INVESTMECH (Pty) Ltd

Reg. No. 1998/07367/07 ● VAT Reg.No. 4490189489
PostNet Suite #251, Private Bag X1015, Lyttelton, 0140 ● www.investmech.com
Tel +27 12 664-7604 ● Fax +27 86535-1379● Cell: +27 82 445-0510
E-mail address: mheyns@investmech.com

Introduction to Fracture Mechanics

IIW Section 3.11

Prepared for

Universities

DIRECTORS:
M Heyns Pr.Eng., Ph.D., (Managing)
CJ Botha B.Eng(Hons): Industrial

Document No: IM-TR000
Revision: 0.0
Date: 13 August 2018

Document information

Project Name:	IIW Section 3.11
Title:	Introduction to Fracture Mechanics
Author:	Author
Project Engineer:	Project Engineer
Document Number:	IM-TR000
Filename:	Investmech (Introduction to Fracture Mechanics) TR R0.0
Creation Date:	22 August 2013
Revision:	0.0
Revision Date:	13 August 2018

Approval

Responsibility	Name	Designation	Signature	Date
Checked by:				
Approved by:				

Distribution list

Name	Company / Division	Copies
Weld fatigue students	Universities	1
DCC (Tenders & Quotations)	Investmech (Pty) Ltd	Original

Summary

This document presents the study notes for an introduction to Fracture Mechanics.

Table of Contents

List of Tables

List of Figures

1. INTRODUCTION

This document presents the class notes for an introduction to Fracture Mechanics according to IAB Module 3.11.

Figure 1: Example of fracture in the stick of an excavator

2. STUDY MATERIAL

The student shall arrange access to the following documents:

BS EN 1993-1-9:2005. Eurocode 3: Design of steel structures. Part 1-9: Fatigue. *British Standards Institution.*

Hobbacher, A. 2007. *Recommendations for the fatigue design of welded joints and components.* IIW Bulletin 520.

3. INTRODUCTION TO FRACTURE MECHANICS

3.1. Objectives

The objective of this section is to understand in detail the use of Fracture Mechanics for welded structures.

3.2. Scope – Teaching hours = 4

The scope of theory covered is:

1. Viewpoint of Fracture Mechanics
2. Application of Fracture Mechanics
3. Linear Elastic Fracture Mechanics (LEFM)
4. Fundamentals of Elastic-Plastic Fracture Mechanics (EPFM)
5. Critical flaw size, K_{IC} plane strain fracture toughness
6. Fracture Mechanics testing (CTOD, etc.)
7. Different assessment method
8. Sub-critical crack growth
9. Fatigue testing
10. Standards global (ISO), regional (CEN) and National

3.3. Outcome

After completion of this section you will be able to:

1. Explain fully the principles of linear elastic and elastic-plastic Fracture Mechanics.
2. Detail the influence factors for linear-elastic and elastic-plastic Fracture Mechanics.
3. Explain fully the use of Fracture Mechanics for dynamically loaded structures.
4. Detail Fracture Mechanics testing and assessment methods.

4. LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

The purpose of this section is to introduce the principles of linear elastic Fracture Mechanics.

Presentation used in class:	Investmech – Fracture Mechanics (Linear Elastic Fracture Mechanics) R0.0

4.1. Crack growth and fracture

Crack growth is usually slow and can be caused by:

- Fatigue due to cyclic loading
- Stress corrosion due to sustained loading
- Creep
- Hydrogen induced cracking
- Liquid metal induced cracking
- Etc.

The fracture surface is due to repeated blunting and sharpening of the crack tip that results in beach marks.

Fracture is rapid, uncontrolled crack propagation and result in catastrophic failure in most cases.

Typical surface marks are shown in Figure 2.

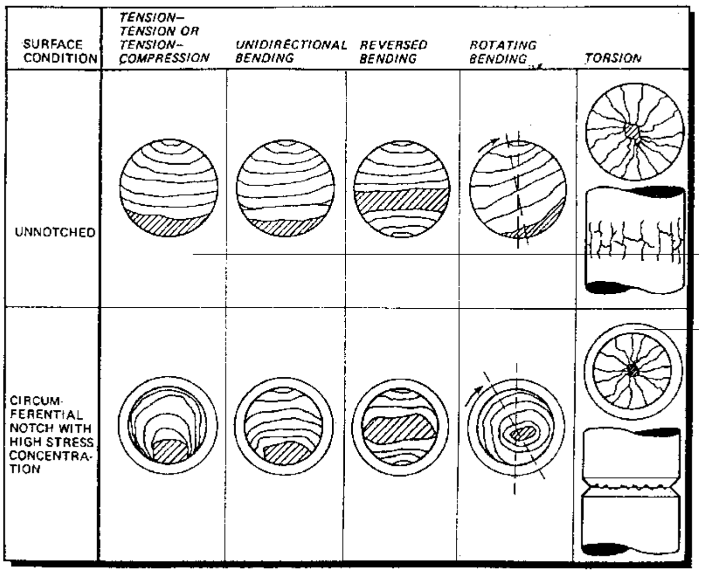


Figure 2: Typical fracture surface marks

4.2. Failure of a component under tensile load

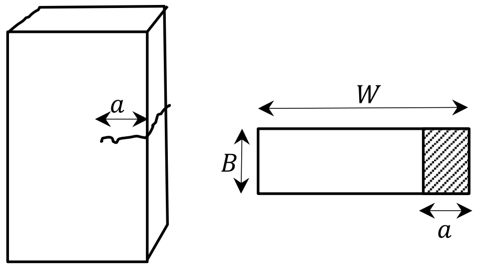
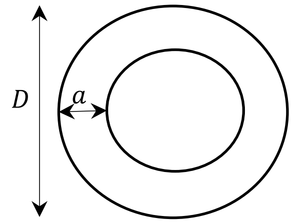
4.2.1. Plastic collapse

This is where the stress in the cross section becomes equal to the yield strength. In a cracked specimen, plastic collapse of the remaining area in the cracked plane can occur.

$$F_{pc} = \frac{f_y}{A_{nett}} \qquad (1)$$

The nett area for two typical scenarios are given in Table 1. The crack size that result in plastic collapse at any force is that crack size that satisfies Equation **Error! Not a valid link.Error! Not a valid link.**.

Table 1: Plastic collapse equations for a rectangular and round cross section

Rectangular cross section	Round cross section
 	
$\frac{F_{pc}}{B(W-a)} = f_y$ $F_{pc} = f_y B(W-a)$	$\frac{F_{pc}}{\frac{\pi}{4}(D-2a)^2} = f_y$ $F_{pc} = f_y \frac{\pi}{4}(D-2a)^2$

4.2.2. Plastic collapse at a notch

- Entire cross section is yielding
- Plane stress collapse or limit force:

$$F_{pc} = B(W - 2_a)f_{pc}$$

- Collapse strength
  - $f_{ty} < f_{pc} < f_{tu}$
  - Determine collapse strength by test on notched sample
  - If $f_{pc} > f_{ty}; F_{pc} = B\ (W - 2_a)f_{ty}$
- Plane stress equation also used for plane strain because the average stress in the section cannot become much higher than in the case of plane stress
- Brittle behaviour: In this case for the Collapse Load:
  - If little overall plastic deformation occurs, the fracture is brittle
  - Plastic deformation is always restricted to the notched section
  - Fracture mechanism = cleavage or rupture

4.2.3. Measurement of collapse strength

- Center cracked panel for explanation
- Measure maximum load, F_{pc} at fracture (f_{pc} = plastic collapse strength [Pa])

$$F_{pc} = B(W - 2a)f_c\sigma_{fc}$$
$$= \frac{W - 2a}{W} f_{pc}$$

4.2.4. Fracture

In this case the stress intensity factor due to the stress on the cracked component equals the fracture toughness of the material. The plane strain fracture toughness is the lowest of the different types, and will be used in the remainder of this course as limiting factor. According to the universal fracture mechanics equation, the stress intensity factor is:

$$K = \beta\sigma\sqrt{\pi a} \quad (2)$$

Fracture occurs when:

$$K = K_{Ic}$$
$$K_{Ic} = \beta\sigma\sqrt{\pi a_f} \quad (3)$$

$$a_f = \frac{1}{\pi}\left(\frac{K_{Ic}}{\beta\sigma}\right)^2$$
$$\sigma_f = \frac{K_{Ic}}{\beta\sqrt{\pi a_{fr}}}$$

Where:

a	is the crack size [m]
a_f	is the crack size where fracture occurs [m]
K	is the stress intensity factor [$MPa\sqrt{m}$]
K_{Ic}	is the plane strain fracture toughness [$MPa\sqrt{m}$]
β	is the geometric stress concentration of the crack
σ	is the nominal stress [MPa]
σ_f	is the fracture stress [MPa]

4.2.5. *Critical crack size*

The critical crack size is the smallest of the crack size that results in plastic collapse and the crack size that result in fracture.

$$a_{cri} = \min\begin{cases} a_{pc} & for\ plastic\ collapse \\ a_f & for\ fracture \end{cases} \qquad (4)$$

4.2.6. *Example*

An example was done in class. Download the class notes from the website to get the detail.

4.3. Damage tolerance and residual strength

Cracks reduce strength, as shown in the equations above. Damage tolerance is the evaluation of the residual strength diagram and the crack growth curve with the purpose of designing structures that can safely tolerate cracks.

In practise we want to prevent unscheduled fracture of components and structures. However, in some cases we are challenged by components already cracked. In this case, the diminishing strength of the component need to be considered.

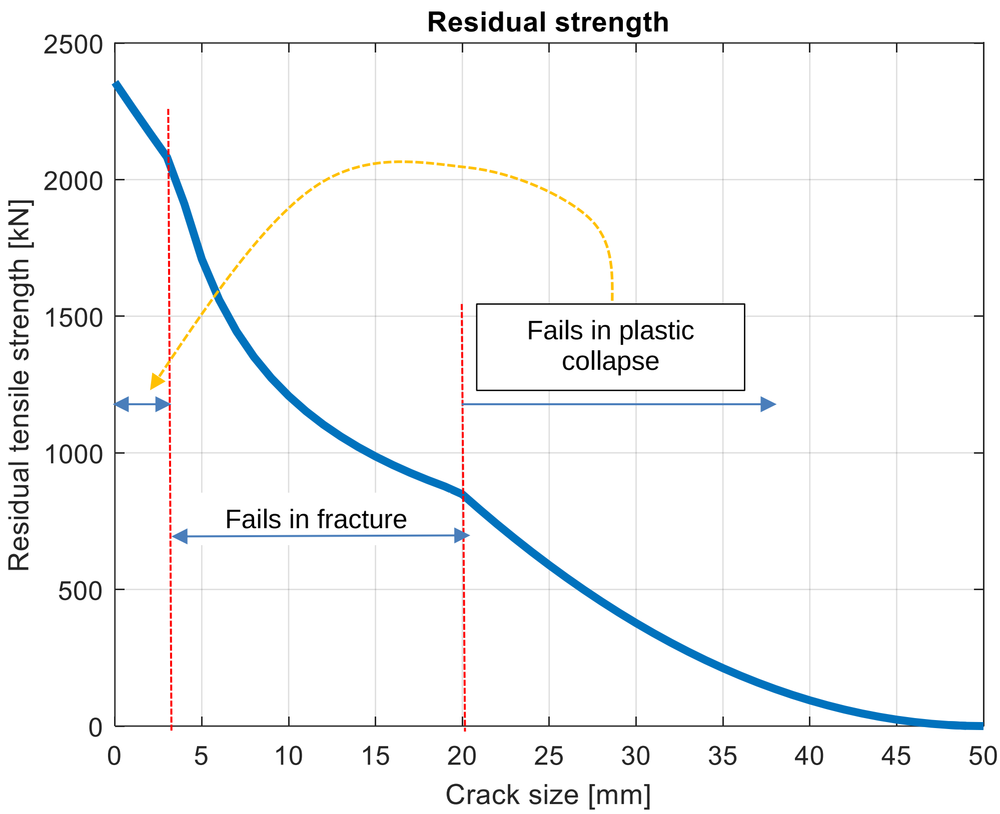
In the initial design, an attempt is made to predict overloads as accurate as possible, and, with the use of factors of safety, overload is not expected to occur. Structural flaws cause stress concentrations and initiation points for cracks. Once initiated, cracks propagate resulting in diminishing cross section to resist tensile stress causing higher tensile stress on the remaining section, increased crack growth rate and diminishing residual strength of the component.

