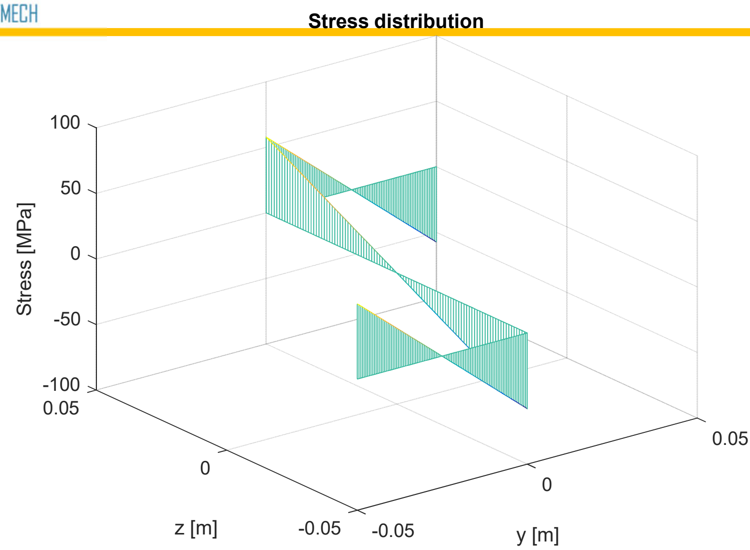

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Normal stress distribution

- Is uniform

$$\sigma_n = \frac{F}{A}$$

Shear stress distribution-solid rectangular bar

The first moment of area around the x- and y-axes are:

$$Q_x = \int y dA$$

$$Q_y = \int x dA$$

From which the shear stress is:

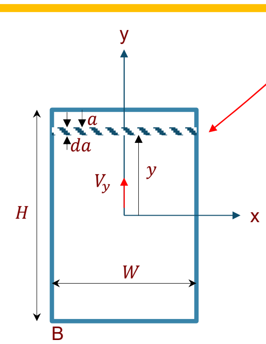
$$\tau_y = \frac{V_y Q_x}{I_{xx} W}$$

$$\tau_x = \frac{V_x Q_y}{I_{yy} H}$$

$$\tau = \tau_x + \tau_y$$

Average shear stress:

$$\tau_{ave} = \frac{V}{A}$$



$$dA = W dy$$

Shear flow equation

$$Q_x(y) = -\int_{H/2}^{y} yW dy$$

$$= -\left[\frac{1}{2} W y^2\right]_{H/2}^{y}$$

$$= -\frac{1}{2} W \left[y^2 - \frac{H^2}{4}\right]$$

$$= \frac{1}{2} W \left[\frac{H^2}{4} - y\right]$$

With maximum value at $a = \frac{H}{2}$:

$$Q_{x,max} = \frac{1}{2} W \frac{H^2}{4}$$

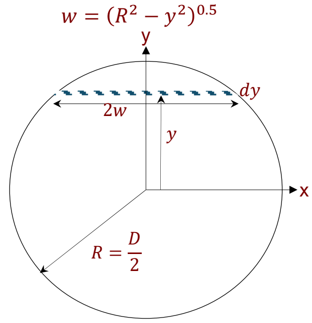
$$\tau_{max} = \frac{V_y \frac{WH^2}{8}}{\frac{1}{12} W^2 H^3}$$

$$= \frac{3}{2} \cdot \frac{V_y}{WH}$$

$$= 1.5 \times \frac{V_y}{A}$$

Shear stress: Circular section

$w = (R^2 - y^2)^{0.5}$



The first moment of area around the x-axis (the minus sign is because the integral from R to y is in the opposite direction as shown in the figure):

$$Q_x(y) = \int_R^y -y dA$$
$$= \int_R^y -y \cdot 2w dy$$
$$= 2\int_R^y -y(R^2 - y^2)^{0.5}\ dy$$
$$= 2 \cdot \frac{1}{3}[(R^2 - y^2)^{1.5}]_R^y$$
$$= \frac{2}{3}(R^2 - y^2)^{1.5}$$

The shear stress is maximum when $y = 0$:

$$\tau = \frac{VQ}{I_{xx}(2R)}$$
$$I_{xx} = \frac{\pi}{4}R^4$$
$$\tau_{max} = \frac{V \cdot \frac{2}{3}R^3}{\frac{\pi}{4}R^4 \cdot 2R}$$
$$= \frac{4}{3}\left(\frac{V}{\pi R^2}\right) = \frac{4}{3}\tau_{ave}$$

Average shear stress:

$$\tau_{ave} = \frac{V}{A}$$
$$= \frac{V}{\pi R^2}$$

Consult http://integral-table.com/ for integrals

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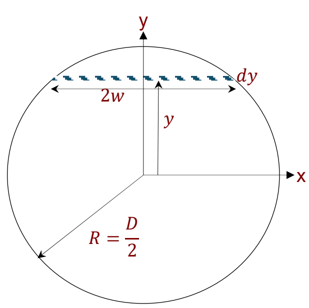
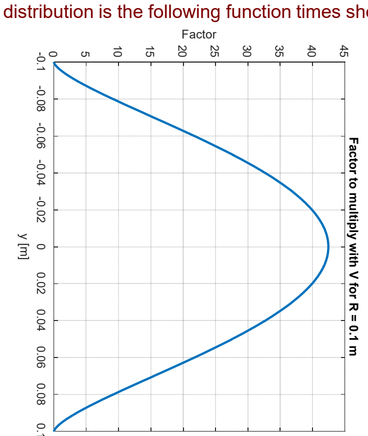
Stress distribution

For radius $R = 100\ mm$, the shear stress distribution is the following function times shear force V:





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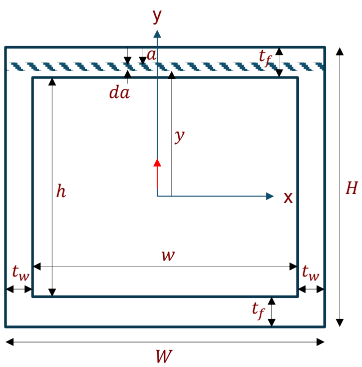
Shear stress distribution in boxed section



Area:

$$A = WH - wh$$

Second moment of area:

$$I_{xx} = \frac{1}{12}(WH^3 - wh^3)$$

First moment of area:

$$Q_x = \int y dA$$

$$= \overbrace{\int_0^{t_f}\left(\frac{H}{2} - a\right)W da}^{Top\ flange} + \overbrace{\int_{t_f}^{H-t_f}\left(\frac{H}{2} - a\right)2t_w da}^{Webs}$$

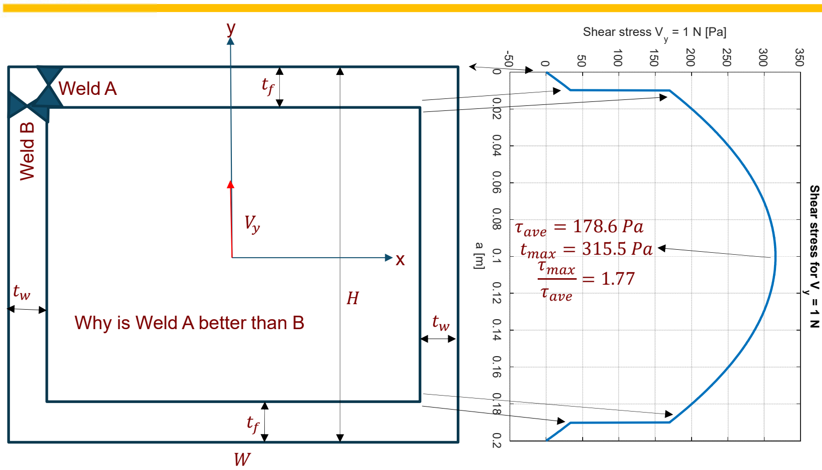
$$+ \overbrace{\int_{H-t_f}^{H}\left(\frac{H}{2} - a\right)W da}^{Bottom\ flange}$$