To explain, take a round cross section with concentric cracking from the outside with crack size, a. The following applies:

- Crack size: 0 to 30 mm.
- Outer diameter, $d_o = 100\ mm$
- Yield strength, $f_y = 300\ MPa$
- Plane strain fracture toughness, $K_{Ic} = 30\ MPa\sqrt{m}$
- Geometrical stress concentration factor, $\beta = 1.1$, and was assumed for this example to stay constant

The equations were programmed in Matlab function residualstrength.m that give the residual strength vs crack size shown in Figure 3, from which the following may be concluded:

1. Cracks reduce strength, as we have seen before.
2. For crack size below 4 mm, the failure mode is plastic collapse. This is expected in the initial parts of the curve, because, if there is no crack the stress intensity factor is 0 $MPa.m^{1/2}$.
3. For crack size between 4 mm and 20 mm, the mode of failure is fracture (stress intensity exceeds the plane strain fracture toughness).
4. For crack size more than 20 mm the mode of failure is plastic collapse.



Source: Investmech Matlab function: residualstrength.m

Figure 3: Residual strength

4.4. Fracture control

A simulation was done to indicate crack propagation in a steel component ($C = 1.65 \times 10^{-11}, m = 3, K_{Ic} = 50\ MPa\sqrt{m}$) subject to a completely reversed nominal stress with amplitude 100 MPa as shown in Figure 4. The geometric stress concentration factor, β, was assumed a constant 1.1 for this simulation. As expected, the crack propagates initially very slowly and accelerates with increasing crack size. The critical crack size is that crack size that first result in:

1. Plastic collapse of the remaining area.
2. Fracture.

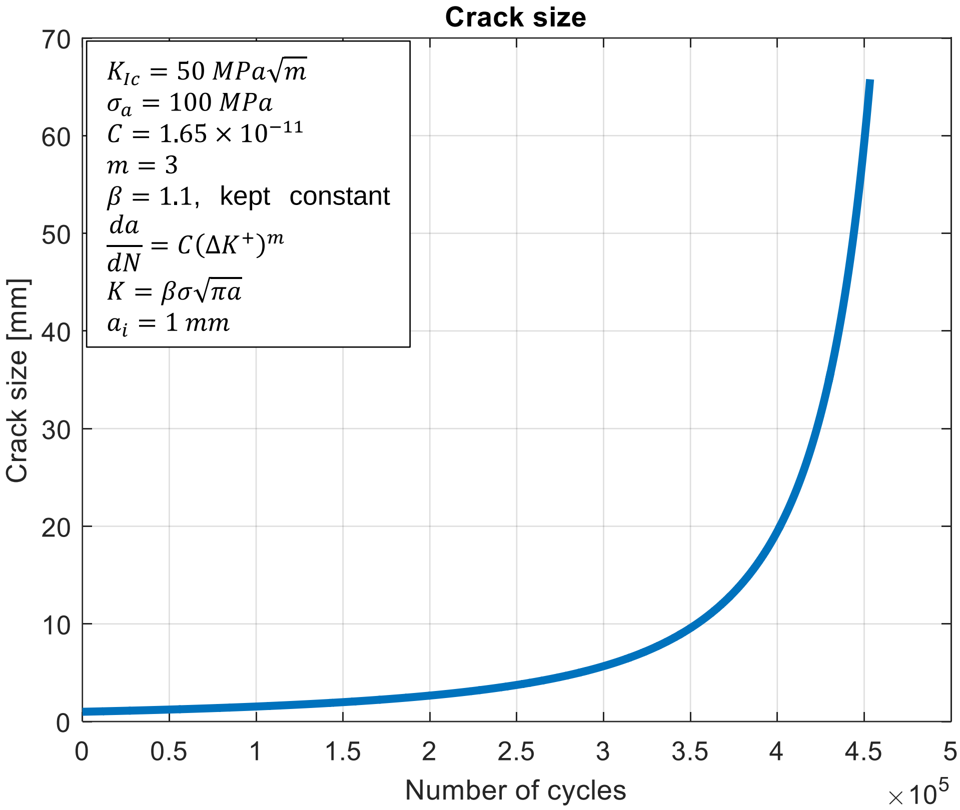
4.4.1. Fracture control programme

This is where the crack size vs. time curve is used to determine the ideal inspection interval where the crack size is monitored. Depending on the criticality of the equipment, the inspection interval, I, is typical one fourth to one tenth of the remaining life, H, calculated from the point of the first detection of the crack.

$$I = \frac{H}{4\ to\ 10\ and\ even\ higher} \quad (5)$$

When the monitored crack size is below, but still close to the predicted values, all goes well. If the monitored crack size is above or far below the curve, a repetition of the analysis must be done, because one or more of the following could not have been accurate:

- Crack growth parameters
- Loads
- Etc.



Source: Investmech Matlab function crackgrowthcurve.m

Figure 4: Typical crack propagation in steel under completely reversed nominal stress with amplitude 100 MPa (no stress intensity threshold was modelled)

4.5. Fracture

Fracture, which is in most instances the final event and rapid, can occur due to the following mechanisms:

- Cleavage: Splitting apart of the atomic planes, surface reflects incident light, and, takes place at 1 600 m/s
- Rupture: Results in widely spaced holes, surface looks dull grey, and, this takes place at 500 m/s
- Intergranular fracture (which can be represented by cleavage and rupture) occurs along the grain boundaries

Fracture is dependent in the mode loading, for which there are typically three as shown in Figure 5. Final failure of the component can be due to plastic collapse or fracture, and both must be considered. It is wrong to speak of fatigue fracture, or, stress corrosion fracture, etc

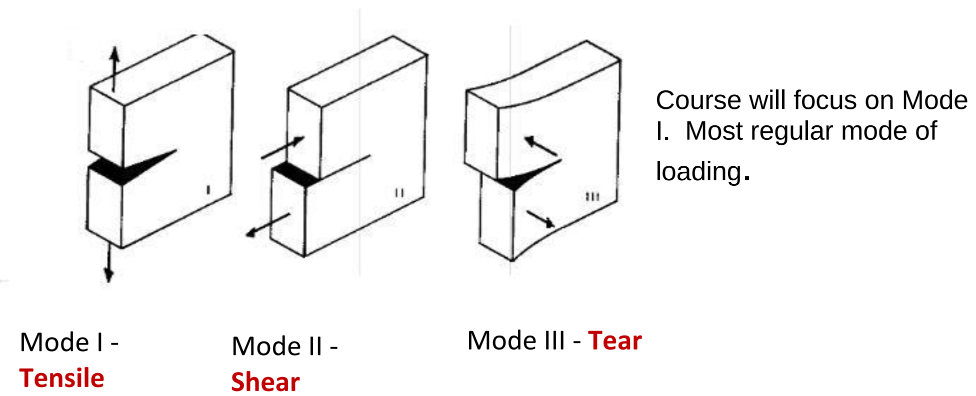


Figure 5: Three modes of loading

4.6. Need for analysis

Cracks result in:

- Collapses and accidents
- Explosions in pressure vessels, etc.

Maintenance can be scheduled from the crack size dependent remaining life.

The emphasis in this course in how we carry out damage tolerance and ensure that structures can safely tolerated cracks.

Structural inputs are essential to ensure accurate modelling. Remember GIGO – Garbage In = Garbage Out. Fracture mechanics and damage tolerance analysis are not ideal, but, provide answers that can be used in design and maintenance. Adjusting your crack growth function with crack monitoring can significantly improve the accuracy of analysis. The probabilistic nature of crack initiation and growth must also be considered.

4.7. Symbols and unit conversions

The following applies:

- Fracture stress = residual strength
- 1 kips = 4.5 kN
- 1 ksi = 6.89 MPa
- f_{tu} is the tensile strength [Pa]
- f_{ty} is the yield strength [Pa]
- f_{pc} is the plastic collapse strength [Pa]
- $1\ ksi\sqrt{inch} = 1.09\ MPa\sqrt{m}$

4.8. Effect of cracks and notches

- Failure criteria from strength of materials
  - Maximum normal stress
  - Maximum normal strain
  - Total strain energy
  - Von Mises
  - Tresca
  - Mohr circles
  - Buckling
- Factored Resistance is used in standards
- LEFM: parameter representing crack tip stress field
- EPFM: strain energy release rate and crack tip opening displacement (CTOD)
- Important – Plastic collapse and fracture are competing conditions-one satisfied first will prevail

4.9. Stress concentration at the crack tip

The presence of a notch interrupts the load paths at the crack tip. The transverse stress is biaxial of nature due to the three-dimensional shape. The crack tip is subject to shear loading. The stress in the stress concentration area (with theoretical stress concentration factor, k_t) is $\sigma_1 = k_t \sigma_{nom}$.

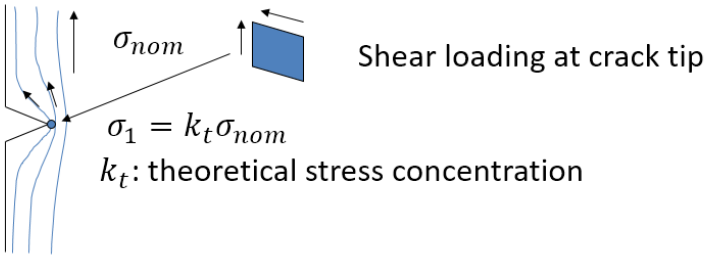


Figure 6: Interrupted load paths at a crack tip

The state of stress at a stress concentration are dependent on which of the following two conditions apply:

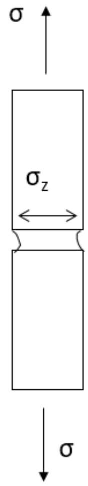
- Plane stress: This is where $\sigma_z = 0$. The stress-strain relationship is then:

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) \quad (6)$$

- Plane strain: This is the case where the strain the z-direction $\varepsilon_z = 0$:

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = 0$$
$$\sigma_z = \nu(\sigma_x + \sigma_y) \quad (7)$$

In this case a stress is produced in the z-direction.



Even at low nominal stresses, yielding can occur at the notch. Plastic deformation occurs in the stress concentration area that take place by slip and shear stresses.

In ideal tri-axial stress state ($\sigma_1 = \sigma_2 = \sigma_3$), there is no shear stress and no resulting slip and plastic deformation.

In sharp notches the highest principle stress is the yield stress under plane stress conditions. However, under plane strain conditions the stress can increase to $3f_{ty}$.

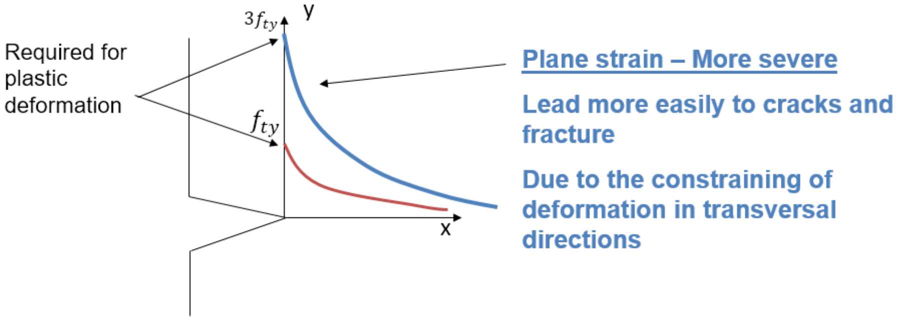


Figure 7: Stress at the tip of a stress concentration

During yielding at the notch, the stress distribution is slightly different. Because, at the surface plane stress conditions apply.

4.9.1. Stress distribution at the crack tip

Linear elastic fracture mechanics (LEFM) makes it possible to obtain the residual strength diagram and maximum permissible crack size. Materials with low fracture resistance fail below their plastic collapse strength and include material like:

- High strength and hard (brittle) materials
- High strength low alloy steels
- Cold worked stainless steels, etc.