The equation for the shear stress:

$$t = \frac{V_y Q_x}{I_{xx} b}$$

Where b is the thickness, ranging from W to $2t_f$, depending on a

Shear stress distribution for V_y = 1 N for H = 0.2; W = 0.1; t_w = 0.01; t_f = 0.01



Investmech algorithm used: stressshearboxed.m

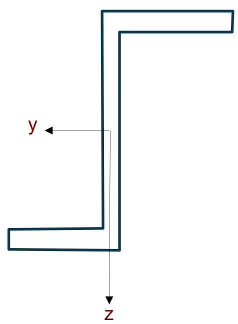
Shear stress in unsymmetric sections

The shear stress for unsymmetric sections is:

$$\tau = \frac{I_{zz}Q_y - I_{yz}Q_z}{b\left(I_{zz}I_{yy} - I_{yz}^2\right)}V_z + \frac{I_{zz}Q_z - I_{yz}Q_y}{b\left(I_{zz}I_{yy} - I_{yz}^2\right)}V_y$$

$$Q_y = \int zdA$$

$$Q_z = \int ydA$$



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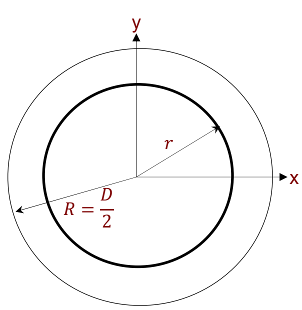
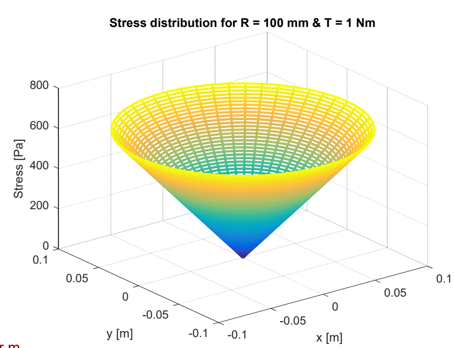
Torque

Shear stress due to torque:

$$t_T = \frac{Tr}{I_{zz}}$$





Investmech algorithm: stresscircular.m

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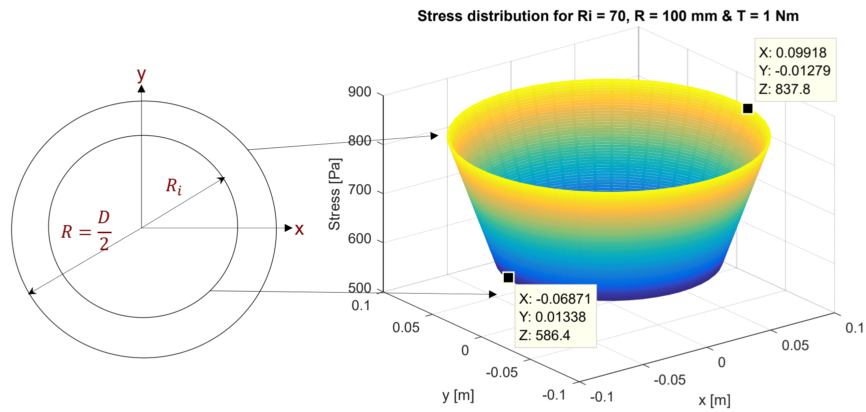
Circular tube



Investmech algorithm: stresscircular.m

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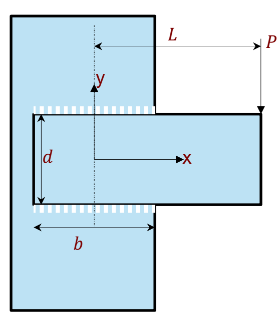
Weld group subjected to torsion and shear



Problem statement
The attachment is subjected to the force P as shown. Compile equations from which the weld throat area, a, can be calculated.