Linear elastic fracture mechanics can be successfully applied to model fracture of these materials. To simplify the modelling, the nominal stress away from the stress crack, σ_{nom}, is used with a geometric stress concentration factor, β, in the following equation:

$$K = \sigma_{nom}\beta\sqrt{\pi a} \qquad (8)$$

The equation above is called the universal equation for the stress intensity factor. Fracture occurs when the stress intensity factor, K, becomes equal to the plane strain fracture toughness, K_{Ic}, for the applicable mode. The crack size at fracture can then be calculated as below.

$$\begin{aligned} K &= K_{Ic} \\ K_{Ic} &= \beta\sigma_{nom}\sqrt{\pi a_f} \\ a_f &= \frac{1}{\pi}\left(\frac{K_{Ic}}{\beta\sigma_{nom}}\right)^2 \end{aligned} \qquad (9)$$

The equations below describe the theoretical stress concentration factor at the crack tip of a linear elastic material (no yielding).

$$\begin{aligned} \sigma_x &= \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\cdot\sin\frac{3\theta}{2}\right] \\ \sigma_y &= \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\cdot\sin\frac{3\theta}{2}\right] \\ \sigma_{xy} &= \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left[1 - \cos\frac{\theta}{2}\cdot\cos\frac{3\theta}{2}\right] \end{aligned} \qquad (10)$$

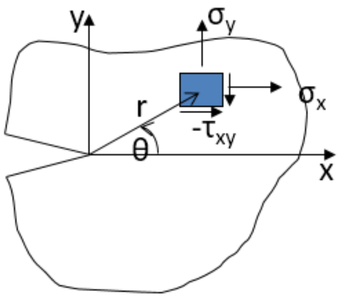


Figure 8: Parameters for theoretical stress concentration at the crack tip

These equations were programmed in Matlab function cracktipstress.m where the stress concentration factor distribution for $\theta = 0$ is as shown in Figure 9. The stress distribution shows that there is stress at yield for even low applied nominal stress.

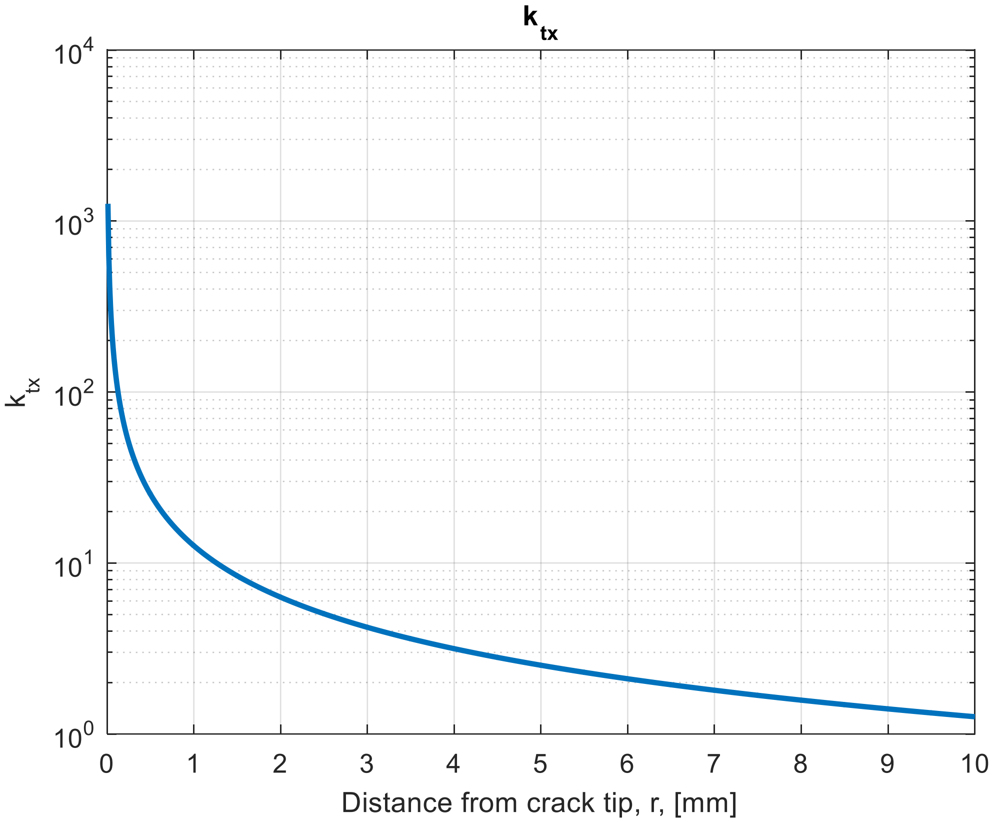
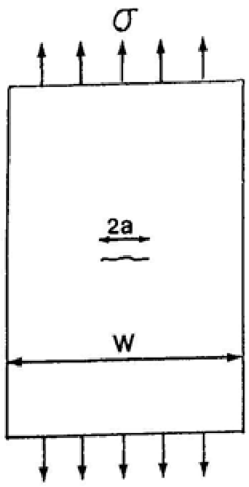


Figure 9: Theoretical stress concentration factor in an infinitely sharp crack tip, $\theta = 0°$

4.10. Geometric stress concentration factors

There are Geometric stress concentration factors for almost any linear elastic fracture mechanics problem. See Figures 10 to 13 and update with information shared in class.

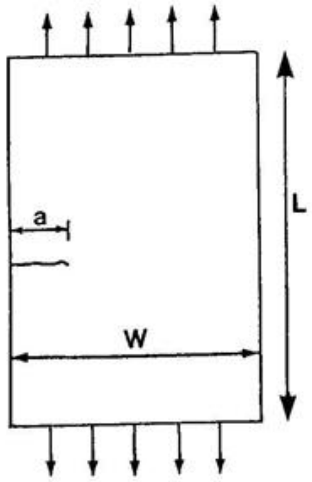


$$\beta = \sqrt{\sec \frac{\pi a}{W}}$$

or

$$\beta = 1 + 0.256 \frac{a}{W} - 1.152 \left(\frac{a}{W}\right)^2 + 12.200 \left(\frac{a}{W}\right)^3$$

Figure 10: Geometric stress concentration factor for a centre cracked plate



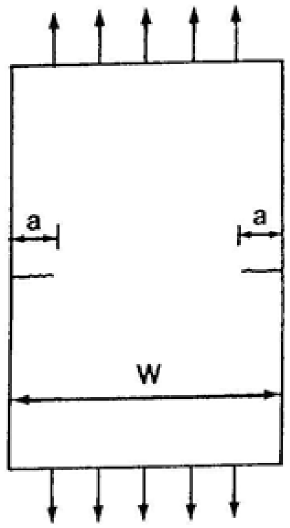
L/W = 2 ONLY:

$$\beta = 1.12 - 0.23\frac{a}{W} + 10.56\left(\frac{a}{W}\right)^2 - 21.74\left(\frac{a}{W}\right)^3 + 30.42\left(\frac{a}{W}\right)^4$$

(SEE FIGURE 8.12)

D

Figure 11: Geometric stress concentration factor for a single edge cracked specimen



$$\beta = 1.12 + 0.43\frac{a}{W} - 4.79\left(\frac{a}{W}\right)^2 + 15.46\left(\frac{a}{W}\right)^3$$

Figure 12: Geometric stress concentration factor for double edge cracked specimen

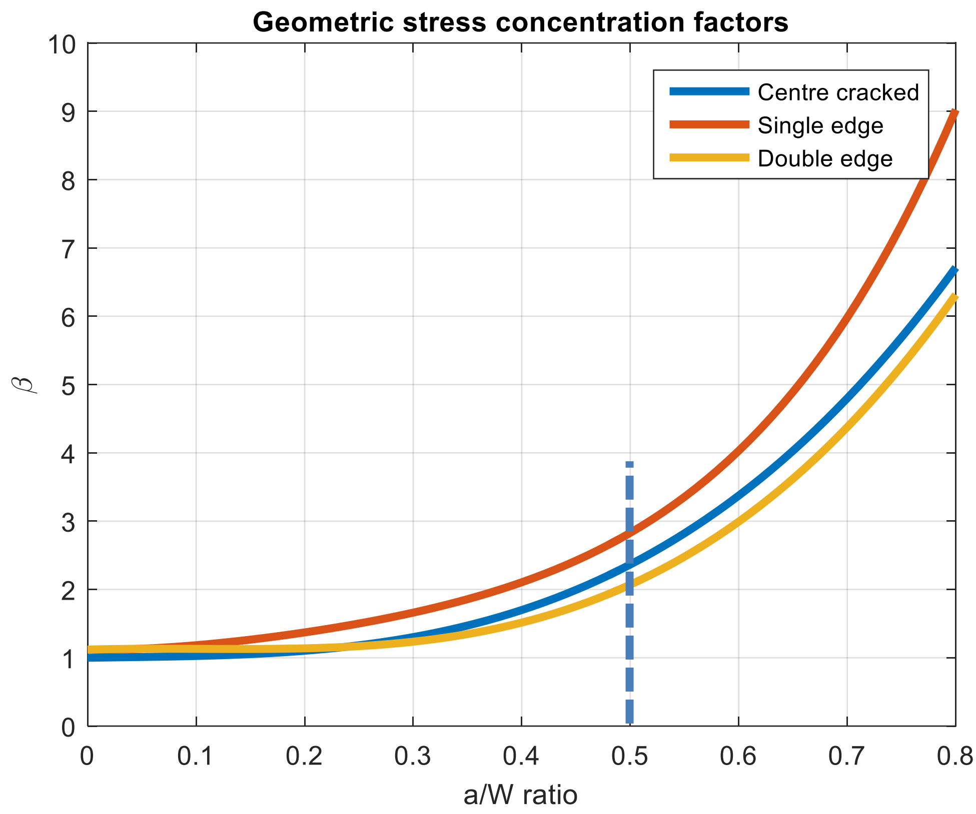


Figure 13: Geometric stress concentration factor vs a/W ratio for centre cracked, single edge cracked and double edge cracked specimens

Now, we know how to model the stress at the stress concentration area of a crack. Let's now look at toughness.

4.11. Fracture toughness

This is the highest stress intensity that the material of a cracked component can resist. It is a material property. Figure 14 shows typical Mode I plane strain fracture toughness for a range of materials. There are several sources on the Internet, e.g. www.engineeringtoolbox.com .

Later in the course you will be introduced to equations that relate impact test results with the plane strain fracture toughness.