Solution
The second moments of area for the weld group is:

$$I_{xx} = 2ab\left(\frac{d}{2}\right)^2$$
$$= \frac{1}{2}abd^2$$
$$I_{yy} = 2 \cdot \frac{1}{12}ab^3$$
$$= \frac{1}{6}ab^3$$
$$I_{zz} = I_{xx} + I_{yy}$$
$$= a\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)$$

The area of the weld group is:

$$A = 2ab$$

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- Shear stress due to shear force
  - Assume uniform distribution

$$\tau_{yV} = -\frac{P}{A} = -\frac{P}{2ab}$$

- Shear stress due to torque

$$\tau_{yT} = -\frac{PLx}{I_{zz}}$$

$$\tau_{xT} = \frac{PLy}{I_{zz}}$$

- From direction of stresses, critical point is Point A

$$x_A = \frac{b}{2};\, y_A = \frac{d}{2}$$

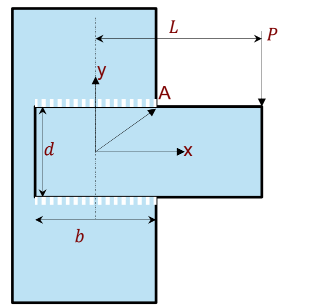
$$\tau_{y,A} = -\frac{PLb}{2a\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)} - \frac{P}{2ab}$$

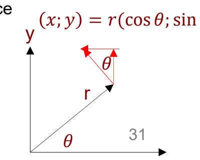
$$\tau_{xT,A} = \frac{PLd}{2a\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)}$$

  - Calculate residual stress on the surface and compare with factored resistance

$$\tau = \sqrt{\tau_{y,A}^2 + \tau_{x,A}^2} \le 0.67^2 x_u$$

Shear stress due to **shear force** and **torque** to be considered





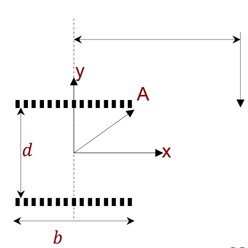
- Substitute further:

$$\tau_{y,A} = -\frac{PLb}{2a\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)} - \frac{P}{2ab}$$

$$= \frac{P}{a}\left[-\frac{Lb}{2\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)} - \frac{1}{2b}\right]$$

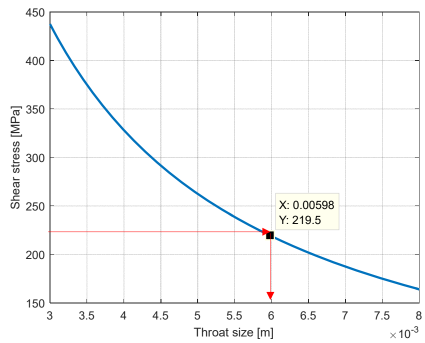
$$\tau_{x,A} = \frac{PLd}{2a\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)}$$

$$= \frac{P}{a}\left[\frac{Ld}{2\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)}\right]$$



Using the idea

- Say, $d = 100\ mm, b = 50\ mm, L = 400\ mm, P = 15\ 000\ N$
- Weld metal is E70XX, with ultimate tensile strength $490\ MPa$
  - Factored resistance: $\tau_R = 0.67^2 \cdot 490 = 220\ MPa$
  - Calculate stress and find throat size. From graph it is $a \geq 6\ mm$
  - $e \geq 8.5\ mm$