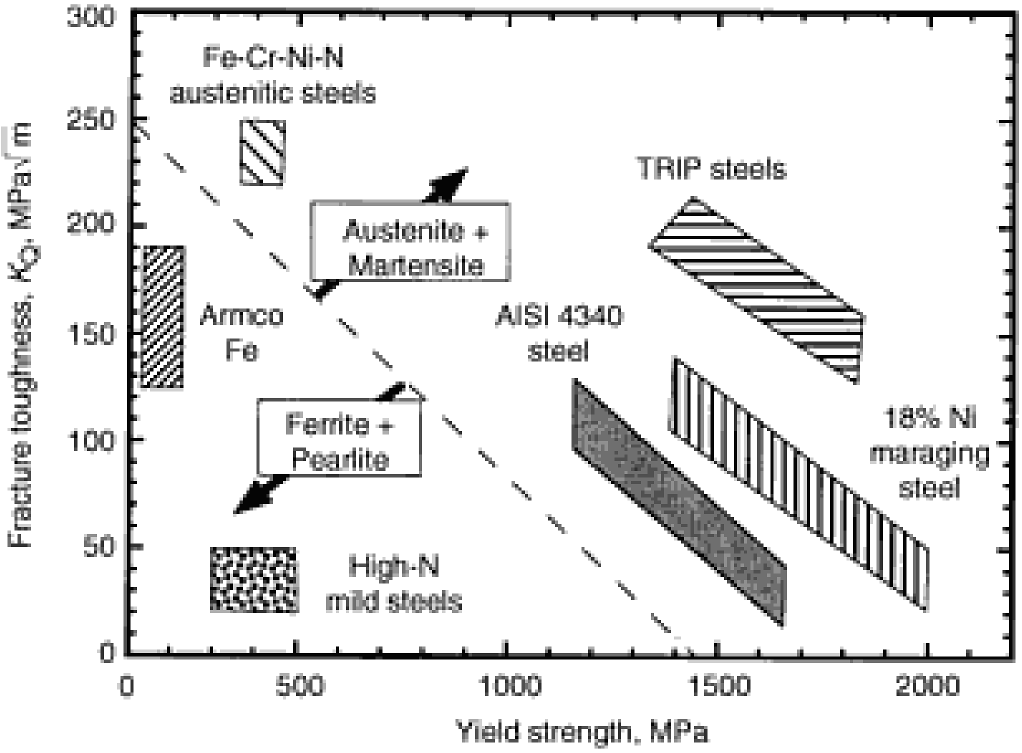
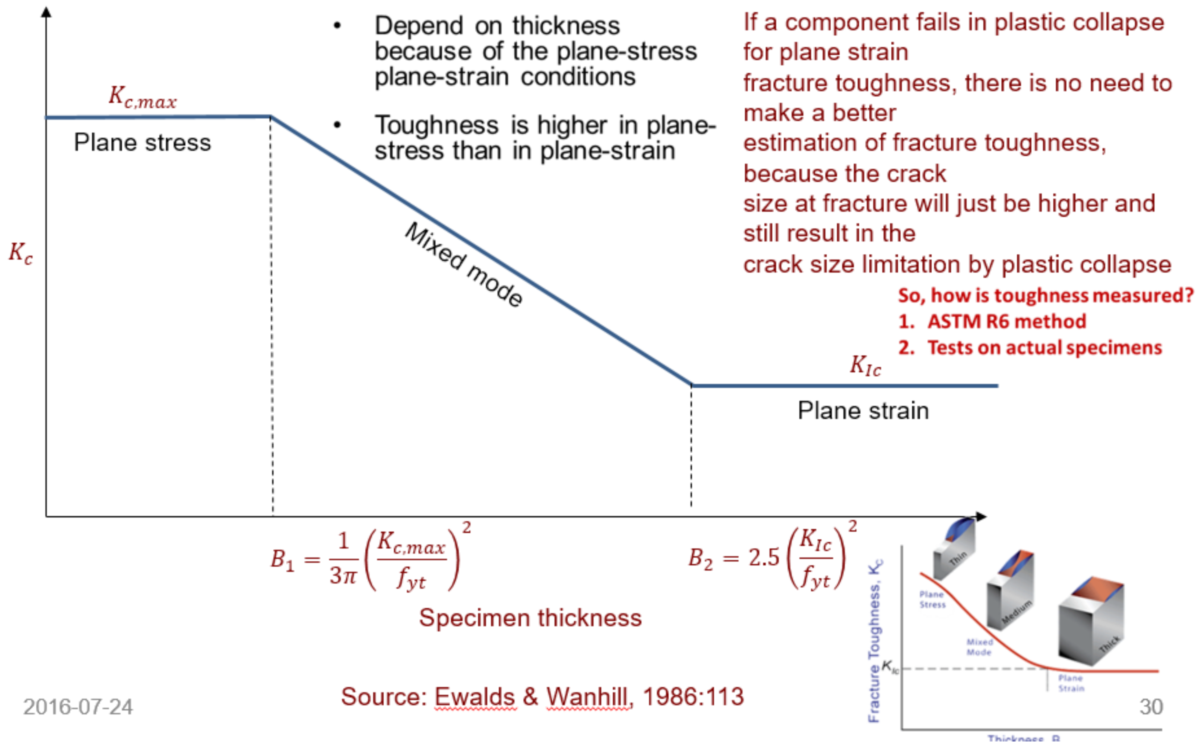


Figure 14: Typical Mode I plane strain fracture toughness for materials

4.11.1. Fracture toughness and specimen thickness



4.11.2. Size application range of LEFM

The plane strain fracture toughness is less than the plane stress fracture toughness. The application region for the plane strain fracture toughness based fracture mechanics is

$$B, W - a, a \geq 2.5\left(\frac{K_{Ic}}{f_{yt}}\right)^2 \qquad (11)$$
$$W \geq 5.0\left(\frac{K_{Ic}}{f_{yt}}\right)^2$$

4.12. Conclusion

The application of linear elastic fracture mechanics (LEFM) is done to comply with the following:

- Size application range

$$B, W - a, a \geq 2.5\left(\frac{K_{Ic}}{f_{yt}}\right)^2$$
$$W \geq 5.0\left(\frac{K_{Ic}}{f_{yt}}\right)^2$$

- Universal equation for the stress intensity factor

$$K = \beta\sigma_{nom}\sqrt{\pi a}$$

- Fracture occurs when the stress intensity factor becomes equal to the plane strain fracture toughness of the same mode of loading

$$K = K_{Ic}$$

  - From this equation, the crack size at fracture, or the fracture stress at any crack size for a specific load, or the fracture load at any crack size can be calculated

- The critical crack size is the smallest crack size between fracture and plastic collapse

$$a_{cri} = \min\begin{cases} a_{pc} & for\ plastic\ collapse \\ a_f & for\ fracture \end{cases}$$

4.13. References

Ravichandran, k.s. & Vasudevan, a.k. 1996. Fracture Resistance of Structural Alloys, *Fatigue and Fracture,* Vol 19, *ASM Handbook,* 1996, p 381-392.

Hobbacher, A. 2007. *Recommendations for fatigue design of welded joints and components*. IIW Bulletin 520.

Ewalds, H.L. & Wanhill, R.J.H. 1986. *Fracture Mechanics*. Delftse Uitgevers Maatschappij, Delft.

5. ESTIMATION OF FRACTURE TOUGHNESS

The purpose of this section is to provide techniques to estimate the fracture toughness of materials.

Presentation used in class:	Investmech – Fracture Mechanics (Estimation of fracture toughness) R0.0

5.1. Background info

- Not all cases are covered in available material data like:
  - www.efunda.com
- Situation is worse for most widely used materials: common structural steels
- On other hand – One often have Charpy data
  - Charpy-value essentially a fracture energy
  - Can be expected to correlate with K_{IC}
  - Charpy test:
    - Measures total fracture energy of specimen
    - = Integral of $R(a)$ over ligament
    - If R-curve would be horizontal, value of integral divide by ligament would be R
    - Then $K = \sqrt{ER}$
- For the low toughness at the lower shelf the R-curve will be nearly horizontal
- Correlation between K_{Ic} and Charpy energy is indeed likely
- Differences in Charpy & Fracture test:
  - Difference in loading rate
    - Affecting the yield strength
  - Difference in notchacuity
    - Affecting the state of stress at the notch root and as such the stress at yield

5.2. Charpy test data

- For the low toughness at the lower shelf the R-curve will be nearly horizontal
- Correlation between K_{Ic} and Charpy energy is indeed likely
- Differences in Charpy & Fracture test:
  - Difference in loading rate
    - Affecting the yield strength
  - Difference in notchacuity
    - Affecting the state of stress at the notch root and as such the stress at yield
- An empirical correlation for plain strain fracture toughness (Broek, 1989:217):
  - for C_v in Joules and K_{IC} in MPa.m$^{0.5}$

$$K_{IC} = 11.4\sqrt{C_v}$$

  - for C_v ft.lbs and K_{Ic} in ksi.in^{½}

$$K_{IC} = 12\sqrt{C_v}$$

- A conservative lower bound is claimed to be (Broek, 1989:217):
  - for C_v in Joules and K_{IC} in MPa.m$^{0.5}$

$$K_{IC} = 21.6(C_v)^{0.17}$$

  - for C_v ft.lbs and K_{Ic} in ksi.in^{½}

$$K_{IC} = 22.5(C_v)^{0.17}$$

5.3. Loading rates & Temperature shifts

Loading rates

- Toughness values on previous slide would be for the same high loading rates (impact) as prevalent in the Charpy test
- Toughness for slower loading rates may be obtained from same equations

Temperature shifts

- Account for temperature shift in the test as follows:
  - For $250\ MPa < f_y < 965\ MPa$

$$\Delta T = 119 - 0.12 f_y\ °C$$

  - For $f_y > 965\ MPa$

$$\Delta T = 0\ °C$$

  - As shown, materials with low yield strength can tolerate a larger temperature shift than on with high yield strength, as is expected (brittleness and ductility)
- Slower loading will cause transition temperature to be lower (lower f_y)
  - Estimate of toughness will be useful – if loading rate is low
  - Even at low temperatures of T-ΔT (°C)
- There are several other empirical equations in literature to model temperature and loading rate effects on plane strain fracture toughness
- Comparison: A fizzer sweet will fail brittle at low T and ductile at high T

5.3.1. Example

5.3.1.1. Problem statement

- A material is available with:
  - A Charpy value of 20ft.lbs
  - Yield strength of 60ksi
  - Temperature of 65F
- Estimate the toughness for high loading rates and comment

5.3.1.2. Solution

- Convert to SI units as follows:
  - 1 psi (or PSI of lb/in^2) = 6 894.76 Pa
    - 1 ksi = 6.895 MPa
  - 1 ftlbs (or ft-lbf) = 1.35582 J
  - 65 Fahrenheit is 18.3°C
- Step 1: Calculate plain strain fracture toughness:

$$\begin{aligned} K_{IC} &= 11.4\sqrt{C_v} \\ &= 11.4\sqrt{1.35582 \times 20} \\ &= 59.4\ MPa\sqrt{m} \end{aligned}$$

- Step 2: Calculate the conservative lower bound:

$$\begin{aligned} K &= 21.6 C_v^{0.17} \\ &= 21.6(1.35582 \times 20)^{0.17} \\ &= 37.9\ MPa\sqrt{m} \end{aligned}$$

- Comments:
  - The K_{Ic} value of 59.4 MPa.m^{½} would be safe value to use:
    - Because toughness would be higher for slower loading
    - Applicable even at low temperature of:
      - 18.3 – (119-0.12x6.895x60) = -47.2°C

5.4. Fracture mechanism

- The Charpy impact energy is affected by changes in the fracture mechanism
- Metals usually fracture by *microvoid coalescence*
  - In which the plastic strain causes void nucleation around inclusions
  - These grow and link up until failure occurs

- In body centered cubical (bcc) metals, failure can also occur by *cleavage*
- As the yield strength of the metal is increased, the tensile stress in the plastic zone can become sufficiently high for cleavage to occur
- The fracture mechanism in a ferritic carbon steel therefore ***changes*** from microvoid coalescence to cleavage as the yield strength increases
  - This can be caused by an **increase in strain rate** or a **decrease in temperature**
- The work of fracture of cleavage is much less than the work of fracture of microvoid coalescence since it involves much less plastic deformation
- The change in fracture mechanism therefore causes a sharp *ductile to brittle transformation* in the Charpy impact energy

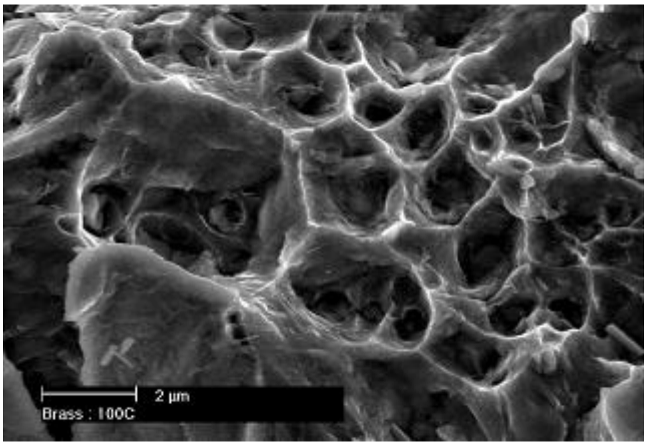


Figure 15: Microvoid coalescence

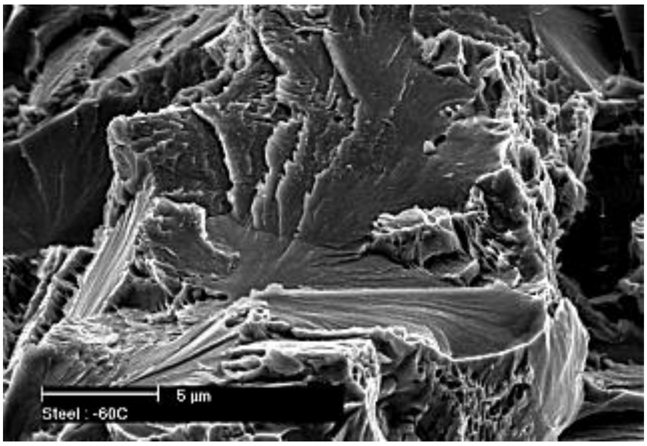


Figure 16: Cleavage

5.5. Ferguson & Sargisson for Comsteel EN25

For 0.976 correlation coefficient:

$$K_{IC} = -0.53S_{ut} + 188.12$$

K_{IC} is the plane strain fracture toughness [$ksi\sqrt{inch}$]