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Using the equation

$$\tau_{y,A} = \frac{P}{a}\left[-\frac{Lb}{2\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)} - \frac{1}{2b}\right]$$

$$= -\frac{1}{a} \cdot 7.0385 \times 10^5$$

$$\tau_{x,A} = \frac{P}{a}\left[\frac{Ld}{2\left(\frac{1}{2}bd^2 + \frac{1}{6}b^3\right)}\right]$$

$$= \frac{1}{a} \cdot 1.1077 \times 10^6$$

$$\tau = \frac{1}{a}\sqrt{(7.0385 \times 10^5)^2 + (1.1077 \times 10^6)^2}$$

$$= \frac{1.3214}{a}\ MPa \leq 220$$

$$a = 0.006\ m$$

$$e \geq 8.4\ mm$$

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Test with forces

- The factored shear resistance of 8 mm weld is:

$$\tau'_R = 0.67^2 \cdot \frac{8}{\sqrt{2}} \cdot 490 = 1.24\ kN/mm$$

- The weld lengths are 50 mm, giving:

$$V_R = 62.2\ kN \text{ per weld}$$

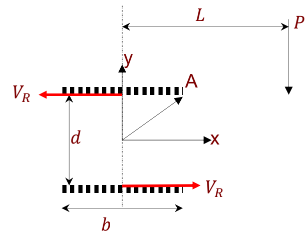
- The torque that can be resisted is:

$$T_R = 2\frac{d}{2}V_R = 2 \cdot 0.05 \times V_R$$
$$= 6.2\ kNm$$

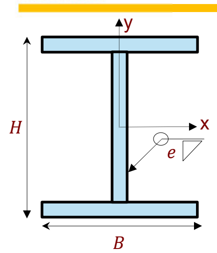
- The applied torque is:

$$T = PL = 0.4 \times 15 = 6\ kNm$$

- This is less than the resistance
- The shear force resistance is 112.4 kN, more than the shear force or 15 kN
- This is not ideal, because at Point A the weld is subject to the highest shear stress due to torque and shear force combined
  - However, above approximation using forces give quick check



Welded end-connection



Problem statement

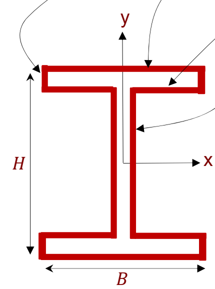
An I-beam with height, H, width B, flange thickness, t_f and web thickness, t_w is joined at the end by the all around weld of size e. A force, F is applied at the other end.

Give the equations from which the following can be calculated in the weld group:

1. Bending stress
2. Normal stress
3. Shear stress
4. Combined stress

Assume for all calculation that the weld throat is positioned at the outside lines of the cross-section. This is a conservative assumption and will reduce calculations substantially.

The weld throat area



Area:

$$A = 2a_w(H - 2t_f) + 2a_w(B - t_w) + 2a_w B + 4a_w t_f$$

Second moments of area:

$$I_{xx} = \frac{1}{12} a_w \left(2(H - 2t_f)^3 + 4t_f^3\right)$$
$$+\left(\frac{H - t_f}{2}\right)^2 4a_w t_f$$
$$+2a_w(B - t_w)\left(\frac{H}{2} - t_f\right)^2$$
$$+2a_w B\left(\frac{H}{2}\right)^2$$
$$I_{yy} = \frac{1}{12} a_w(4B^3 - 2t_w^3) + 2a_w(H - 2t_f)\left(\frac{t_w}{2}\right)^2$$
$$+4a_w t_f\left(\frac{B}{2}\right)^2$$
$$I_{zz} = I_{xx} + I_{yy}$$

Using these equations, the normal and shear stress can be calculated at any point in the weld group

For the torque, calculate x- and y-components to add or subtract as needed

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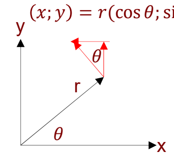
Stress at any point

General equations:

$$\sigma_i = \frac{M_x y_i}{I_{xx}} - \frac{M_y x_i}{I_{yy}} + \frac{F_z}{A}$$
$$\tau_y = k_y \frac{V_y}{A} + \frac{Tx}{I_{zz}}$$
$$\tau_x = k_x \frac{V_x}{A} - \frac{Ty}{I_{zz}}$$
$$\alpha = \operatorname{atan}\left|\frac{y}{x}\right|, x \neq 0$$
$$\tau_w = \sqrt{(\tau_x^2 + t_y^2) + \sigma^2}$$

$(x; y) = r(\cos\theta; \sin\theta)$



Where:

- F_z: Normal force [N]
- V_x, V_y: Shear force in the x- and y-directions [N]
- $T = M_z$: Moment around the z-axis = torque [Nm]
- x_i: x-coordinate of the point where stress is required
- y_i: y-coordinate of the point where stress is required
- I_{xx}: Second moment of area around the x-axis [m^4]
- I_{yy}: Second moment of area around the y-axis [m^4]
- I_{zz}: Second moment of area around the z-axis [m^4]

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- Do an example with numbers in class
  - Not with MSV 780

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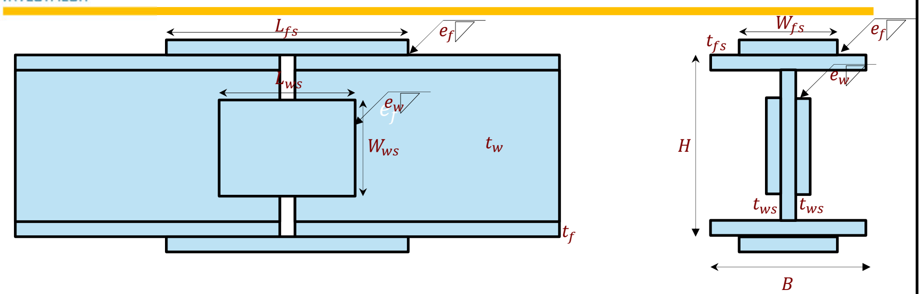
Welded splice



Problem statement:
An I-beam with height, H, width B, flange thickness, t_f and web thickness, t_w is joined by the welded splice as shown in the figure above. The dimension of the strengthening on the flange is L_{fs}, W_{fs} and t_{fs} respectively. The double plates on the web each has dimensions: is L_{ws}, W_{ws} and t_{ws}.
Type equation here. The gap is assumed to be g. Assume the weld size on the flange and web to be e_f and e_w respectively.
Calculate the weld size required on the flange and web, assume them of same size, to resist the shear force and bending moment over the cross section.

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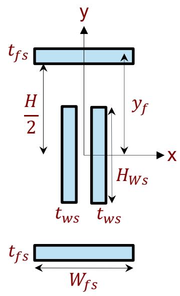
The inserted metals in the gap



The second moment of area around the x-axis:

$$I_{xx} = \frac{1}{12}(t_{ws}W_{ws}^3) \times 2 + 2\left[y_f^2(W_{fs}t_{fs}) + \frac{1}{12}(W_{fs}t_{fs}^3)\right]$$

$$y_f = \frac{H}{2} + \frac{t_{fs}}{2}$$

The area:

$$A = 2[W_{fs}t_{fs} + W_{ws}t_{ws}]$$

The bending stress distribution:

$$\sigma_b = \frac{M_x y}{I_{xx}} - \frac{M_y x}{I_{yy}}$$

The bending force in the top and bottom section for M_x:

$$dF_{top} = \frac{M_x y}{I_{xx}} \cdot W_{fs} \cdot dy$$

$$F_{top} = \frac{M_x}{I_{xx}} \int_{\frac{H}{2}}^{\frac{H}{2}+t_{fs}} y\, W_{fs} dy$$

$$= \frac{M_x}{I_{xx}} \cdot \frac{W_{fs}}{2}\left[\left(\frac{H+2t_{fs}}{2}\right)^2 - \left(\frac{H}{2}\right)^2\right]$$

$$= \frac{M_x}{I_{xx}} \cdot \frac{W_{fs}}{8}\left[(H^2+4Ht_{fs}+4t_{fs}^2) - H^2\right]$$

$$= \frac{M_x}{I_{xx}} \cdot \frac{W_{fs}}{2}\left[Ht_{fs} + t_{fs}^2\right]$$

This force must be resisted by the weld on the flanges

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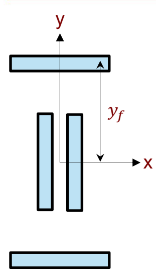
% bending moment resisted by the flange plates