S_{ut} ultimate tensile strength [ksi]

5.6. Harhn & Rosenfield and Sudhakar & Murty

Hahn & Rosenfield

$$K_{IC} = \left(\frac{2ES_y\varepsilon_f}{3}\right)^{\frac{1}{n}}$$

Sudhakar & Murty

$$\frac{K_{IC}^2}{E} = 0.22C_v^{1.5}$$

Where:

K_{IC} plane strain fracture toughness $[MPa\sqrt{m}]$
E modulus of elasticity in $[MPa]$
C_v Charpy energy in $[MPa.m]$
S_y yield strength $[MPa]$
n strain hardening exponent
ε_f fracture strain in uniaxial tension

5.7. Toughness at upper shelf

- Upper shelf Charpy energy is not directly related to toughness
- If R-curve rises steeply – which it does at the upper shelf – integral of $R(a)$ over ligament is not universally relatable to R
- Much of Charpy energy on upper shelf used for general specimen deformation rather than fracture:
  - In extreme case, Charpy specimens do not fracture at all, are simply folded double – only deformation energy is measured!
- Nevertheless, empirical correlations may be the only way to arrive at estimate

5.8. Internet sources

- http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19680020500.pdf
- Handbook of Damage Tolerant Design: http://www.afgrow.net/applications/dtdhandbook/sections/page7_2_1_1.aspx

6. ELASTIC-PLASTIC FRACTURE MECHANICS (EPFM)

This section introduces the principles of elastic-plastic Fracture Mechanics.

Presentation used in class:	Investmech – Fracture Mechanics (Elastic Plastic Fracture Mechanics) R0.0

Include the CTOD technique.

6.1. Background

- Cases where there is considerable plastic deformation
- Use the J-integral = strain energy release rate
- Use the non-linear stress-strain equation to fit non-linear stress strain relationship
- Energy criterion:
  - Work done by loads = strain energy + fracturing work
  - Fracture will occur when enough energy can be delivered to provide for the fracture energy $\frac{dW}{da}$

6.2. Energy release rate

- The energy release rate = the absolute change of the strain energy $\left(\frac{DU}{da}\right)$
- The fracture criterion is:

$$\frac{dU}{da} = \frac{dW}{da}$$
$$\therefore G = R$$

where

$G = \frac{dU}{da}$ is the energy release rate

$R = \frac{dW}{da}$ is the fracture resistance

6.3. Energy criterion for plastic fracture

The energy conservation criterion is as follows:

$$\begin{aligned}\sigma_{fr} &= \sqrt{\frac{ER}{\beta^2 \pi a}} \\ &= \sqrt{\frac{K_{IC}^2}{\beta^2 \pi a}} \\ &= \frac{K_{IC}}{\beta\sqrt{\pi a}} \\ and \\ R &= \beta^2 \sigma \varepsilon \pi a \\ &= \frac{\pi \beta^2 \sigma^2 a}{E}\end{aligned}$$

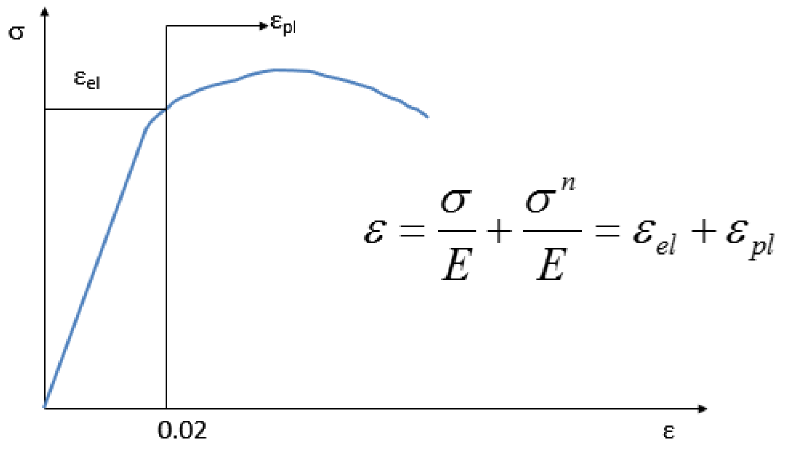
But, β will change when plastic deformation occur. Then use a new geometry factor, H:

$$R = H\sigma\varepsilon a = J$$

Failure occurs when:

$$H\sigma\varepsilon a = J_R$$

6.4. Ramberg-Osgood

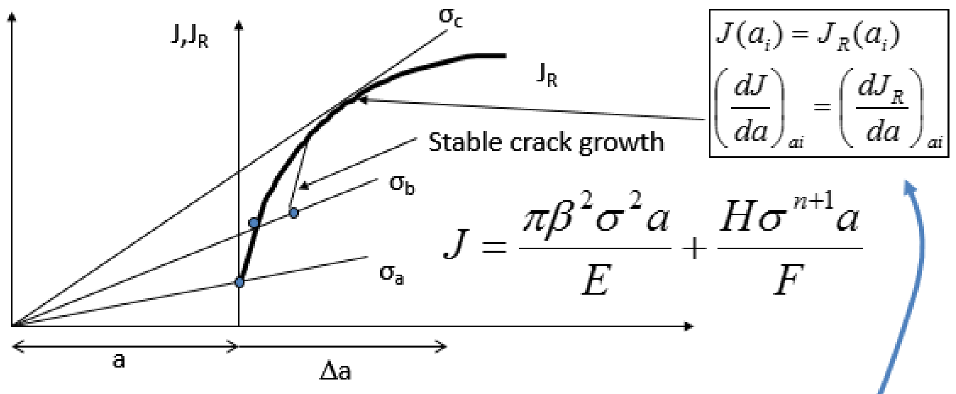


- Failure if:

$$\frac{\pi\beta^2\sigma^2 a}{E} + \frac{H\sigma^{n+1}a}{F} = J_R$$

  - Expressed in terms of stress
  - Find n and F from tensile test
  - J_R – Material's fracture resistance
  - Same as LEFM
  - J_R is measured in a test
  - Geometry factor H must be found in literature

6.5. Rising fracture energy

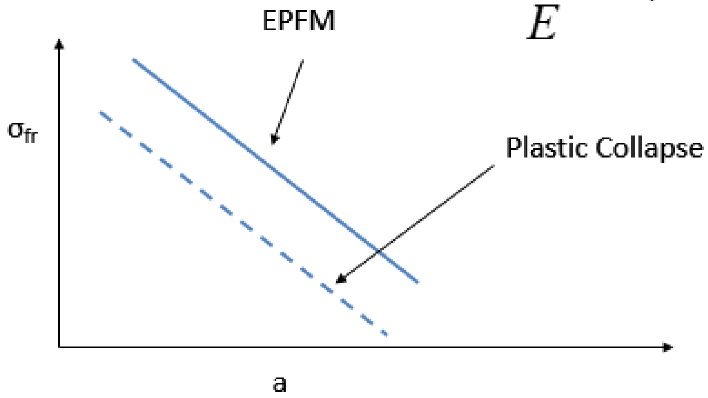


EPFM deals with high toughness materials

Failure and instability when

6.6. Residual strength diagram in EPFM-collapse

$$\frac{\pi\beta^2\sigma_{fr}^2 a}{E} + \frac{H\sigma_{fr}^{n+1} a}{F} = J_R$$



6.7. The h-functions in EPFM

The geometry factor H was used in EPFM thus far. The h-functions simplify plastic collapse calculations as follows:

$$J_{pl} = \alpha\sigma_0\varepsilon_0 c h_1 \left(\frac{P}{P_0}\right)^{n+1}$$

Where

P is the load [N]

P_0 load at collapse [N]

σ_0 Collapse strength [Pa]

$c = \left(\frac{W}{2} - 1\right) a$ factor

h_1 is the geometry factor

There are many configurations for which the h_1 geometry factors are given.

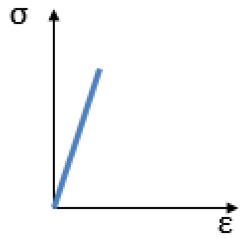
Additional geometry factors, g, k and l can be introduced to have:

$$J_{pl} = \frac{\sigma_o^n}{\varepsilon_o F}\sigma_o\varepsilon_o l a h_1 \left(\frac{g}{k}\right)^{n+1}\left(\frac{\sigma}{\sigma_o}\right)^{n+1}$$
$$= \frac{H\sigma^{n+1}a}{F}$$
$$= \frac{H\sigma^{n+1}a}{S_f^n}$$
$$with\ H = lh\left(\frac{g}{k}\right)^{n+1}$$

Where

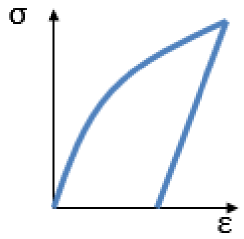
$$P = g\sigma$$
$$P_o = k\sigma_o$$
$$c = la$$

This result in

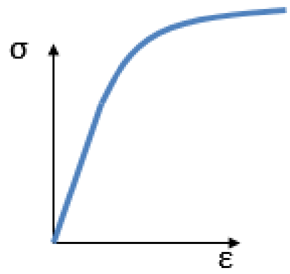


Linear elastic

LEFM



Elastic Plastic



Non-linear elastic

EPFM

6.8. Accuracy

- J depends on σ^n
- Most alloys satisfy the Ramberg-Osgood equation
- Other $\sigma - \varepsilon$ fitting techniques may also be used

6.9. Class problem

A problem will be done in class. Please make your own notes here.

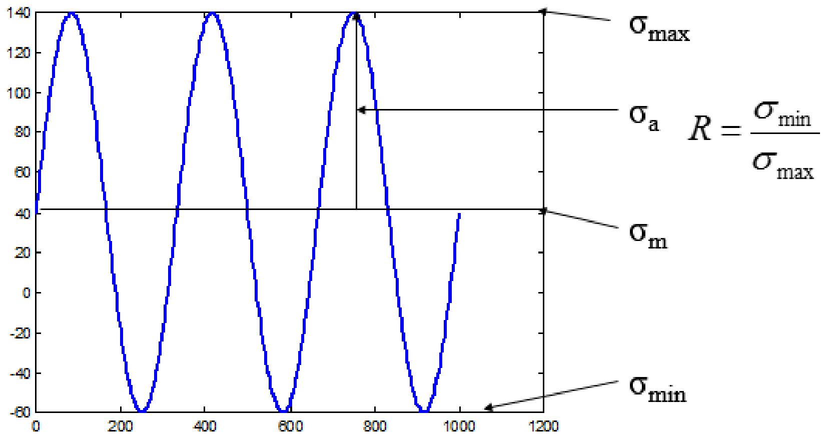
7. CRACK GROWTH

The purpose of this section is to provide a technique to model crack propagation in a material.

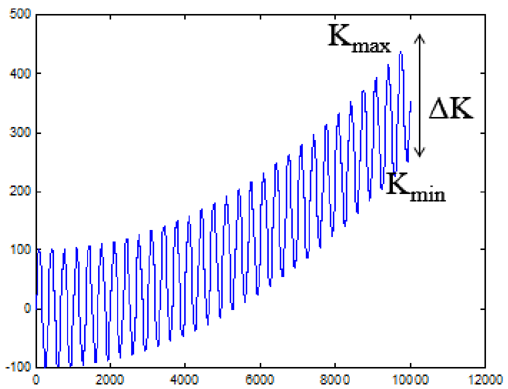
Presentation used in class:	Investmech – Fracture Mechanics (Crack growth) R0.0

7.1. Fatigue to Fracture Mechanics

- Fatigue analyses the life from new to a detectable crack
- Fracture Mechanics determine the maximum allowable crack size and how long it will take to grow to there
- The total life of a component is its fatigue life + life to plastic collapse or fracture



7.2. Stress intensity cycle



$$K = \beta\sigma\sqrt{\pi a}$$

β becomes larger as the crack increases in size

7.3. To accelerate crack growth

$$\frac{da}{dN} = f(\Delta K, R)$$

- Increase maximum stress intensity – or maximum stress
- Reduce minimum stress intensity or minimum stress
- Increase the stress range, stress intensity range, and R

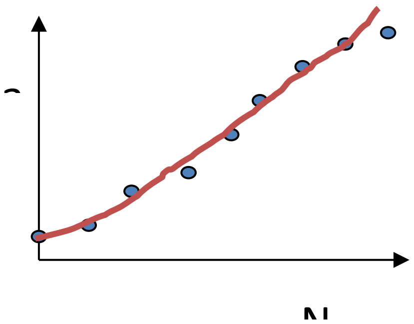
Crack growth

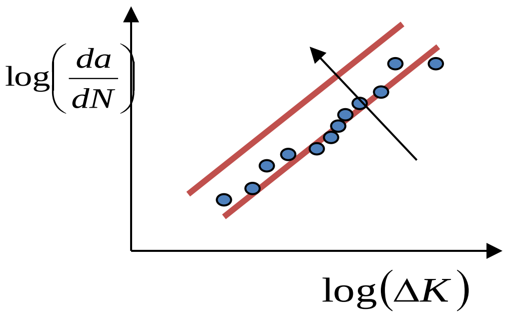
- Experiments are done with $R = 0.01$ or lower (even -1)

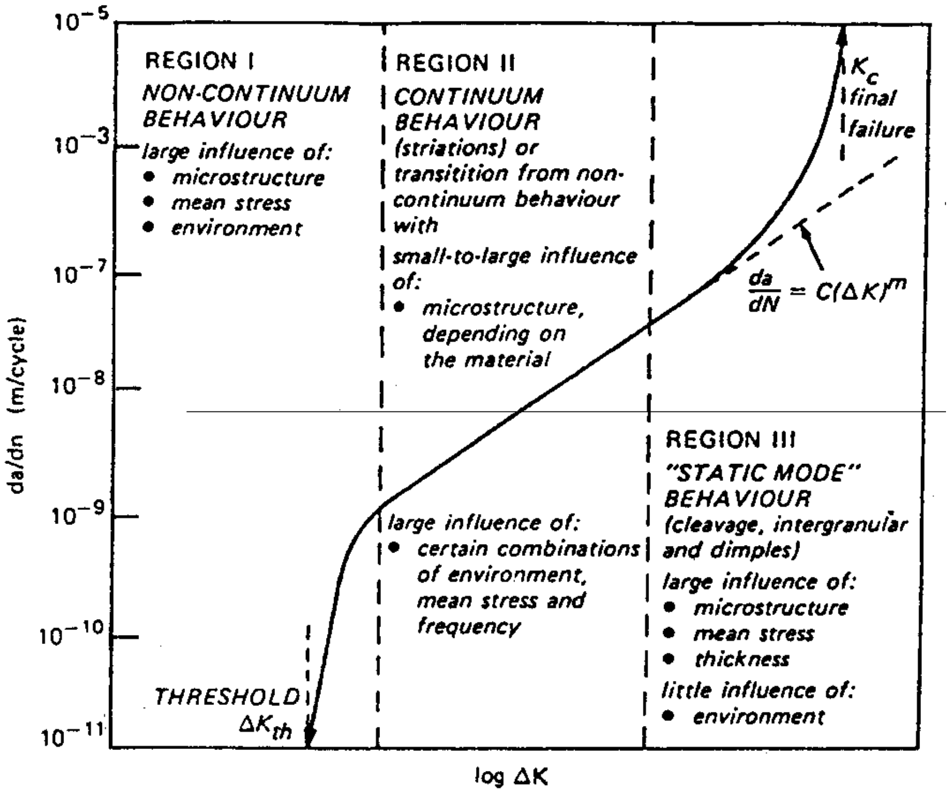
$$\Delta K = \Delta\sigma\beta\sqrt{\pi a}$$

- Measure crack length (α) after several cycles N

- β must be known
  - Centre cracked panels often used







7.4. Crack growth rate

- Curve fitting was done to fit a model on experimental values. Paris equation:

$$\frac{da}{dN} = C_p(\Delta K^+)^{m_p}$$

C_p& m_p determined from two points on the da/dN curve

- Subscript p refer to Paris.

– For $R \neq 0$ see Broek for Walker and Forman, not in the scope of this course

7.4.1. Paris equation in action

- Suppose for a certain component with crack and geometry stress intensity factor $\beta = 2.838$ is subjected to a sinusoidal constant amplitude loading of $\Delta\sigma = 50\ MPa$ – which is only the positive component of the signal
  - That is, if this signal had a mean of zero, the stress range would have been 100 MPa, but, the positive stress part is only 50 MPa
  - Tensile stress propagates a crack
- Take the material constant as $C_p = 1.6 \times 10^{-13}$ and $m_p = 4$ for stress in MPa and crack size in m
- How many cycles will it take to grow a crack from 5 mm to 10 mm?

7.4.1.1. Solution

The stress intensity is:

$$\Delta K^+ = \beta \Delta\sigma^+ \sqrt{\pi a}$$

The Paris equation gives:

$$\begin{aligned} \frac{da}{dN} &= C_p \Delta K^{m_p} = C_p\left(\beta\Delta\sigma\sqrt{\pi a}\right)^{m_p} \\ &= 1.6 \times 10^{-13}\left(2.838 \times 50 \times \sqrt{\pi a}\right)^4 \\ &= 640.25 \times 10^{-6} a^2 \end{aligned}$$

Integration of the Paris equation by separation of terms:

$$\begin{aligned} \int_0^N dN &= \frac{1}{640.25 \times 10^{-6}} \int_{0.005}^{0.010} a^{-2} da\, N \\ &= \frac{1}{640.25 \times 10^{-6}}\left[-\frac{1}{0.01} + \frac{1}{0.005}\right] \\ &= 156{,}200\ cycles \end{aligned}$$

This can be solved by the following Matlab commands:

```
N=@(a) a.^(-2)/640.25e-6;
integral(N,0.005,0.01)
```

7.5. Forman equation for non-zero mean

The Forman equation:

$$\frac{da}{dN} = C_f \frac{\Delta K^{m_f}}{(1-R)K_c - \Delta K}$$

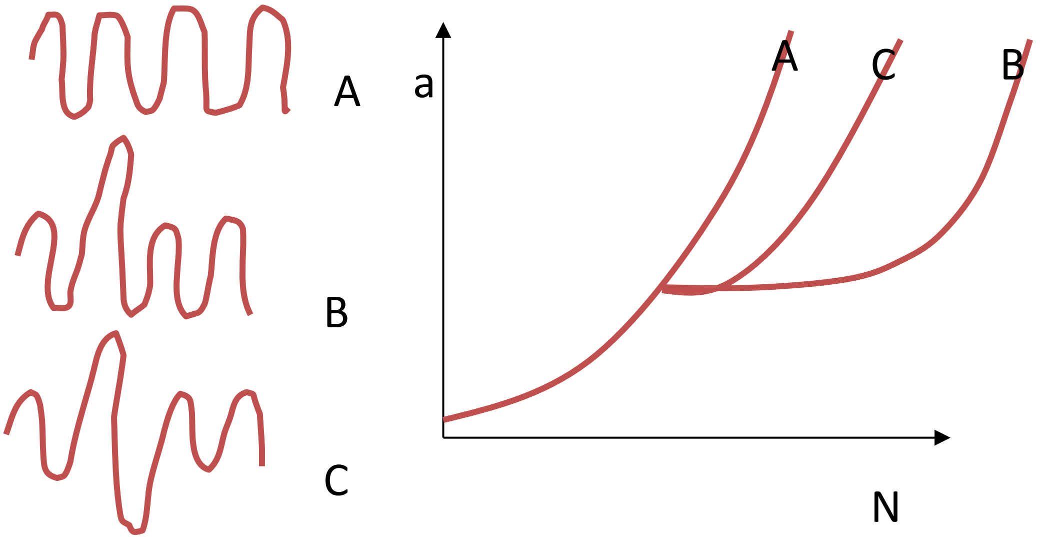
The equation can be solved analytically, or numerically using Excel or Runge-Kutta in Matlab

Accurate answers are possible with even great steps

7.6. Crack retardation

- Crack retardation takes place when a plastic zone is developed at the crack tip
- When the load is released, a residual compressive stress will prevail at the crack tip
- The crack takes longer to grow through the plastic zone
- This is normally done by overloading the structure for a cycle (pressure vessel, cranes)
  - Can also cold work the hole
- Grinding removal of crack
- Stop drilling
- Drilling crack-flank holes near the crack
- Artificial crack closure by infiltrating
- Epoxy resin
- Alumina powder

- Mixture of above
- Patching
- Overloading in tension
- Spot heating (Ray, et al., 2002)
- Cold working



7.7. Patching & Infiltration

Patching

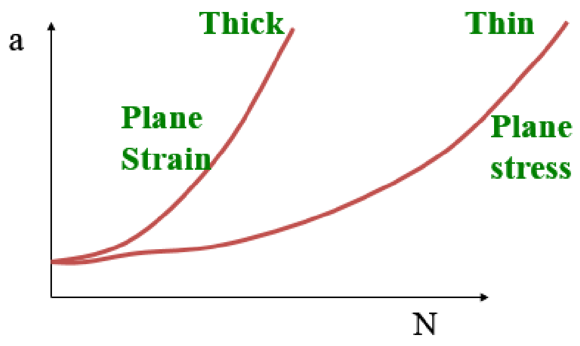
- Crack patched with composite materials
  - The loads is now shared by the composite material patched over the crack
  - This reduces the stress intensity over the crack
- Effective in thin-walled components

Retarding by infiltration

- Epoxy resin into fatigue crack
  - Reduce crack growth to 4-10% of original value (Shin, et al., 1996)
  - Order of reduction same as 100% overload

7.8. Thickness and crack growth

A crack grows faster in a thick component than in a thin component – plane strain resulting in higher stresses.