If assumed that the forces on the flanges act at the centroid of the top and bottom plates, y_f, the percentage of the bending moment carried by the top and bottom plates is:

$$y_f = \frac{H}{2} + \frac{t_{fs}}{2}$$

The moment by the top and bottom flange plates:

$$M = F_{top}y_f + F_{bot}y_f$$

$$= 2 \cdot F_{top}y_f$$

The force in the top section (same as bottom) from previous slide:

$$M = 2y_f \frac{M_x}{I_{xx}} \cdot \frac{W_{fs}}{8}\left[4Ht_{fs} + 4t_{fs}^2\right]$$

$$= y_f \frac{M_x}{I_{xx}} \cdot \frac{W_{fs}}{4}\left[4Ht_{fs} + 4t_{fs}^2\right]$$

Ratio in moment by top and bottom flanges (M) vs. total moment (M_x):

$$\frac{M}{M_x} = \frac{y_f}{I_{xx}} \cdot \frac{W_{fs}}{4}\left[4Ht_{fs} + 4t_{fs}^2\right]$$

$$= \frac{y_f}{I_{xx}} \cdot W_{fs}\left[Ht_{fs} + t_{fs}^2\right]$$

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254 x 254 x 73 parallel flange H-section

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Designation	h	b	t_w	t_f	r_1	m	A	I_x	Z_x	r_x
mmxmmxkg/m	mm	mm	mm	mm	mm	kg/m	10^3 mm²	10^6 mm⁴	10^3mm³	mm
254x254x73	254.2	254	8.6	14.2	12.7	73.1	9.29	114	896	111

I_y	Z_y	r_y	J	C_w	Z_{plx}	Z_{ply}	h/t_f	h_w
10^6 mm⁴	10^3mm³	mm	10^3mm⁴	10^9 mm⁶	10^3mm³	10^3mm³		mm
38.8	306	64.6	578	559	990	463	17.9	200

Source: https://www.macsteel.co.za/files/saisc_structural_steel_section_properties.xls

From the tables above, it is clear the H-beam has the following dimensions:

$$H = 254.2\ mm$$
$$B = 254.2\ mm$$
$$t_f = 14.2\ mm$$
$$t_w = 8.6\ mm$$

Assume the splice is made with the following dimensions:

$$W_{fs} = 200\ mm$$
$$t_{fs} = 12\ mm$$
$$H_{ws} = 120\ mm$$
$$t_{ws} = 10\ mm$$

Calculate the length of plate required on the flanges to resist a bending moment $M_x = 179.2\ kNm$. That is, calculate the weld length required to resist the force in the outer flanges for a weld size of 5 mm.

Flange weld calculation

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Cross-section parameters								
H	0.2542	m	B	0.2542	m	I_{xx}	1E-04	m^4
t_f	0.0142	m	t_w	0.0086	m	A	0.009	m^2
Flange plate								
W_{fs}	0.2	m	t_{fs}	0.012	m	A_{fs}	0.002	m^2
Web plate								
W_{ws}	0.12	m	t_{ws}	0.01	m	A_{ws}	0.001	m^2
Cross-section parameters for the plates								
y_f	0.1331	m	I_{xx}	8.79725E-05	m^4			
g	0.005	m						
Bending moment:			M_x	1.79E+05	Nm			
Force in top plate:			F_z	738 698	N			
				739	kN			
% Moment by flange plates:				97%				
Welding on flanges								
x_u	4.90E+08	Pa	f_{weld}	1.73E+03	N/mm			
e_{fs}	0.005	m	L_{weld}	4.26E+02	mm			
a_{fs}	0.00354	m	$L_{fs}/2$	213	mm			
			L_{fs}	426	mm	Gap added		

The plates on the flanges resist 97 % of the bending moment

To be continued in 2018

Shear flow and shear stress in membraned sections

- Not in your scope

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References

- Cross section properties

https://www.macsteel.co.za/files/saisc_structural_steel_section_properties.xls

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