7.9. Short cracks

- Strain control fatigue tests are done at $R = -1$
- The plastic zone at the hole > crack tip plastic zone
- Crack tip also subjected to completely reversed loading

7.10. Paris rule exponents and coefficients

IIW Bulletin 520:

– No detail fatigue analysis is required for stress intensity factors below the threshold levels:

Steel: $\Delta K_{th} = 2.0\ MPa\sqrt{m} = 63\ N.m^{-\frac{3}{2}}$

Aluminium: $\Delta K_{th} = 0.7\ MPa\sqrt{m} = 21\ N.m^{-\frac{3}{2}}$

No fatigue analysis for: $\Delta K_{s,d} \leq (\Delta K_{th})/\gamma_{Mf}$

$$\frac{N}{m^2} m^{\frac{1}{2}} = N m^{\frac{1}{2}} m^{-2} = N m^{-1.5} = N m^{-\frac{3}{2}}$$

No crack growth for $\Delta K \leq \Delta K_{th}$

For other materials use the following conversion:

$$C = C_o, steel \left(\frac{E_{steel}}{E}\right)^m$$

$$\Delta K_{th} = \Delta K_{th,steel} \left(\frac{E_{steel}}{E}\right)^m$$

For the assessment of cracks without detected cracks, use an initial crack depth of $\alpha_i = 0.15$ mm and aspect ratio $\alpha : c = 1:10$ in calculations

For root cracks, e.g. fillet welds, the root gap should be taken as an initial crack

$$R = \frac{\Delta K_{min}}{\Delta K_{max}}$$

– Paris data for steel:

Units	Paris power law parameters	Threshold values ΔK_{th}			
		R≥0.5	0≤R≤0.5	R<0	Surface crack depth <1mm
$K\left[N \cdot mm^{-\frac{2}{3}}\right]$ da/dN [mm/cycle]	$c_0 = 5.21 \times 10^{-13}$ $m = 3.0$	63	170-214R	170	≤63
$K\left[MPa\sqrt{m}\right]$ da/dN [m/cycle]	$C_0 = 1.65 \times 10^{-11}$ $m = 3.0$	2.0	5.4-6.8R	5.4	≤2.0

– Paris data for aluminium:

Units	Paris power law parameters	Threshold values ΔK_{th}			
		R>0.5	0<R<0.5	R<0	Surface crack depth <1mm
$K\left[N \cdot mm^{-\frac{2}{3}}\right]$ da/dN [mm/cycle]	$c_0 = 1.41 \times 10^{-11}$ $m = 3.0$	21	56.7-7.72.3-R	567	≤21
$K\left[MPa\sqrt{m}\right]$ da/dN [m/cycle]	$C_0 = 4.46 \times 10^{-10}$ $m = 3.0$	0.7	1.8-2.3-R	18	≤0.7

7.11. Assessing defects according to IIW Bulletin 520

- Calculate stress range for initial crack size $\Delta\sigma_i$
- Calculate stress range for the critical crack size $\Delta\sigma_c$
- The stress range $\Delta\sigma_C$, or the detail category belonging to a crack propagation from a_i to a_c at $N = 2 \times 10^6$ cycles is then:
  - For steel:

$$\Delta\sigma_{C,steel} = \sqrt[3]{\Delta\sigma_i^3 - \Delta\sigma_c^3}$$

- For aluminium:

$$\Delta\sigma_{C,al} = \frac{\Delta\sigma_{C,steel}}{3}$$

7.12. Crack-like defects

- Identified by NDT
  - Idealize NDT indications as elliptical cracks
- Embedded cracks
  - Shape idealized by circumscribing ellipse, measured by a and c
    - Crack depth parameter a is the half-axis of the ellipse in the direction of crack growth
    - Remaining half-axis is c
  - Wall thickness parameter t is the distance from the centre of the ellipse to the nearest surface
  - For $\frac{a}{t} > 0.75$ categorize the defect as a surface defect

Surface cracks

- For $\frac{a}{t} > 0.75$
- Regarded as being fully penetrating
- Categorize as a centre crack or an edge crack

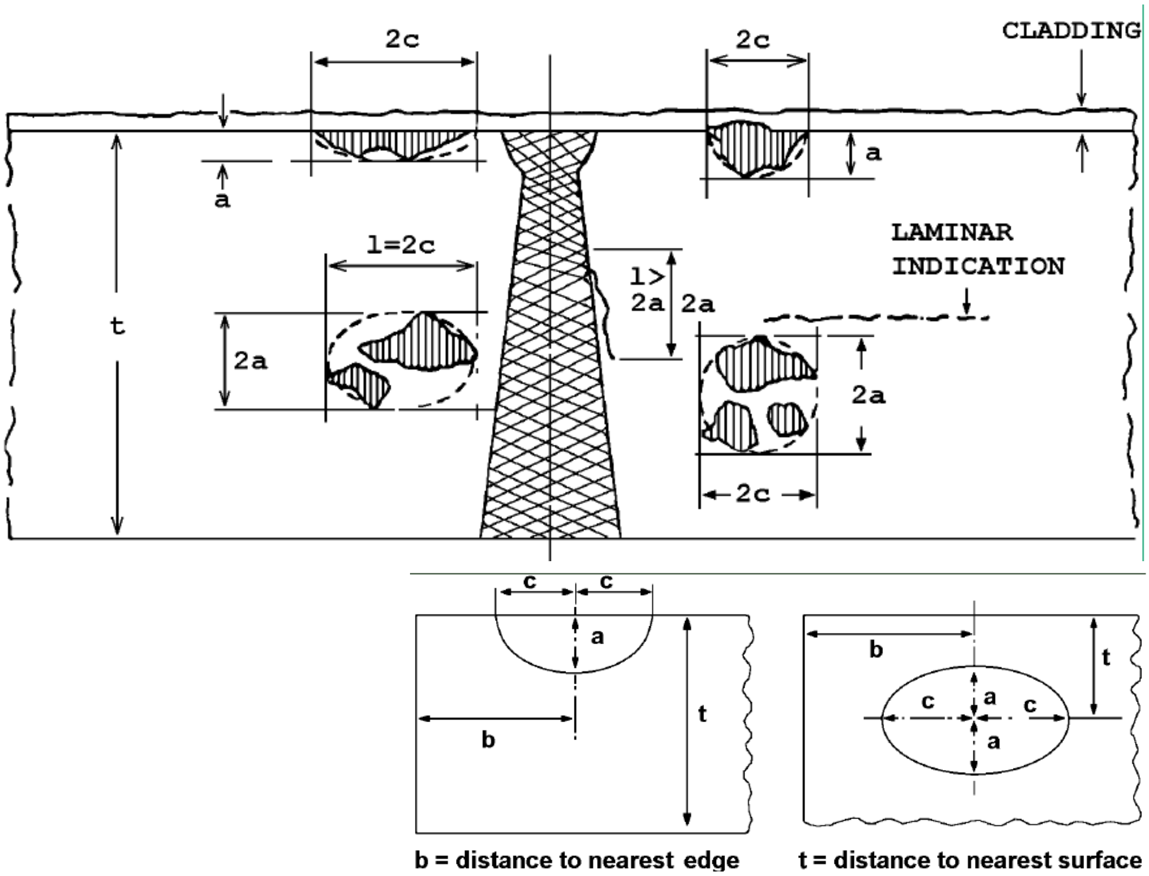


Figure 17: Crack-like defects

8. DAMAGE TOLERANT STRUCTURES

The purpose of this section is to introduce the design of damage tolerant design structures.

Presentation used in class:	Investmech – Fracture Mechanics (Damage tolerance and fracture control) R0.0

8.1. Damage tolerant structure

- Has developed primarily within aerospace industry
- Characterized by structural configurations that are designed to minimize loss of aircraft because of:
  - Propagation of undetected flaws, cracks or similar damage
- Two major design objectives:
  - Controlled safe flaw growth, or safe life with cracks
  - Positive damage containment
- Design objective that must be met:
  - Controlled safe flaw growth, or safe life with cracks
  - Positive damage containment

Implies safe remaining or residual strength

These design objectives are interlinked, their combination ensures effective fracture control

8.2. Damage tolerant design

- Not substitute for careful fatigue analysis!
- Requires careful :
  - Stress analysis
  - Geometry selection
  - Detail design
  - Material selection
  - Surface finish
  - Workmanship

Mission and user profile definition

8.3. Goals of damage tolerant design & fracture control

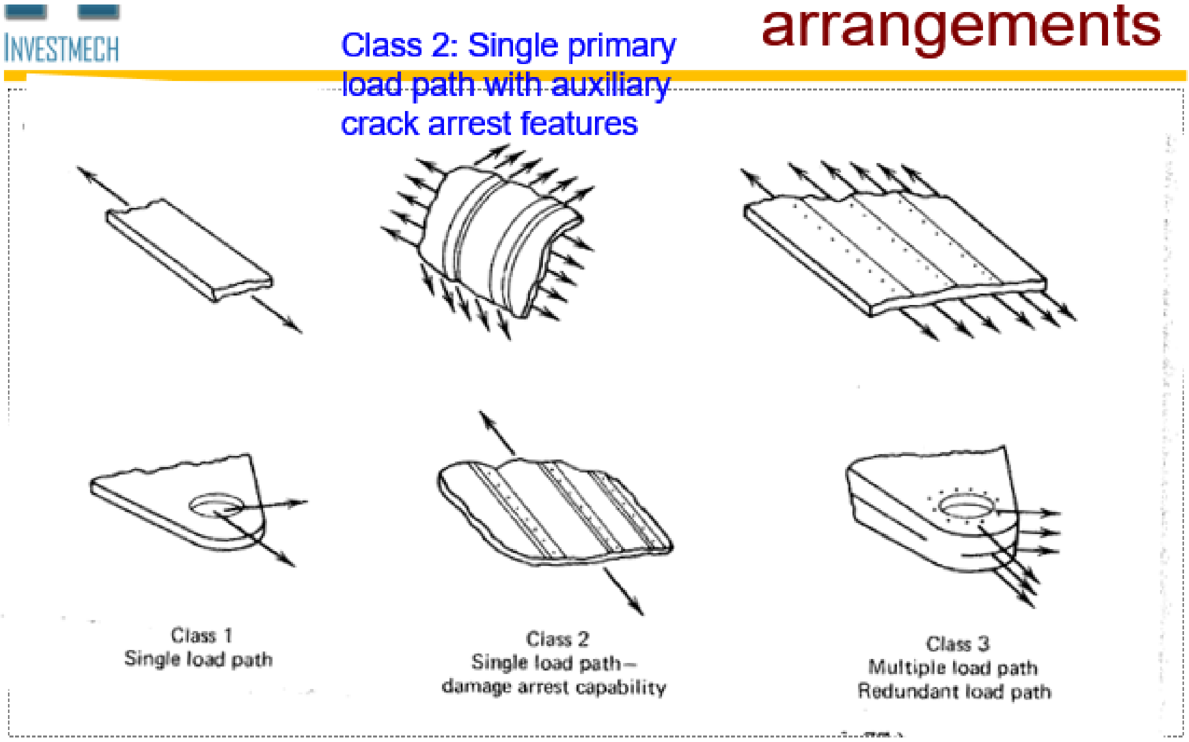
- Selection of fracture-resistant materials and manufacturing processes
- Design for inspectability
- Use of damage-tolerant structural configurations like:
  - Multiple load paths
  - Crack stoppers

8.4. Fracture control

- Basic assumption:
  - Flaws do exist even in new structures and that they may go undetected
- First major requirement for damage tolerance:
  - Any member in the structure, including each element of a redundant load path group, **must have a safe life with assumed cracks present**
- Primary factors influencing design:
  - Type or class of structure
  - Quality of Non Destructive Inspection (NDI)
  - Techniques used in production assembly
  - Accessibility of structure to inspection
  - Assurance that the member will be inspected on schedule when in service

– Probability that a flaw of sub-critical size will go undetected, even for scheduled NDI

8.5. Classification of structural arrangements



8.6. Class requirements

- Class 1: Essential to satisfy **safe-life-with-cracks** requirement
  - Failure is catastrophic
- Class 2 (including pressurized cabins & pressure vessels): Essential to satisfy **leak-before-break** design characteristics
  - Relative large amounts of damage may be contained with tear straps or stiffeners
  - Usually high probability of damage detection
    - Because system will leak!
- Class 3: **Fail-safe** type structure
  - Designed to provide specific percentage of original strength, that is, residual strength during and subsequent to failure of **one** element
  - Usual to assume smaller initial flaw size
    - Appropriate to take larger risk of operating with cracks if multiple load paths are available

Therefore, we have **fail-safe** and **safe-life** design approaches

8.7. Safe-life design approach

- Evaluate expected lifetime
  - Employ traditional design methods
- Followed by full scale testing
- Account for uncertainties and scatter with carefully selected safety margin
- Safety factor sometimes referred to as scatter factor

8.8. Fail-save design approach

- Provide redundant load paths
  - Secondary member will carry the load when one member fails

8.9. Fracture control program

- Inspection programme
  - Important part
  - Appropriate inspection procedures
    - For each structural element
    - Regions within element may be classified with respect to required NDI sensitivity

8.10. Inspection intervals

- Assure that an undetected flaw will not grow to critical size before next inspection
  - With comfortable margin of safety
- Intervals usually picked so that
  - Two inspections will occur before crack reach critical size
- Cracks larger than the detectable flaw size presumed to be discovered and removed
- Assumption that
  - All critical points are checked at every inspection
  - Cracks larger than detectable flaw size are found
    - At latest during second inspection
  - Inspections are performed on schedule
  - Inspection techniques are truly non-damaging

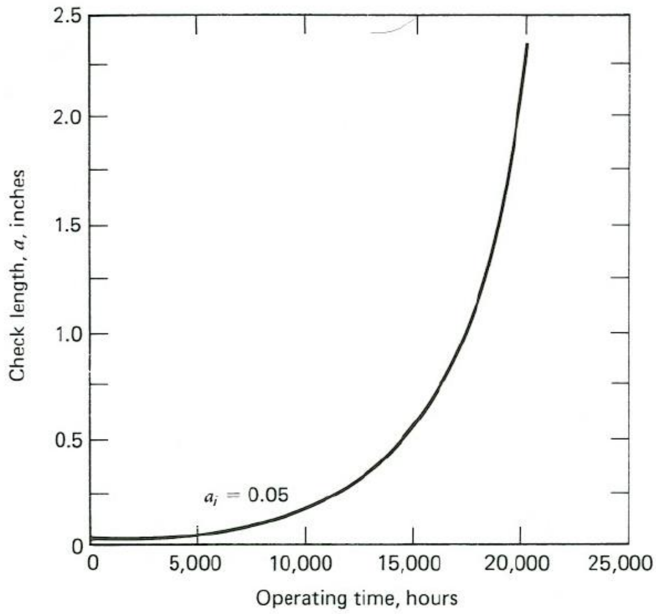
Unfortunately, these assumptions are sometimes violated in field practice

- Example of violation
  - Some large aircraft may contain as many as 20,000 critical fastener holes in lower wing surface alone
  - Complete inspection tedious and time consuming
    - Subject to error borne of the boredom of inspecting 20,000 holes with no serious problems, only to miss one hole with a serious crack
      - Sometimes referred to as a “rogue” crack
- Uncertainties associated with best NDI techniques remain significant
  - Even for large rogue cracks reliability of detection is only about 80% with high probability that the same crack may be missed on second inspection
- Efforts to improve NDI state of the art
  - Have concentrated on lowering the detection threshold to smaller crack sizes
    - But, giving insufficient attention to the largest crack size that can be missed
- NDI in practice
  - May not necessarily be used to check every critical point at every inspection
  - May miss cracks it is supposed to find
  - May not always be performed on schedule
  - May sometimes be partially damaging
- HOWEVER, use of NDI techniques & establishment of appropriate inspection intervals
  - Represent significant advances in the state of the art
  - Should be implemented in all high-performance fatigue-critical designs
- Requires knowledge of:
  - Initial flaw size
  - Detectable crack length
  - Critical crack size – at which fracture or plastic collapse occur

- Crack growth versus life relationship
- Or crack length vs. operating hours (or cycles)

8.11. Example

- For this hypothetical example:
  - Initial crack size a_i = 0.050 inch
  - Detectable crack size a_{det} = 0.15 inch
  - Critical crack size a_{cr} = 1.5 inches
  - Safety factor SF = 2
- Calculate
  - Initial inspection interval
  - Second and subsequent intervals



Source: (Collins, 1981)

Figure 18: Crack growth dependent inspection intervals example

8.11.1.1. Solution

The initial inspection interval, I_1, is

$$I_1 = \frac{t_{a_{cr}=1.5} - t_{a_i=0.05}}{SF} = \frac{18000 - 5000}{2} = 6\,500\ h$$

The second and subsequent inspection intervals, I_2, is:

$$I_2 = \frac{t_{a_{cr}=1.5} - t_{a_i=0.15}}{SF} = \frac{18000 - 9\,500}{2} = 4\,250\ h$$

8.12. Damage tolerance criteria

- Depend upon:
  - Degree of inspectability
  - Frequency of inspection
  - Class of structure
- Structures that are less inspectable, less frequently inspected or less fail safe

- Damage tolerance criteria, including initial flaw size, minimum required residual strength, service-induced flaw size, crack growth rate, must be more conservative

8.13. Fracture control programme

Encompass and interact with design, materials selection, fabrication, inspection, and operational phases in the development of any high-performance engineering system

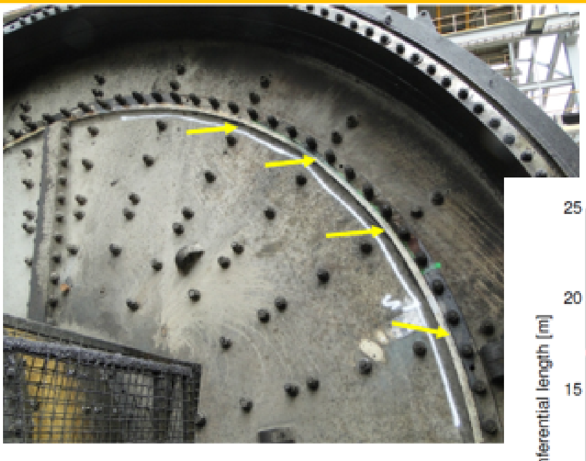
- Design
  - Determine stress and strain distributions
  - Determine flaw tolerance for regions of greatest fracture hazard
  - Estimate stable crack growth for typical service periods
  - Recommend safe operating conditions and specify intervals between inspections
- Materials
  - Determine yield and ultimate strengths
  - Determine fracture parameters: K_c, K_{Ic}, K_{ISCC}, da/dN
  - Establish recommended heat treatments
  - Establish recommended welding methods
- Fabrication
  - Control residual stress, grain growth, and grain direction
  - Develop or protect strength and fracture properties
  - Maintain fabrication records
- Inspection
  - Inspect part prior to final fabrication
  - Inspect fabrication factors such as:
    - Welding current and welding speed
  - Inspect for defects using NDI techniques
  - Proof test
  - Estimate largest crack-like defect sizes
- Operation
  - Control stress level and stress fluctuations in service
  - Protect part from corrosion
  - Inspect part periodically

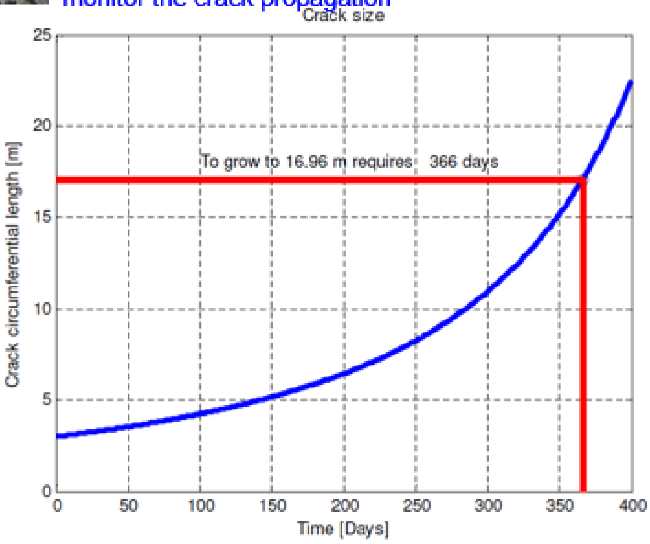
8.14. Case study: Cracked ball mill flange

Case study: Cracked ball mill

INVESTMECH

The discharge end of a ball mill cracked
Crack size at inspection was 3.5 m
Fracture mechanics indicated critical crack size 17 m
Crack propagation was calculated as shown below
Calculation proved to be accurate and was used to monitor the crack propagation





Fracture control: Weekly crack sizes were plotted on the crack propagation curve
If it should be above the curve, Investmech would repeat calculations with more accurate inputs and material properties

2016-08-10

9. FAILURE ASSESSMENT DIAGRAMS

The purpose of this section is to introduce the use of failure assessment diagrams.

Presentation used in class:	Investmech – Fracture Mechanics (Failure Assessment Diagrams - FAD) R0.0

9.1. Background

- Assessment levels:
  - Level 1 FAD
  - Level 2 FAD
  - Level 3 FAD
- Fitness-for-service codes:
  - API 579-1/ASME FFS-1 (previously API RP 579)
  - BS 7910
  - British Energy R5/R6 procedure
  - SINTAP/FITNET
  - ASME B31.G
  - Investmech in-house procedures
  - Other?

9.2. Level 1 FAD according to BS 7910

Typical procedure:

- Determine the flaw size and shape dependent geometric stress concentration factor
- Calculate the stress intensity factor
- Calculate the stress on the uncracked section
- Calculate the nearness to plastic collapse
- Calculate the nearness to brittle fracture
- Plot on the FAD or verify mathematically

- Plastic collapse:

$$S_r = \frac{\sigma_{ref}}{\sigma_{flow}}$$

  - For Level 2 and 3:

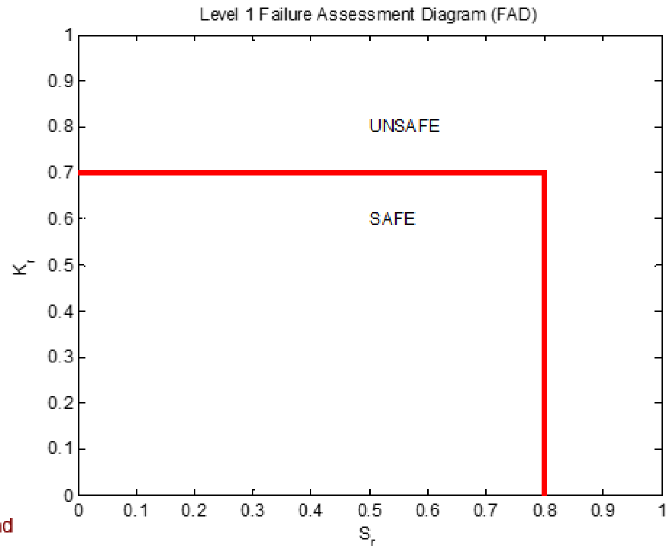
$$L_r = \frac{\sigma_r}{\sigma_{ys}}$$

- Fracture
  - Nearness to brittle fracture, K_r

$$K_r = \frac{K_I}{K_{mat}}$$

List of symbols:

S_r: Nearness to plastic collapse
σ_{ref}: Stress at the cracked section
σ_{flow}: Flow stress of the material, which is yield strength in most cases
L_r: Nearness to plastic collapse for Level 2 and Level 3
σ_{ys}: Material yield strength
K_I: Applied stress intensity factor
K_{mat}: Fracture toughness of the material



Matlab Code to produce the Level 1 FAD

```
x=[0 0.8 0.8];
y=[0.7 0.7 0];
plot(x,y,'r','LineWidth',3)
axis([0 1 0 1])
xlabel('S_r')
ylabel('K_r')
title('Level 1 Failure Assessment Diagram (FAD)')
text(0.5,0.8,sprintf('UNSAFE'))
text(0.5,0.6,sprintf('SAFE'))
```

2016-08-10 3

10. CASE STUDY – FRACTURE OF AN ARM

Information will be provided in class. Please make your own notes here.

11. REFERENCES

ASM Handbook Volume 1, 1993. *Properties and selection: Irons, Steels and High Performance Alloys.* s.l.:ASM International.

Collins, J., 1981. *Failure of materials in mechanical design.* s.l.:John Wiley & Sons Inc..

Green, D. & Perry, R., 2008. *Perry's Chemical Engineer's handbook.* 8th ed. New York: McGraw-Hill.

Heyns, M., 2016. *Demonstration on Citations,* s.l.: Investmech.

Heyns, M., 2016. *E-mail with subject: Demonstration of this or that,* s.l.: Investmech or Company Name..

Ray, P., Verma, B. & Mohanty, P., 2002. Spot heating induced fatigue crack growth retardation. *International Journal for Pressure Vessels and Pipin,* Issue 79, pp. 373-376.

SANS 10162-1, 2005. *The structural use of steel. Part 1: Limit-state design of hot-rolled steelwork,* Pretoria: Standards South Africa.

Shin, C., Wang, C. & Song, P., 1996. Fatigue damage repair: a comparison of some possible methods. *International Journal of Fatigue,* 18(8), pp. 535-546.

12. APPENDIX A. External file references

12.1. Calculation files

The following calculation files were used in the analysis:

Description	File Name	Revision
Calculate the crack growth curve	Crackgrowthcurve.m	0.0
Calculate stress distribution at the crack tip	Cracktipstress.m	0.0
Calculate residual strength curves and crack propagation	Residualstrength.m	0.0